CONSOLIDATION OF COMPACTED AND UNSATURATED CLAYS

by

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SYNOPSIS

A review is made of published evidence on the physical properties of unsaturated and compacted clays with the object of recognizing the more important factors affecting the consolidation process in such soils. An attempt is then made to formulate and solve a set of general equations to describe the consolidation process. Because of the complexity of these equations it is found desirable to sub-divide the process into a number of less general treatments, each being relevant to a particular range of saturation values. The treatments include the effects of decreasing permeability and a compressible pore fluid, and the resulting non-linear partial differential equations are solved by finite difference approximations on a digital computer. The computed results exhibit the two important characteristics found in consolidation tests on unsaturated clay, viz. a continuously curved settlement/root-time plot and a mid-plane pore pressure dissipation plot which is flatter than the theoretical Terzaghi plot. As a result of this preliminary investigation it is tentatively concluded that a major cause of this difference between Terzaghi theory and the observed behaviour of compacted clay, in the region of or wetter than optimum, is the marked decrease in permeability that occurs during the consolidation process, the effect of the compressibility of the pore fluid being of secondary importance.

On passe en revue les renseignements publiés sur les propriétés physiques des argiles non saturées et compactées afin de reconnaître les facteurs importants affectant le procédé de consolidation dans de tels sols. Puis on essaye d'exprimer et de résoudre un groupe d'équations générales pour décrire le procédé de consolidation. A cause de la complexité de ces équations il est utile de subdiviser le procédé en un certain nombre de traitements moins généraux, chacun concernant une série particulière de valeurs de saturation. Les traitements comprennent le décroissement de la perméabilité ainsi qu'un fluide interstitiel compressible, et les équations différentielles partielles non-linéaires en résultant sont résolues sur une calculatrice digitale par des approximations de différence déterminées. Les résultats computés font ressortir les deux caractéristiques importantes découvertes dans les essais de consolidation sur de l'argile non saturée, à savoir un tracé graphique en courbe continue du tassement et de la racine de la durée, et un tracé graphique de la dissipation de la pression interstitielle en plan moyen qui est plus plat que le tracé théorique de Terzaghi. A la suite de cette enquête préliminaire on conclut à titre d'essai qu'une cause principale de cette différence entre la théorie Terzaghi et le comportement observé de l'argile compactée, dans le voisinage de l'optimum ou plus humide, est le décroissement marqué en perméabilité qui se produit pendant le procédé de consolidation, l'effet de la compressibilité du fluide interstitiel étant d'une importance secondaire.

INTRODUCTION

The prediction of the rate of dissipation of construction pore pressures in earth dams and embankments is a problem of considerable practical importance. The extremely complex nature of the physical properties of a three-phase material such as a compacted clay containing both air and water as pore fluids renders an exact description of the consolidation process rather remote. Consequently the Terzaghi theory designed for saturated clay is commonly applied as a first approximation to the case of compacted clays. The following is an attempt to review the physical properties involved in the consolidation process in unsaturated clay, with the object of recognizing the more important factors.

This is followed by a relatively simple treatment of the consolidation process in compacted clays of various degrees of saturation which necessarily contains a number of simplifying assumptions. In this preliminary investigation solutions are obtained only for the practical case of one-dimensional vertical consolidation.

PHYSICAL PROPERTIES OF COMPACTED CLAY

One general difference between artificially compacted and naturally occurring unsaturated clay is that compacted clay will probably contain a proportion of much larger air voids than are likely in a natural deposit that has dried out slightly since deposition. However, it seems

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reasonable to accept certain similarities in these two materials and to use evidence reported in the extensive literature of agricultural and soil science (Collis-George, 1953; Childs, 1956; Orlob and Radhakrishna, 1958; Marshall, 1959; Remson and Randolph, 1962). In the literature of soil mechanics excellent discussions of this topic have been presented by Aitcheson (1956), Hilf (1956) and Yoshimi and Osterberg (1963). Although the details are not fully understood there appears to be general agreement between the various disciplines on the following points.

In the following the suffix 'a' refers to air and 'w' to water.

(1) The porosity and the structure of the clay depend on the method of compaction and the moisture content.

(2) Water preferentially wets the surface of the clay particles and even in dry soils there will be a thin layer of water separating the air from the clay surface. If sufficient water is present there is a gradual transition from the highly viscous adsorbed water firmly attached to the clay surface to the free pore water which can move freely under potential gradients. According to Carman (1953) the water will redistribute itself until the curvature of the air–water menisci are equal. The presence of curved menisci and surface tension forces is the cause of a (capillary) pressure difference between the pore air and water pressures and if the air pressure is atmospheric this causes considerable suctions in the pore water. The menisci can also cause water to be trapped in cavities and hence to be attached to the clay skeleton. Thus the water can be considered as made up of 'free' water which can flow through the pores under potential gradients and 'dead' water, either trapped or adsorbed, which cannot flow, the relative amounts of each type depending on the degree of saturation.

(3) Similarly the air can exist in different conditions and may be effectively attached to the clay skeleton or free to flow through the clay. Because of surface tension water will flow from larger to smaller pores; also water vapour will move from large to small pores (Carman, 1953). Hence there is a tendency for air to congregate in the larger pores while the smaller ones are full of water. For a high enough air content these larger pores are continuous and the air is free to flow through the clay. However, as the water content increases the thin necks between pores tend to close and the air ceases to be continuous over any distance and is said to be occluded. Excellent evidence of the phenomenon of occlusion is presented by Gilbert (1959) who showed that for Vicksburg silty clay the air voids are fully continuous at a moisture content 4% below optimum and are occluded at a moisture content of 3% above optimum.

The main disagreement concerns the condition of the occluded air, and whether at high degrees of saturation it can exist as small bubbles in the free pore water and hence flow with the water. Hilf (1956) does not favour the presence of air as small bubbles and on p. 37 states 'under equilibrium conditions it follows that bubbles can exist only when the water is saturated with air at the pressure of the bubble'. His argument in support of the above statement is sound, but during a consolidation process equilibrium conditions do not exist and on p. 39 Hilf states 'their transitory existence during compression is recognized'.

Hilf's further two arguments on p. 41 and p. 42 however do not apply if the air is occluded. The basis of these arguments is that as saturation is approached the bubbles become very small and hence the pressure difference $(p_a - p_w)$ would become very large. Assuming p_a is atmospheric throughout this implies a drop in p_w and since p_w does not fall, but actually rises as saturation is approached, Hilf concluded that the mechanism cannot involve the compression of small bubbles. The fallacy is the assumption that p_a equals atmospheric pressure throughout, since if the bubbles are occluded there is no means of measuring their pressures, which will in any case vary depending on the size of the bubble, and which, as saturation is approached, will probably reach very high values greatly in excess of atmospheric.

In the present treatment of the transient state it is assumed that in the occluded state air can exist as small bubbles that are free to move with the pore water, as well as isolated air-

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filled voids that are generally static. There is a possibility that free bubbles may become trapped by the skeleton as they flow through the clay.

Having considered briefly the nature of the material it is necessary to consider the possible forms of the consolidation process. In general it will be treated as the transient flow of two immiscible pore fluids through a compressible porous medium. The secondary processes of the diffusion of water vapour through air and the diffusion of dissolved air through water will be neglected as their effects will be small and their complexity great.

In the present state of knowledge the flow of fluids through a porous medium is best treated in terms of Darcy's law. To apply Darcy's law to the present problem of vertical flow a large number of assumptions are implicit, the more important being as follows.

(1) The form of the expression involves a force potential rather than a velocity potential (Schiedegger, 1957)

$$v = \gamma \frac{K}{\mu} \frac{\partial \phi}{\partial z} \quad \quad (1)$$

where v is the macroscopic velocity, K the permeability of the medium, μ the viscosity of the fluid, and ϕ the total potential.

(2) In the expression for total potential ϕ the osmotic pressure potential and the adsorption potential will be ignored. The gravitational potential will also be neglected at this stage as the inclusion of gravity would require the treatment of a non-homogeneous soil with many of the properties varying with depth.

For experimental results at laboratory scale this effect is negligible, but it could be important at field scale and should be included in a more refined treatment. The expression for Darcy's law therefore simplifies to

$$v = \frac{K}{\mu} \frac{\partial p}{\partial z} \quad \quad (2)$$

where p is the absolute pressure of the fluid and γ its density. In the following K/μ will be written as k with a suitable suffix where necessary to indicate the appropriate fluid.

(3) A theoretical treatment by Slepicka (1960) based on dimensional analysis indicates that a more general form of Darcy's law is $v = k(i)^n$ with $n > 1$ for very small velocities, $n = 1$ for a considerable range of intermediate velocities, $n < 1$ for high velocities. This confirms the experimental findings of Hansbo (1960), Swartzendruber (1962) and Abelev and Tsytovich (1964), at very low velocities; and of a large number of workers at high velocities (Muskat and Botset, 1931; Muskat, 1946).

Thus in assuming the simplest form with $n = 1$ there is a probability of error under the low velocities common at the field scale, whereas the effect will be negligible at laboratory scale. The use of a power law will not be attempted here and Darcy's law is applied as $v = k\partial p/\partial z$.

(4) Darcy's law has been fully verified for the steady state flow of water through saturated clay, except under very low gradients, and also for the steady flow of air through dry porous media provided the flow was laminar. There is also a considerable amount of experimental evidence in soil science and petroleum engineering literature to confirm that it also gives at least a close approximation in the case of the flow of two or even three immiscible fluids (air, water and oil) through a porous medium (Wykoff and Botset, 1936; Richardson *et al.*, 1952; Rose, 1954). There is, nevertheless, a great deal of experimental work yet to be done in studying the application of Darcy's law to the simultaneous flow of air and water through an unsaturated clay which is undergoing a change in both structure and degree of saturation. In the meantime it is reasonable to assume that Darcy's law can be applied to each of the separate fluids, the main problem being to establish the approximate manner in which the permeability to each fluid varies during a consolidation process with the consequent compression of the skeleton and the change in the degree of saturation.

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Thus for combined flow of air and water:

$$v_a = k_a \frac{\partial p_a}{\partial z} \quad \text{. . . . . (2a)}$$

$$v_w = k_w \frac{\partial p_w}{\partial z}. \quad \text{. . . . . (2b)}$$

(5) For the flow of water through saturated clay it is known that permeability is a complex function of both porosity n and structure λ (Lambe, 1954):

$$k = f(n, \lambda).$$

Theoretical attempts to predict $f\,(n, \lambda)$ will have little general success for materials with a complex structure such as clay.

For the flow of both water and air through porous media, usually sands of constant porosity and structure, it is known that permeability is a complex function of degree of saturation s (Wyckoff and Botset, 1936; Orlob and Radhakrishna, 1958; Christiansen, 1944):

$$k = f\,(s).$$

This function is not unique but depends on whether s has been reached by wetting or drying, and again attempts to predict it on theoretical grounds have not been successful.

For the present problem we must expect:

$$k_a = f_a\,(n, \lambda, s) \quad \text{. . . . . (3a)}$$

$$k_w = f_w\,(n, \lambda, s) \quad \text{. . . . . (3b)}$$

where f_a and f_w are not single-valued functions and will have to be established experimentally in every case. Experimentally it is very difficult to separate the relative effects of changes in n, λ, and s (Lambe, 1954; Sibley and Miller, 1962). Further consideration will be given to f_a and f_w in a later section.

GENERAL EQUATIONS FOR THE CONSOLIDATION PROCESS

In common with the majority of consolidation theories the pore pressures will be expressed in terms of a pore pressure excess u rather than in terms of the absolute pressure p. In the present case, since gravity has been neglected, this merely implies in excess of atmospheric pressure p_o and hence $u = p - p_o$. This procedure allows the use of the familiar boundary condition $u = 0$ at the free drain and makes the computation slightly simpler than if $p = p_o$ at the free drain.

Thus
$$p_a = p_o + u_a \quad \text{and} \quad \frac{\partial p_a}{\partial z} = \frac{\partial u_a}{\partial z};$$

$$p_w = p_o + u_w \quad \text{and} \quad \frac{\partial p_w}{\partial z} = \frac{\partial u_w}{\partial z}.$$

Consider separately the continuity of the mass of the air and the water phases flowing vertically through an element of clay, under isothermal conditions. In the case of the mass continuity of the air phase the solubility of the air in the water may be an appreciable factor. Based on Henry's law the mass of air m dissolved in unit volume of water is given by the expression

$$m = h\,p_a = h\,(p_o + u_a)$$

where h is a constant related to Henry's coefficient. In the case of the mass continuity of the water phase the mass of water vapour flowing in the air phase is neglected.

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Continuity of mass in air phase gives

$$\frac{\partial}{\partial z}(v_a \gamma_a + m\, v_w) = \frac{\partial}{\partial t}(m\, s\, n + \gamma_a (1-s)\, n). \quad \quad (4)$$

Continuity of mass in the water phase gives

$$\frac{\partial}{\partial z}(v_w \gamma_w) = \frac{\partial}{\partial t}(n\, s\, \gamma_w) \quad \quad (5)$$

where v is the macroscopic velocity given by Darcy's law, γ the density of the fluid, s the degree of saturation of voids by water, and n the porosity of the clay.

Assuming the water to be incompressible, γ_w = constant. Assuming air to obey the ideal gas law $\gamma_a = c\, p_a = c\,(p_o + u_a)$ where c is a constant. Equations (4) and (5) become:

$$\frac{\partial}{\partial z}\left[c(p_o+u_a)\, k_a \frac{\partial u_a}{\partial z} + h\,(p_o+u_a)\, k_w \frac{\partial u_w}{\partial z}\right] = \frac{\partial}{\partial t}\left[h(p_o+u_a)\, sn + c(p_o+u_a)\,(1-s)\, n\right] \quad (6)$$

$$\frac{\partial}{\partial z}\left[k_w \frac{\partial u_w}{\partial z}\right] = \frac{\partial}{\partial t}[n\, s] \quad \quad (7)$$

where k_a and k_w are given by equation (3). To solve for the four independent variables u_a, u_w, s and n it is necessary to produce two further equations. For a given pair of pore fluids the capillary difference can also be expressed as an empirical function of n, λ and s,

$$(u_a - u_w) = f_c\,(n, \lambda, s). \quad \quad (8)$$

Assuming that there is no time dependence (creep) in the void ratio–effective stress relation,

$$n = f_n\,(\sigma'). \quad \quad (9)$$

According to Bishop (1960) the effective stress in an unsaturated soil can be expressed as

$$\sigma' = \sigma - u_a + \chi\,(u_a - u_w). \quad \quad (10)$$

In considering stress–volume change relationships the stress paths of both components $(\sigma - u_a)$ and $(u_a - u_w)$ must be considered and 'this places a severe restriction on the type of prediction which can be made' (Bishop and Blight, 1963). Since in equation (10) χ is again a complex function of (n, λ, s) equation (9) can be written

$$n = f_n[\sigma - u_a + f_\chi\,(n, \lambda, s)]. \quad \quad (11)$$

For certain simple materials such as uniform rounded sands, where structure is not so important, it may be possible to derive theoretical expressions for f_a, f_w, f_c and f_χ, but this approach seems very unlikely. For compacted clays it is obviously impossible and these functions will have to be found experimentally in each case.

If experiments on a wide range of clays did show that the empirical expressions for any particular function were essentially of the same form and differed only in the values of the coefficients involved, then direct progress might be possible. Equations (6), (7), (8) and (11) could probably be solved for the four main variables u_a, u_w, s and n using numerical methods and with the help of a computer to provide a very comprehensive solution. Unfortunately the number of empirical coefficients necessary to describe fully the complex functions f_a, f_w, f_c and f_χ is certain to be high. The number of combinations of practical values of these coefficients necessary to provide an adequate cover of the problem would require a very large number of separate solutions and would make it impossible to present the results in a convenient form.

In view of this difficulty it appears relevant to consider alternative treatments, less general than the above, but which may be simpler to apply in practice. The following simplified treatment is based on the classification of any particular consolidation process as one of five somewhat idealized processes, whose differing characteristics depend mainly on the range of

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s-values involved. The complex functions f_a (n, λ, s) etc, are replaced by simple functions of a single variable and for this purpose pore pressure seems the most relevant parameter as it strongly influences all three parameters (n, λ, s).

IDEALIZED CLASSIFICATION OF CONSOLIDATION PROCESS FOR COMPACTED CLAY

The values of s quoted as boundary values and the terms 'dry' and 'wet' of 'optimum' are intended merely as a guide to the state of the soil and will vary from clay to clay. It is not suggested that $s = 0{\cdot}9$ at the optimum for all clays.

(1) Extremely dry clays ($s < 0{\cdot}5$). It is unlikely that compacted clay fill will be placed in such a dry condition but the material will be included for the sake of completeness. The water is firmly attached to the skeleton by capillary forces. The air voids are completely interconnected and only air will flow from the consolidating clay (Yoshimi and Osterberg, 1963). Because of the low value of s the parameter χ in equation (10) will be small (Bishop *et al.*, 1964), and hence equation (10) can be applied as $\sigma' = \sigma - u_a$.

(2) Clay dry of optimum ($0{\cdot}5 < s < 0{\cdot}9$). The water still does not flow from the clay to any appreciable degree, possibly because the capillary difference $(u_a - u_w)$ is still large enough to ensure that u_w rarely exceeds zero. The air voids are still continuous and air is again the only fluid to flow from the compressed clay (Yoshimi and Osterberg, 1963). In this case equation (10) cannot be simplified.

(3) For clays in the region of the optimum moisture content there is probably a short transition stage (between the flow of air only in process 2 and the flow of liquid in process 4) when both air and water flow simultaneously and separately according to equations (6), (7), (8) and (11). At this transition stage the value of $(u_a - u_w)$ has probably reduced sufficiently to give $u_w > 0$ and water can now drain. Also the increase in s will have reduced k_a and increased k_w to the extent that the drainage rates of the two fluids are of similar order.

(4) For clay wet of optimum ($s > 0{\cdot}9$) the value of k_a drops off rather abruptly due to the sealing by water of the thin necks between the air filled larger voids. The air then exists in its occluded state and cannot flow as a separate continuous fluid. Certain smaller air bubbles may be free to flow with the free pore water forming a homogeneous compressible fluid flowing under the gradient $\partial u_w/\partial z$. The majority of the air will be static, trapped by the skeleton. Because of the high value of s the parameter χ in equation (10) will approach unity and hence equation (10) can be applied as $\sigma' = \sigma - u_w$ (Bishop *et al.*, 1964).

(5) For very wet clays ($s > 0{\cdot}95$) it can be concluded that the small amount of air present will be trapped by the skeleton and the fluid flowing from the clay will be fairly incompressible.

The above five processes have each been treated separately using simple parameters to describe their respective physical conditions. It is implied that any consolidation process in compacted clay can be satisfactorily treated by assuming it to follow only one of these idealized processes. In the region of optimum moisture content, however, it is possible that a large load increment could cause an overlap. For example, consolidation might start on the dry side with only air flowing from the clay, which could cause s to increase to the value at which the air became fully occluded; thus process 2 could change, via process 3, into process 4. Such *apparent* discontinuity cannot be taken into account in the following treatment, any consolidation process being considered as essentially one of the above idealized processes 1 to 5. The only way to remove such *apparent* discontinuity is to revert to the much more difficult solution of the general equations (6), (7) and (11), which requires knowledge of the complex functions f_a, f_w, f_c and f_χ.

TREATMENT OF PROCESS 1

For very dry clays the water phase can be considered as effectively bound to the clay skeleton. The air voids can be treated as continuous and only air will flow from the clay.

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The value of k_a is so high that despite the compressibility of the air the transient process will in general not take long. It is therefore unlikely that equilibrium will be maintained between air and water according to Henry's law which will be neglected from the mass continuity equation (4) which will be expressed as

$$\frac{\partial}{\partial z}(\gamma_a v_a) = \frac{\partial}{\partial t}(\gamma_a n_a) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

where n_a is the porosity with respect to air filled voids. For air flow the effect of gravity is small, regardless of the scale, and so the error will be negligible in applying Darcy's law in the previously adopted approximate form,

$$v_a = k_a \frac{\partial u_a}{\partial z}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

The variation of $k_a = f_a(n, \lambda, s)$ throughout a consolidation process is unknown. For simplicity it will be assumed to vary linearly with u_a, as both n and $(1-s)$ decrease as u_a dissipates, whence

$$k_a = k_{af}(1 + b\,u_a) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where k_{af} is the final value of k_a when the pore pressure excess u_a has finally dissipated to zero. Assuming air to behave as an ideal gas under isothermal conditions,

$$\gamma_a = \frac{\gamma_o}{p_o}\cdot p_a = \frac{\gamma_o}{p_o}(p_o + u_a) = \gamma_o(1 + d\,u_a) \quad . \quad . \quad . \quad . \quad . \quad (15)$$

where $d = 1/p_o$ = constant. For a dry soil the parameter χ becomes small and equation (10) can be applied with little error as $\sigma' = \sigma - u_a$.

Assuming a linear relation between porosity and effective stress,

$$n_a = n_{af}(1 + a\,u_a) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

where n_{af} is the final value of n_a when the pore pressure excess u_a has finally dissipated to zero.

On substituting equations (13), (14), (15) and (16) into equation (12),

$$\frac{\partial}{\partial z}\left[\gamma_o(1 + d\,u_a)\,k_{af}(1 + b\,u_a)\frac{\partial u_a}{\partial z}\right] = \frac{\partial}{\partial t}[n_{af}(1 + a\,u_a)\,\gamma_o(1 + d\,u_a)]. \quad . \quad (17)$$

To reduce to a dimensionless form put

$$z = z\,H, u_a = u\,u_o, \quad \alpha = a\,u_o, \quad \beta = b\,u_o, \quad \delta = d\,u_o$$

where H is the length of the vertical drainage path. u_o is the initial $t=0$ value of the pore-pressure excess, but which in an unsaturated soil is not equal to the increment of total stress, since $B \neq 1$. α, β and δ are characteristics which establish the degree of variation of n, k and γ respectively

$$\frac{n_o}{n_f} = (1+\alpha), \quad \frac{k_o}{k_f} = (1+\beta), \quad \frac{\gamma_o}{\gamma_f} = (1+\delta).$$

Equation (17) on expansion and rearranging gives

$$(\beta + \delta + 2\delta\beta u)\frac{\delta^2 u^2}{\partial z^2} + 2\,(1 - \delta\beta u^2)\frac{\partial^2 u}{\partial z^2} = 2\left(1 + \frac{\delta}{\alpha} + 2\delta u\right)\frac{\partial u}{\partial T} \quad . \quad . \quad (18)$$

where the time factor

$$T = \frac{k_f\,u_o\,t}{n_f\,\alpha\,H^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

Since $(a\,n_f)$ is equivalent to the conventional coefficient of volume decrease m_v, equation (19) can be written

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$$T = \frac{k_f\, t}{m_v\, H^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19a)$$

which is identical in form to the Terzaghi expression.

For an incompressible fluid $\delta = 0$ and for a constant value of permeability $\beta = 0$ and equation (18) reduces to $\frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial T}$ as required.

Equation (18) for the rate of dissipation of u in terms of T is non-linear and cannot be solved analytically. In a subsequent section it is solved for the usual boundary conditions of one-dimensional vertical consolidation, using finite difference approximations.

Since the compression in this case is related very simply to the pore-pressure excess u_a by equation (16) the rate of settlement can easily be obtained, and so the complete consolidation process is defined.

It can be seen that in the above simple treatment the main source of error is likely to appear in equation (14) which is clearly an oversimplification of the complex function f_a (n, λ, s).

TREATMENT OF PROCESS 2

Again water does not flow from the clay in any appreciable amount (Yoshimi and Osterberg, 1963), possibly for the two following reasons. Provided the air phase remains continuous, k_a is considerably greater than k_w; and unless u_a is very high the capillary difference $(u_a - u_w)$ ensures that u_w is rarely greater than zero. Neglecting the effect of Henry's law as in process 1 mass continuity again gives equation (12). The process 1 expressions for v_a and γ_a can also be assumed in this case. However, for the variation of n_a the effective stress must be obtained using equation (10) in full, and the term corresponding to equation (16) will be much more complex.

On considering equations (17) or (18) it can be seen that if the compressibility of the fluid (air in this case) is much greater than the compressibility of the skeleton $(\delta > \alpha)$, then the solution of (18) for the rate of dissipation of u_a is not going to be greatly affected by a variation in the form of the parameter α, caused by the use of equation (10) in full. Thus in the present case of air flow only, the rate of dissipation of u_a with T will again be given by the numerical solution of equation (18), to an accuracy sufficient for many engineering purposes.

Unless the values of χ and $(u_a - u_w)$ are known at each value of u_a the details of the rate of volume change or rate of increase of shear strength remain unknown and hence the above solution is far from complete, and merely serves as an estimate of the overall length of the dissipation process.

TREATMENT OF PROCESS 3

At higher values of s, in the region of the optimum, it is possible that, just before the air voids become occluded, the value of k_a will drop and the value of k_w will rise until they are of similar magnitude. Also at such high values of s the capillary difference $(u_a - u_w)$ will be less. It is therefore possible that a short range of s values exists during which both air and water can flow from the clay as separate fluids, but as the air flows out and s increases there is likely to be a gradual transition to process 4 conditions. To treat process 3 fully would require the solution of the general equations (6) to (11) which was abandoned earlier. However, in the present case certain simplifications may be made to these general equations.

In the mass continuity equations (6) and (7) the secondary effect of Henry's law could be neglected. Since the parameter χ approaches unity $\sigma' = \sigma - u_w$ and hence $n = n_f\,(1 + a\, u_w)$.

The variation of k_a and k_w during the consolidation process is not understood, apart from the fact that both decrease as pore pressures dissipate. The simplest expressions are linear:

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$$k_a = k_{af}\,(1+b_a\,u_a)$$
$$k_w = k_{wf}\,(1+b_w\,u_w).$$

The simplified forms of equations (6) and (7) can be written

$$\frac{\partial}{\partial z}\left[(1+d\,u_a)\,k_{af}\,(1+b_a\,u_a)\,\frac{\partial u_a}{\partial z}\right] = \frac{\partial}{\partial t}[(1+d\,u_a)\,(1-s)\,n_f\,(1+a\,u_w)] \quad . \quad (20)$$

$$\frac{\partial}{\partial z}\left[k_{wf}\,(1+b_w\,u_w)\,\frac{\partial u_w}{\partial z}\right] = \frac{\partial}{\partial t}[n_f\,(1+a\,u_w)\,s]. \quad . \quad . \quad . \quad . \quad (21)$$

The main problem is the variation throughout the process of the capillary difference $(u_a - u_w)$. It might be argued that at such high values of s near the optimum the value of $(u_a - u_w)$ is small and as a first approximation $u_a = u_w$. However, this observed effect may be more apparent than real, since at occlusion u_a cannot be measured by the usual direct means. Instead u_w will probably be registered on the air pressure measuring device with the apparent result that $u_a - u_w$ approaches zero; whereas in fact the pressure in the occluded air will depend on the size of the individual bubble and could be much higher than u_w before all air finally goes into solution.

The solution of equations (20), (21) and some form of equation (8) for the three variables u_a, u_w and s is not easily accomplished and will not be attempted until more reliable experimental information is obtained, particularly on the true behaviour of the capillary difference $(u_a - u_w)$.

TREATMENT OF PROCESS 4

In this state the air is present as occluded bubbles. It is probable that near the optimum certain of these bubbles are interconnected locally, but not sufficiently to allow the air to flow as an independent fluid as in process 3. For the present simple treatment the bubbles will be classified as 'free' or 'trapped' although it is recognized that some free bubbles may become trapped in their progress through the clay.

(*a*) Free bubbles are entirely surrounded by free pore water and will be transported by this water as it drains from the clay. The pressures of the air in the individual bubbles, being an internally balanced pressure caused by the curvature of the menisci, is not effective in providing the potential gradients causing flow. The free air bubbles exert an influence on the process mainly by their effect on the compressibility of the fluid, flow being brought about by the pressure gradients $\partial u_w/\partial z$ in the continuous water phase.

(*b*) Trapped bubbles occupy 'dead' pores, which are no longer in the general flow channels through the clay, or are held by capillary forces or the viscous forces of the adsorbed double-layer. These bubbles are effectively bound to the clay skeleton and can be treated as a component of the skeleton rather than of the mobile fluid. The important effect of the trapped bubbles is on the permeability of the skeleton to the compressible fluid. Since they tend to occupy the larger voids and since free bubbles may become trapped when they expand as pressures dissipate it is suggested that the effect on overall permeability is disproportionately great.

To treat the above system in a simple manner it is necessary to assume that the mobile fluid is homogeneous to the extent that its compressibility characteristics do not vary throughout the mass of the clay and do not change during a consolidation process as a possible consequence of Henry's law.

It is also assumed that since the mass of air is negligible compared with the mass of water in the homogeneous compressible fluid then the effects of Henry's law can be omitted from the following equation for continuity of mass of the homogeneous fluid.

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$$\frac{\partial}{\partial z}(\gamma_e v_e) = \frac{\partial}{\partial t}(\gamma_e n_e) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

where the suffix 'e' refers to the effective mobile fluid (homogeneous, compressible) as in the similar notation employed by Orlob and Radhakrishna (1958). As stated above flow is caused by gradients in u_w alone

$$v_e = k_e \frac{\partial u_w}{\partial z} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

It is convenient to work in terms of u_w as the main dependent variable and to obtain a solution to equation (22) the other dependent variables γ_e, n_e and k_e will be expressed as simple functions of u_w.

Equation (10) can be applied with little error as

$$\sigma' = \sigma - u_w. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

Remembering that n_e is defined in terms of volume of voids occupied by mobile fluid rather than total void volume, n_e will be expressed as

$$n_e = n_{ef}(1 + C_a u_w + C_s u_w). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

C_a is a parameter (which may be pressure dependent) which accounts for the compressibility with respect to u_w of the air bubbles bound to the skeleton. C_s is a parameter (which may be pressure dependent) which accounts for the compressibility of the skeleton with respect to effective stress and hence by equation (24) with respect to u_w.

Accurate expressions accounting for the pressure dependency of C_a and C_s would lead to very complex equations and to avoid this the following approximation is suggested. C_a increases and C_s decreases as u_w decreases during a consolidation process; and consideration of the general order of the pore-pressure parameter B suggests that C_a and C_s are of similar order of magnitude. Hence it is suggested that $(C_a + C_s)$ is sensibly constant during a consolidation process and equation (25) can be applied in the form:

$$n_e = n_{ef}(1 + a\, u_w). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

It should be noted that the parameter a is now larger than the corresponding parameter a in process 1 (equation (16)) which applied only to the compressibility of the skeleton.

For the clay under consideration the volume of air present in the mobile fluid is certainly less than 10% depending on the proportion of air bound to the skeleton. Thus the increase in the density γ_e of the fluid from $u_w = 0$ to full saturation under a high value of u_w must be less than 10% and will usually be nearer 5%. It might be possible to calculate a relation between γ_e and u_w based on Boyle's and Henry's laws and the capillary difference $(u_a - u_w)$, but such a relation would be rather cumbersome. In view of the small variation of some 5 to 10% in γ_e it appears justifiable to adopt a simple linear relation

$$\gamma_e = \gamma_0 (1 + d\, u_w). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

The value of d in the above is much smaller than when the compressible fluid is entirely air as in process 1, equation (15).

The relation between k_e and u_w is much more complex and it is not such a simple matter to postulate a realistic approximation based on the form of equation (3).

The permeability of compacted clay is extremely sensitive to changes in structure (Lambe, 1954). The presence of air in the voids of compacted clay must necessarily decrease the permeability (Christiansen, 1944). If the air tends to occur in the larger voids, which provide the most important flow channels through the clay, the effect of even small changes in s can be expected to be important; this is generally confirmed by the experimental results of Bjerrum and Huder (1957) and of Sibley and Miller (1962). It is very difficult in such experimental work to separate clearly the effects of the various parameters n, λ and s, but fortunately

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what is required at this stage is simply their combined effect on permeability. Since the permeability decrease due to changes in both structure and saturation are governed by u_w it is proposed that for the present simple treatment it is sufficient to postulate

$$k_e = k_{ef}\,(1+b\,u_w). \qquad (28)$$

It is interesting to note that a simple relation of this form is compatible with the permeability variation found by Schmid (1957) for saturated clays. From the discussion above it is apparent that the value of b will be greater for an unsaturated than for a saturated clay.

In a later section the results of introducing expressions more general than equation (28) will be investigated.

Substitution of the approximate relations for v_e, γ_e and n_e in terms of the pore-water pressure excess u_w into the mass continuity equation (22) yields an equation identical to equation (17) which is expressed in dimensionless form as equation (18). As stated previously, equation (18) is subsequently solved for the usual boundary conditions of one-dimensional consolidation, using finite difference approximations, to yield the rate of dissipation of u_w in terms of the dimensionless time factor T. The difference between the predicted behaviour computed for process 1 and process 4 is therefore due to the differences in the magnitude of the parameters α, β and δ.

Since the compression in the present case is very simply related to u_w by means of equation (26), the rate of settlement is easily obtained. Thus the complete consolidation process is defined by the above treatment and it can be applied to both settlement and stability calculations.

TREATMENT OF PROCESS 5

It is assumed that most of the small amount of air present in the clay is bound to the skeleton and very little flows with the water. The treatment is therefore as in process 4 with d negligibly small. The main difference between the consolidation of this type of clay and a fully saturated one is in the disproportionately large effect of the small amount of air on the variation of permeability throughout the process.

As the parameter d is being removed from the analysis it is an opportune place to introduce a more general expression in place of the linear variation of k with u_w (equation (28)).

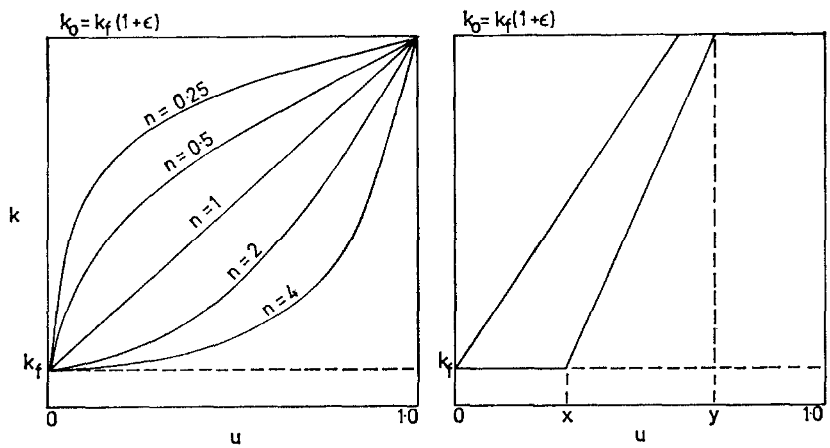

Fig. 1(a). Fig. 1(b).

The following variations of k with u_w, illustrated in Figs 1(a) and 1(b), have been treated. For the variation illustrated in Fig. 1(a) the general equation is

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$$k = k_f\,(1+bu_w^n) \qquad (29)$$

where n is positive, but can be greater or less than unity. For $n=1$ the above reduces to the linear form of equation (28). The equation to be solved in this case is

$$\frac{\partial}{\partial z}\left[k_f\,(1+b\,u_w^n)\,\frac{\partial u_w}{\partial z}\right] = \frac{\partial}{\partial t}[n_f\,(1+a\,u_w)] \qquad (30)$$

Putting into dimensionless form and expanding,

$$(1+\epsilon\,u^n)\cdot\frac{\partial^2 u}{\partial z^2}+n\,\epsilon\,u^{n-1}\left(\frac{\partial u}{\partial z}\right)^2 = \frac{\partial u}{\partial T} \qquad (31)$$

where $T = \dfrac{k_f\,t}{m_v\,H^2}$, $u_w = u\,u_o$ and $\epsilon = b\,(u_o)^n$.

Equation (31) has been solved for various values of n and ϵ using finite difference approximations as outlined in the next section.

For the variations of k with u_w illustrated in Fig. 1(b) the treatment is as follows:

$$k = k_f \quad \text{for} \quad 0<u<x$$

$$k = k_f\left[1+\epsilon\left(\frac{u-x}{y-x}\right)\right] \quad \text{for} \quad x<u<y$$

$$k = (\epsilon+1)\,k_f \quad \text{for} \quad y<u<1.$$

This is easily incorporated into a finite difference treatment, the equations being solved for various values of the parameters ϵ, x and y.

SOLUTION OF THE NON-LINEAR EQUATIONS

In general non-linear partial differential equations cannot be solved by analytical methods and so use must be made of finite difference approximations. There are a number of established methods of solving such equations using finite differences, the two best known being based on the explicit Schmidt and the implicit Crank-Nicholson types of solution.

The size of the finite step intervals in the space–time grid necessary to ensure both stability and accuracy of solution are not readily obtainable for non-linear equations such as (18) and (31) (Keller, 1960). For mildly non-linear equations of the type considered the procedure can be one of repeated trials using smaller and smaller values of the ratio $R=\Delta T/(\Delta z)^2$ until first stability is obtained; and then using finer and finer space intervals (Δz) with the established value of R until the solution is sensibly unaltered by subsequent refinement of the grid. The resulting solution is then a close approximation of the solution of the partial differential equation.

Since the explicit Schmidt method is the more direct and is simpler to programme for a computer it has been adopted for the following, despite the fact that it is probably less economic in the use of computer time.

Using the commonly accepted finite difference approximations for $\partial^2u/\partial z^2$, $\partial u/\partial z$ and $\partial u/\partial T$ the finite difference expressions of the various equations are:

equation (18),

$$u_1 = u_o+\frac{\Delta T}{(\Delta z)^2}\,\frac{\alpha}{2(\alpha+\delta+2\delta\alpha u_o)}\times$$
$$\times[(\beta+\delta+2\delta\beta\,u_o)\,(u_2^2+u_4^2-2\,u_0^2)+2(1-\delta\beta\,u_0^2)\,(u_2+u_4-2\,u_o)] \qquad (32)$$

equation (31),

$$u_1 = u_o+\frac{\Delta T}{(\Delta z)^2}\left[(1+\epsilon\,u_0^n)\,(u_2+u_4-2\,u_o)+\frac{\epsilon\,n\,u_0^{n-1}}{4}\,(u_2-u_4)^2\right] \qquad (33)$$

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A similar approach to the problem of the transient flow of gas has been made by Aronofsky and Jenkins (1964), and consolidation of saturated clay with a variable coefficient of consolidation by Scott (1961).

As anticipated the larger the values of the parameters α, β, δ, ϵ and n the smaller the value of the ratio $R=\Delta T/(\Delta z)^2$ required to ensure stability, i.e. a smoothly decaying pore pressure u with no sign of any superimposed oscillation at any stage.

For the simple Terzaghi solution a value of $R<0{\cdot}5$ is sufficient for stability. In the present solutions a value of $R=0{\cdot}1$ ensured stability for the smaller values of the parameters α, β, etc., and a value of $R=0{\cdot}01$ was usually adequate for the larger practical values.

From the point of view of accuracy of solution a grid with 10 space steps ($\Delta z=0{\cdot}1$) was in general found to give point values which differed by less than 0·5 from the corresponding values on a grid with 20 space steps ($\Delta z=0{\cdot}05$), the agreement increasing as dissipation increased.

Thus the most commonly used grid was one with $\Delta z=0{\cdot}1$ and $R=0{\cdot}1$. This grid required 10^3 time steps (ΔT) to reach even a value of $T=1$, and hence the calculation of 10×10^3 individual grid-point values, which requires the use of a digital computer.

The results of the computations are presented and discussed in the next section.

EXAMPLES OF COMPUTED SOLUTIONS

A representative range of the computed numerical solutions have been plotted in Figs 2–7.

Since secondary or creep effects are not included in this treatment, and the porosity has in all cases a simple linear relation with pore-pressure excess u, the settlement of a confined sample at any value of T is given by $\bar{U}$ the average degree of consolidation, which is the mean of the point values of u. In most oedometer tests the rate of settlement is measured and to allow comparison $\bar{U}$ has been plotted against T. In dissipation tests the mid-plane pore pressure excess u_m is measured, and the point value of u_m has also been plotted against T.

Figs 2(a) and 2(b) present results for constant permeability ($\beta=0$) and various values of the compressibility of the pore fluid.

$\delta<0{\cdot}1$ approximates to the compressibility of the pore fluid in a process 4 soil with $s>0{\cdot}9$. As anticipated the compressibility of the pore fluid slows down the dissipation process, in this case by a factor of 2. In Figs 5(a) and 5(b) the curves have been replotted after fitting them to the standard Terzaghi solutions ($\beta=0$, $\delta=0$) at the 50% dissipation point and it can be seen that their *shapes* are almost identical. This indicates that for such small values of δ the usual curve fitting methods would indicate that the Terzaghi solution gives an exact description of the process, whereas in fact the deduced permeability value would be approximately one-half of the true one.

$\delta=2$ represents the compressibility of the pore fluid in a soil from which only air is flowing (processes 1 and 2). For such a process the shape of the plot apparently depends on the value of the initial pore pressure u_o. In the treatment of processes 1 and 2, $\delta=d\,u_o=\dfrac{u_o}{p_o}$ and thus $\delta=2$ applies for a consolidation process in which the initial pore air pressure excess is twice atmospheric pressure. Due to this high fluid compressibility the dissipation process is greatly retarded. Despite this effect, the very high values of k_a (compared with k_w) will in general ensure that the transient is of much shorter duration than when the fluid is water.

In Figs 5(a) and 5(b) it can be seen that, after fitting at the 50% point, there is a small but perceptible difference in the shape of the above solution ($\beta=0$, $\delta=2$) and the Terzaghi solution ($\beta=0$, $\delta=0$).

Figs 3(a) and 3(b) present results for an incompressible pore fluid ($\delta=0$) for various values of the ratio initial/final permeability $k_o/k_f=(1+\beta)$. The reason why the curves in Fig. 3 for

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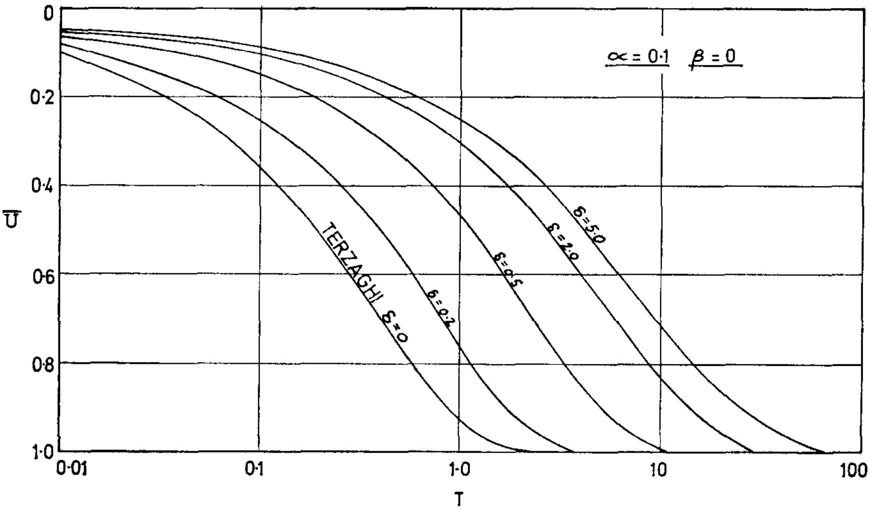

Fig. 2(a).

$\beta > 0$ lie below the Terzaghi $\beta = 0$ curve is that T the time factor is defined in terms of the *final* value of the permeability (see equation (19a)). $\beta = 2$ represents the order of variation in permeability commonly found in the consolidation of *saturated* clay.

In Figs 5(a) and 5(b) it can be seen that after fitting at the 50% point there is a small but perceptible difference in the shape of the ($\beta = 2$, $\delta = 0$) solution and the ($\beta = 0$, $\delta = 0$) Terzaghi solution, the difference being more pronounced for the mid-plane dissipation. Such conventional attempts at curve fitting must result in different values of the coefficient of consolidation from $\bar{U}/T$ and u_m/T plots.

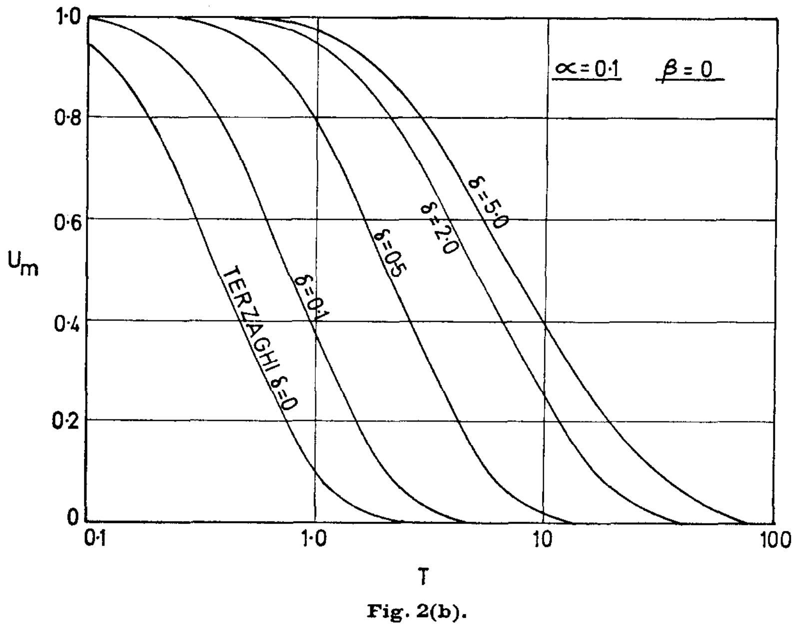

Fig. 2(b).

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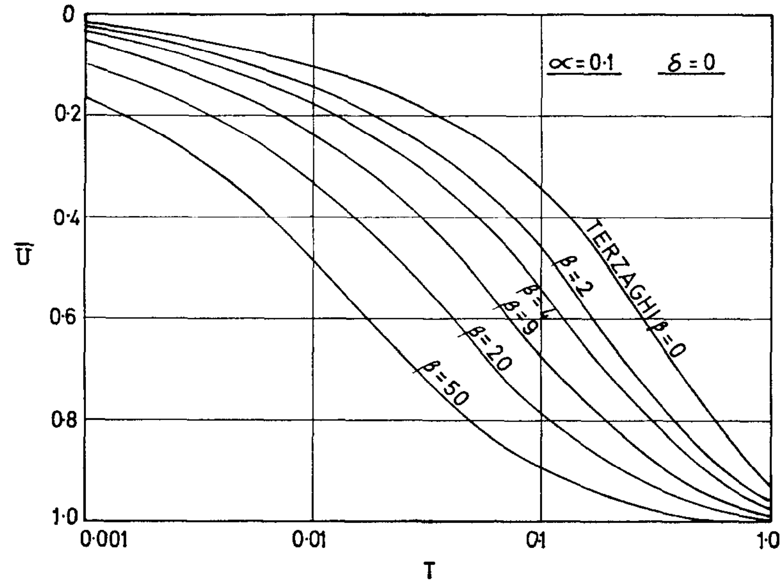

Fig. 3(a).

For large values of β, Figs 5(a) and 5(b) show that there is a considerable difference in the shape of the computed plots and the Terzaghi plot. As could be anticipated, the decreasing value of k causes the process to be accelerated in the early stages and retarded in the later stages. This effect is most marked in the case of rate of dissipation of the mid-plane pore pressure u_m as shown in Fig. 5(b). This departure from the standard Terzaghi shape is also illustrated in Fig. 6, where the conventional Taylor $\sqrt{T}$ plot has been used. Not unexpec-

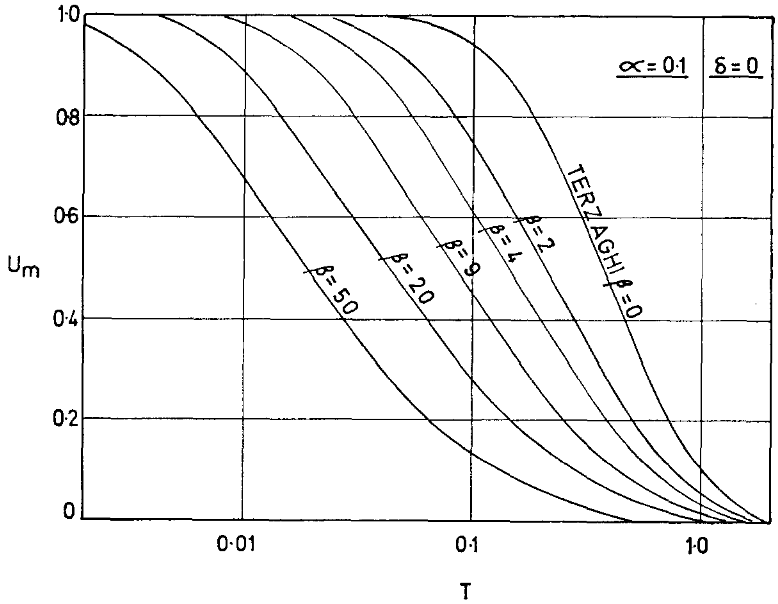

Fig. 3(b).

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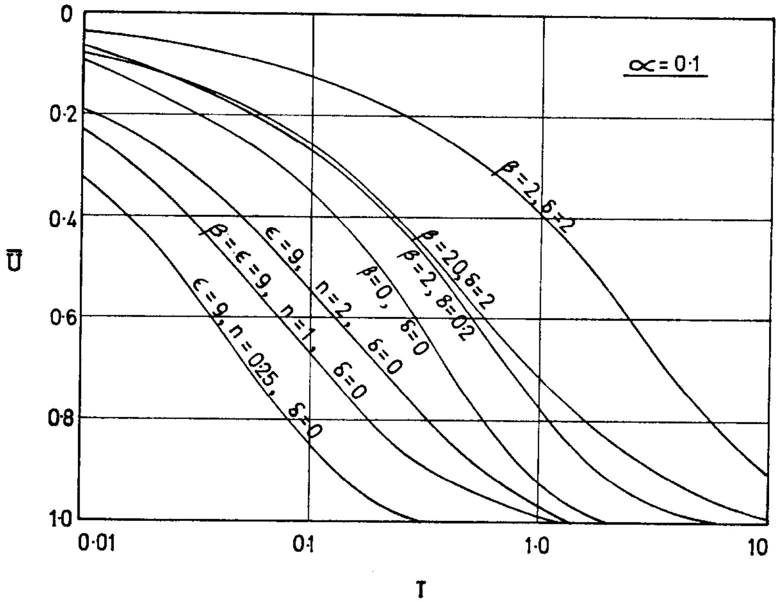

Fig. 4.

tedly, the effect of large values of β is to introduce a curvature into the initial linear portion of the $\sqrt{T}$ plot.

Both of these calculated departures from Terzaghi theory are characteristic of the observed behaviour in laboratory consolidation tests on unsaturated clay. For example, typical results from a 10-in.-dia. oedometer test on field compacted clay fill sampled from the Derwent Dam during construction are plotted in Fig. 8.

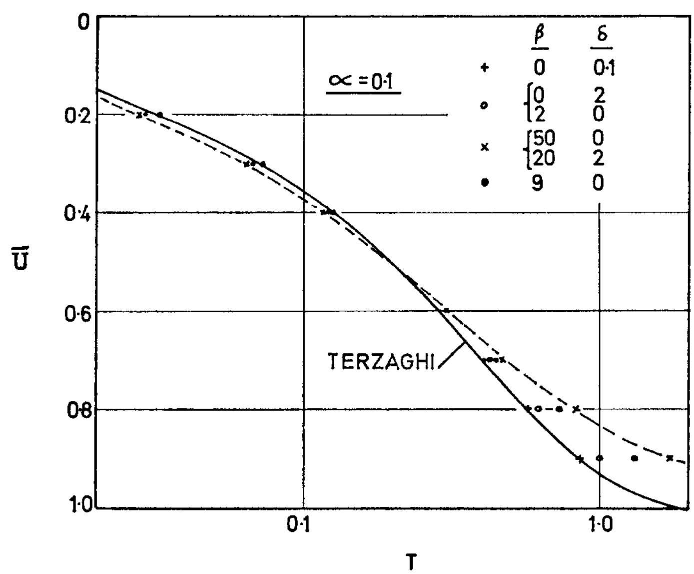

Fig. 5(a).

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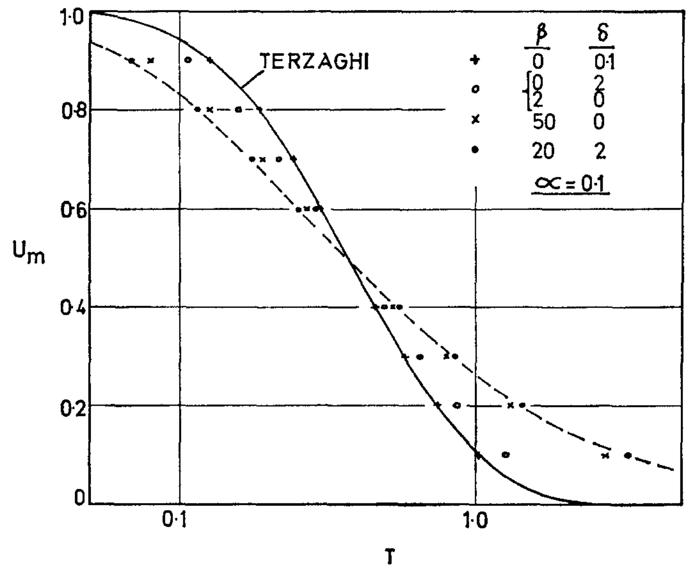

Fig. 5(b).

Fig. 4 presents the results obtained for the more general cases of both $\beta > 0$ and $\delta > 0$, and for the type of non-linear variation of permeability illustrated in Fig. 1(a).

Fig. 7 shows the shape of the isochrones for chosen values of β and δ, at an average degree of consolidation of $\bar{U} = 0{\cdot}64$.

DISCUSSION

For clays dry of optimum with continuous air voids (processes 1 and 2) the value of k_a is very high compared with the usual range of k_w values and the dissipation process will be relatively short despite the effect of the compressibility of the fluid. In fact at the laboratory scale the dissipation of air pressure was so rapid in the tests of Yoshimi and Osterberg (1963) that compression was controlled by the rate of creep of the skeleton. Because of this fact, it is

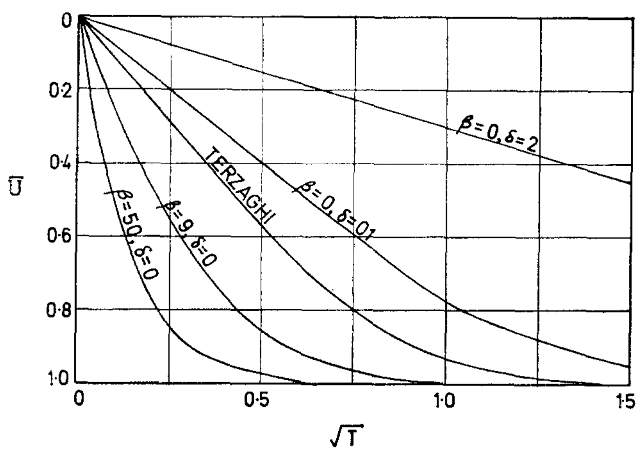

Fig. 6.

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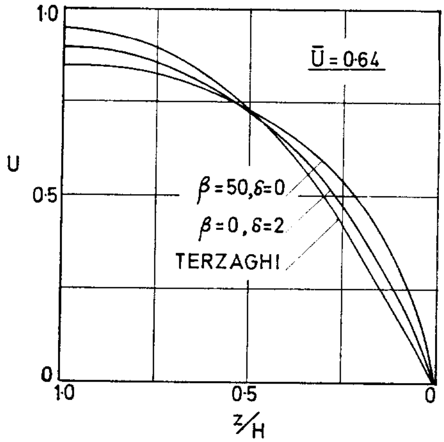

Fig. 7.

unlikely that such compacted clay fills will be capable of building up and maintaining high pore air pressures with consequent stability problems, even at the field scale.

It would appear that stability problems in compacted clay fills will be more common in material close to or wet of the optimum, when the air is occluded. For such clays $\delta < 0·1$ and the effect of fluid compressibility is small (see Figs 5(a) and 5(b)). One of the main reasons for departures from Terzaghi's theory is apparently the large value of β caused in part by the expansion of air bubbles causing blockages in the larger flow channels and by the marked effects of even small changes of structure. The solutions for $\beta > 5$ all exhibit two factors:

(1) A continuously curved $\bar{U}/\sqrt{T}$ plot (Fig. 6).

(2) A u_m/T plot which is flatter than the corresponding Terzaghi plot (Fig. 5(b)).

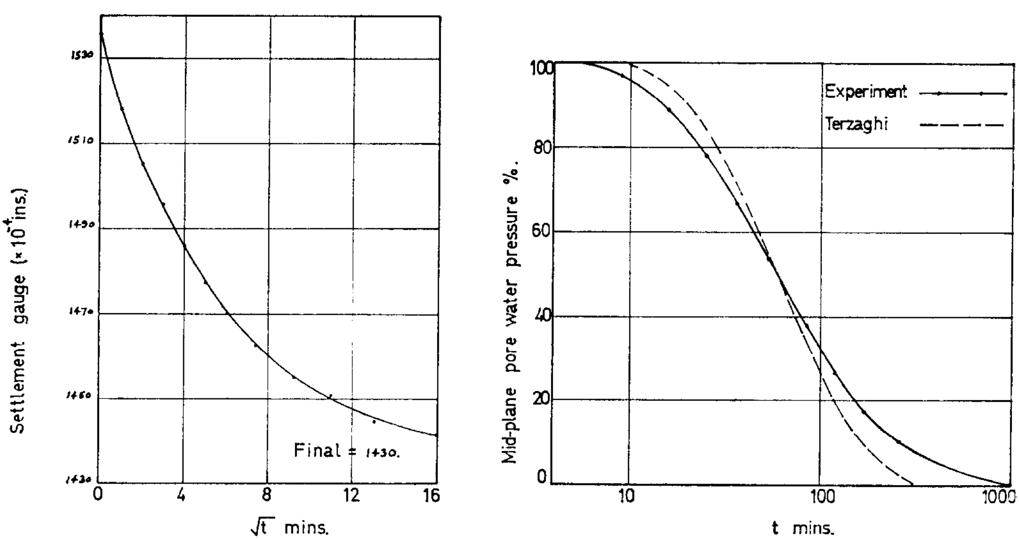

Fig. 8.

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Both of these factors are well known characteristics of the consolidation behaviour of unsaturated and compacted clays (see Fig. 8). It is tentatively concluded that variation of permeability is a major factor in governing the consolidation process in clays compacted *wet of optimum* whereas compressibility of the pore fluid is a secondary factor.

Certain compacted clays show a greater curvature on a $\bar{U}/\sqrt{T}$ plot than illustrated in Fig. 6. Solutions have been computed using variations in k of the type illustrated in Figs 1(a) and 1(b) but they have not yet provided the very marked curvature sometimes found in experimental results. It is therefore possible that some important factor has been completely omitted from the present preliminary treatment.

For *saturated* clays $\delta = 0$ and for small pressure increments $\beta < 3$. Under such conditions pronounced experimental departures from Terzaghi theory are (according to Figs 5(a) and 5(b)) unlikely to be caused by variation of permeability; but are more likely to be a result of structural viscosity or creep effects (Barden, 1964). Nevertheless, even the small departures from Terzaghi theory apparent in Figs 5(a) and 5(b) for such small values of β and δ are enough to cause discrepancies in values of the coefficient of consolidation obtained by fitting either the $\bar{U}/\sqrt{T}$ or $\bar{U}/\log T$ plots on the one hand and the u_m/T plot on the other, with the corresponding Terzaghi solution.

Because of the larger number of parameters necessary to describe the more complex behaviour of unsaturated clays there are greater difficulties involved in applying the theoretical curves presented above, by means of curve fitting techniques, to the interpretation of laboratory test results. Hence at present the most practical approach is to simulate the field problem as closely as possible in a laboratory test (Rowe, 1962). This requires a representative sample of clay, controlled drainage conditions, a suitably scaled loading and dissipation programme and finally the direct measurement of the property under investigation, whether pore pressure, shear strength or deformation. The considerable experimental difficulties involved in this approach suggest that any alternative, possibly curve fitting based on simple theoretical solutions of the type presented, should be investigated.

NOTATION

a	coefficient controlling change of porosity
b	,, ,, ,, ,, permeability
c	,, ,, ,, ,, density of ideal gas
d	,, ,, ,, ,, density of pore fluid
f	final value
k	permeability
m	mass of air in unit volume of water
m_v	coefficient of volume decrease
n	porosity
p	absolute pressure
p_0	atmospheric pressure
s	degree of saturation
t	time
u	pore-pressure excess
u_0	initial value of u
u_m	mid-plane value of u
v	macroscopic velocity
x	coefficient controlling variation of k with u
y	,, ,, ,, ,, ,, ,, ,,
z	vertical space co-ordinate
B	pore-pressure parameter
C	compressibility coefficient
H	length of drainage path
R	$\Delta T/(\Delta z)^2$
T	time factor
$\bar{U}$	average degree of consolidation
α	au_0
β	bu_0
δ	du_0
ϵ	$b(u_0)^n$
γ	density
μ	viscosity
ϕ	potential
λ	parameter representing clay structure
χ	coefficient in effective stress equation

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