第 24 卷第 2 期
2010 年 4 月

模 糊 系 统 与 数 学
Fuzzy Systems and Mathematics

Vol. 24, No. 2
Apr. ,2010

Article ID:1001-7402(2010)02-00-0

A New Form of Fuzzy Compact Spaces and Related Topics via Fuzzy Idealization*

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Abstract: Fuzzy ideals and the notion of fuzzy local function were introduced and studied by Sarkar[12] and by Mahmoud in [9]. The purpose of this paper deals with a fuzzy compactness modulo a fuzzy ideal. Many new sorts of weak and strong fuzzy compactness have been introduced to fuzzy topological spaces in the last twenty years but not have been studied using fuzzy ideals so, the main aim of our work in this paper is to define and study some new various types of fuzzy compactness with respect to fuzzy ideals namely fuzzy L-compact and L^*-compact spaces. Also fuzzy compactness with respect to ideal is useful as unification and generalization of several others widely studied concepts. Possible application to superstrings and E^{∞} space-time are touched upon.

Key words:

CLC number: **Document code**: A

The concepts of fuzzy sets and fuzzy set operations were first introduced by Zadeh[16]. Subsequently, Chang defined the notion of fuzzy topology[7]. Since then various aspects of general topology were investigated and carried out in fuzzy sense by several authors of this field. The local properties of a fuzzy topological space, which may also be in cretin cases the properties of the whole space, are important field for study in fuzzy topology by introducingthe notion of fuzzy ideal and fuzzy local function[9,12]. The concept of fuzzy topology may have very important applications in quantum particles physics particulary in connection with string theory and $E^{(\infty)}$ theory[13-14]. A fuzzy compactness modulo a fuzzy ideal has not been widely studied.

1 Terminologies

Throughout this paper, by (X,τ) we mean a fts in the sense of Chang's[7]. A fuzzy point in X with support $x\in X$ and value $\varepsilon(0<\varepsilon\leqslant 1)$ is denoted by x_{ε}. A fuzzy point x_{ε} is said to be contained in a fuzzy set μ in X iff $\varepsilon\leqslant\mu(x)$ and this will denoted by $x_{\varepsilon}\in\mu$[10]. For a fuzzy set μ in X, $\overline{\mu}$, μ^c and μ^o will respectively denote closure, complement and interior of μ. The constant fuzzy sets taking values 0 and 1 on X are denoted by 0_X, 1_X, respectively. A fuzzy set μ is said to be quasi-coincident with a fuzzy set η, denoted by $\mu q\eta$, if there exists $x\in X$ such that $\mu(x)+\eta(x)>1$[10]. Obviously, for any two fuzzy set μ and η, $\mu q\eta$

* **Received date**:2009-00-07
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will simply $\eta q\mu$. A fuzzy set ρ in a fts (X,τ) is called a q-nbd of a fuzzy point x_ε iff there exists a fuzzy open set ν such that $x_\varepsilon q\nu\subseteq\rho$[7,10]. We will denote the set of all q-nbd of x_ε in (X,τ) by $N(x_\varepsilon)$. A fts (X,t) is said to be a fuzzy extremely disconnected[1] (F.E.D in short) if the closure of every fuzzy open set in X is fuzzy open set. A fuzzy set μ for a fts (X,t) is called fuzzy α-open[1] (resp, β-open[1], preopen[7]) iff $\mu\leqslant\mu^{o-o}$ (resp. $\mu\leqslant\mu^{-o-}$, $\mu\leqslant\mu^{-o}$). A non-empty collection of fuzzy sets L of a set X is called a fuzzy ideal[9,12] iff (i) $\mu\in L$ and $\eta\subseteq\mu\Rightarrow\eta\in L$ (heredity), (ii) $\mu\in L$ and $\eta\in L\Rightarrow\mu\cup\eta\in L$ (finite additivity). Fuzzy closure operator of fuzzy set μ (in short $cl^*(\mu)$) is define $cl^*(\mu)=\mu\vee\mu^*$, and $\tau^*(L)$ be the fuzzy topology generated by cl^* i.e. $\tau^*(L)=\{\mu: cl^*(\mu)^c=\mu^c\}$[12]. The fuzzy local function[12] $\mu^*(L,\tau)$ of a fuzzy set μ is the union of all fuzzy points x_ε such that if $\rho\in N(x_\varepsilon)$ and $\lambda\in L$ then there is at least one $r\in X$ for which $\rho(r)+\mu(r)-1>\lambda(r)$.

2 Fuzzy L-compact spaces

Definition 2.1 Given a fts (X,τ) with fuzzy ideal L on X, a fuzzy set ρ is called fuzzy L-compact iff every open cover $\{\mu_j: j\in J\}$ of ρ has a finite subcover $\{\mu_{j_o}: j_o\in J\}$ such that for each j_o of J, there exists point $y\in X$, $(\rho-\bigvee_{j_o\in J}\mu_{j_o})(y)\leqslant l(y)$ for every $l\in L$.

A fts (X,τ) with fuzzy ideal L on X is fuzzy L-compact as subset is fuzzy L-compact.

Theorem 2.1 A fts (X,τ) with fuzzy ideal L_1 is fuzzy L_1-compactand L_2 is a fuzzy ideal on X such that $L_1\leqslant L_2$. Then (X,τ) is fuzzy L_2-compact.

Proof Obvious.

Theorem 2.2 A fts (X,τ) with fuzzy ideal L is fuzzy L-compact iff every fuzzy closed subset of X is a fuzzy L-compact subset.

Proof Let $\{\mu_j\}_{j\in J}$ be a fuzzy open cover of X and choose $j_o\in J$ such that $0_X\neq j_o\neq 1_X$, $\{\mu_{j_o}\}_{j_o\in J}$, then $\{\mu_j\}_{j\in J-\{j_o\}}$ is fuzzy open cover of $(\mu_{j_o})^c$. Since $(\mu_{j_o})^c$ is fuzzy closed subset of X, there exists a fuzzy finite subset J_o of J such that $(\mu_{j_o})^c(y)-\vee(\mu_j)(y)\leqslant l(y)$ where $(\mu_{j_o})^c_{j_o\in J}$ is fuzzy L-compact space implies $1_X-(\bigvee_{j_o,j\in J_o}(\mu_{j_o},\mu_j))(y)\leqslant l(y)$ then we have $1_X-(\bigvee_{j\in J_o}\mu_{j_o})(y)\leqslant l(y)$. Therefore (X,τ) is fuzzy L-compact subset.

On other hand, let ρ be a fuzzy closed subset of a fuzzy L-compact space (X,τ) and $\{\mu_j\}_{j\in J}$ be a fuzzy open cover of ρ then in each case $\{\rho^c,\mu_j\}_{j\in J}$ is a fuzzy open cover of X, so there exists a finite subset j_o of J such that $1_X-\bigvee_{j\in J_o}(\rho^c,\mu_j)(y)\leqslant l(y)$ for each $l\in L$. Hence $((\rho)(y)-\bigvee_{j\in J_o}(\mu_j)(y))\leqslant l(y)$ and therefore ρ is fuzzy L-compact.

Theorem 2.3 A fts (X,τ) with fuzzy ideal L is fuzzy L-compact iff every fuzzy closed family of fuzzy subsets $\{\rho_j\}_{j\in J}$ of X with $l\in L$, $\bigwedge_{j\in J}(\rho_j)(y)\leqslant l(y)$ $\forall$ $y\in X$ there exists a finite fuzzy subset j_o of j such that $\bigwedge_{j\in j_o}(\rho_j)(y)\leqslant l(y)$ for all $l\in L$.

Proof Let a fts (X,τ) with fuzzy ideal L on be a fuzzy L-compact space and $\{\rho_j\}_{j\in J}$ be family of fuzzy closed subsets of X, then $\{\rho_j^c\}_{j\in J}$ is fuzzy open cover of X, where $\bigwedge_{j\in J}(\rho_j)(y)\leqslant l(y)$ then for fuzzy finite subsets J_o of J we have $\leqslant l(y)(1_X-(\bigvee_{j\in J}(\rho_j^c))(y))$ and hence $\bigwedge_{j\in j_o}(\rho_j)(y)\leqslant l(y)$ for all $l\in L$. On other hand, let $\{\mu_j\}_{j\in J}$ be a fuzzy open cover of X. Then by hypothesis $\bigwedge_{j\in J}(\mu_j^c)(y)\leqslant l(y)$, so there exists a finite fuzzy subset j_o of J such that $\bigwedge_{j\in J_o}(\mu_j^c)(y)=1_X-\bigvee_{j\in J_o}(\mu_j)(y)\leqslant l(y)$ for every $l\in L$. Therefore, X is fuzzy L-compact space.

Theorem 2.4 Let $\{\rho_i\}_{i\in\{1,2,\ldots,m\}}$ be a finite of fuzzy L-compact subsets of the fts (X,τ) with fuzzy ideal L. Then the union of them is fuzzy L-compact subset of (X,τ).

Proof Let $\{\rho_i\}_{i\in\{1,2,\ldots,m\}}$ be a finite of fuzzy L-compact subsets of the fts (X,τ) with fuzzy ideal L and $\{\mu_j\}_{j\in J}$ be a fuzzy open cover of $\bigcup_i\{\rho_i\}_{i\in\{1,2,\ldots,m\}}$. Then $\{\mu_j\}_{j\in J}$ is open cover of each ρ_i. Since $\{\rho_i\}_{i\in\{1,2,\ldots,m\}}$ are fuzzy L-compact, then there exist finite subsets $J_1,J_2,\ldots,J_m$ of J such that $\rho_i(y)-\bigvee_{j_i\in J}(\mu_{j_i})(y)\leqslant l(y)$, therefore $\bigcup_i\{\{\rho_i\}_{i\in\{1,2,\ldots,m\}}\}(y)-\bigcup_i\bigcup_{j_i}\{\mu_{j_i}\}_{j_i\in J_1\cup J_2\cup\ldots\cup J_m}(y)\leqslant l(y)$, where $J_1\cup J_2\cup\ldots\cup J_m$ is a finite subset of J. Then $\bigcup_i\{\rho_i\}_{i\in\{1,2,\ldots,m\}}$ is a fuzzy L-compact subset of X.

Remark 2.1 One can shows that the intersection of two fuzzy L-compact subset of a fts (X,τ) with fuzzy ideal L is fuzzy L-compact.

Definition 2.2 Given a fts (X,τ) with fuzzy ideal L an $\rho\in I^X$, ρ is said to have the finite intersection property modulo L, denoted L-FIP, if for every finite subfamily $\{\mu_j\}_{j\in J}$ of ρ we have $\wedge\mu_j(y)>l(y)$ $\forall\ y\in X$, $l\in L$.

Theorem 2.5 If a fts (X,τ) with fuzzy ideal L. Then the following statements are equivalent;

i. (X,τ) is fuzzy L-compact.

ii. Any family $\{\rho_j\}_{j\in J}$ of fuz{y closed subsets of X having the L-FIP.

iii. Any family of fuzzy closed of X with $\bigwedge_{j\in J}\rho_j(y)\leqslant l(y)$ $\forall\ y\in X$, has J_o of J such that $\bigwedge_{j\in J_o}\rho_j\leqslant l(y)$.

Proof i→ii Let $\{\rho_j\}_{j\in J}$ be family of fuzzy closed subsets and L is fuzzy ideal on (X,τ), $\{\rho_j\}_{j\in J}$ having the L-FIP. And let $\bigwedge_{j\in J}\rho_j(y)\leqslant l(y)$ $\forall\ y\in X$, $l\in L$, then we have $(\bigwedge_{j\in J}\rho_j)^c(y)>l^c(y)\Leftrightarrow(\bigvee_{j\in J}\rho_j^c)(y)>l^c(y)\Leftrightarrow(\bigvee_{j\in J}\rho_j^c)^c\leqslant l(y)$ Since X is fuzzy L-compact then there exists finite subfamily J_o of J such that $(\bigvee_{j\in J_o}\rho_j^c)^c(y)\leqslant l(y)\Leftrightarrow\bigwedge_{j\in J_o}\rho_j\leqslant l(y)$; contradiction.

ii→i Let $\{\mu_j\}_{j\in J}$ be a fuzzy open cover of X. Now if (X,τ) is not fuzzy L-compact, then for any finite subfamily $J_o\in J$, we get $(y)>l(y)\Leftrightarrow(\bigwedge_{j\in J_o}\mu_{j_o}^c)(y)>l(y)\Leftrightarrow(\bigvee_{j\in J}\mu_j)^c>l(y)$; contradiction.

ii→iii Logically obvious.

Theorem 2.6 Every fuzzy compact space is fuzzy L-compact but the converse is not true.

Proof Let L is fuzzy ideal and since $1_X=\bigvee_{j\in J_o}\mu_j$(from the definition of fuzzy compact space) then we have $(\bigvee_{j\in J_o}\mu_j)^c(y)=0_X(y)\in L\Rightarrow 1_X-(\bigvee_{j\in J_o}\mu)(y)\leqslant l(y)$ where $l\in L$.

Remark 2.2 It is clear that the notions of fuzzy compact and fuzzy $<0_X>$-compact spaces are coincide.

Theorem 2.7 Given fts (X,τ) with fuzzy ideal L on X, every fuzzy semi compact space is fuzzy L-compact space.

Proof From the definition of fuzzy semi compact space we have, $\{\mu_j\}_{j\in J}$ is fuzzy semi open cover such that $1_X=\bigvee_{j\in J_o}(\mu_j^{o-})$, therefore $(1_X-\bigvee_{j\in J_o}(\mu_j^{o-}))(y)=0_X(y)$, $y\in X$, hence $(1_X-\bigvee_{j\in J_o}(\mu_j^{o-}))(y)\leqslant l(y)$, $l\in L$ then we have $(1_X-\bigvee_{j\in J_o}(\mu_j))(y)\leqslant l(y)$, $l\in L$ this implies (X,τ) is fuzzy L-compact space.

Theorem 2.8 A fts (X,τ) with fuzzy ideal L on X is fuzzy L-compact if and only if $(\bigwedge_{j\in J_o}\rho_j^o)(y)>l(y)\Rightarrow(\bigwedge_{j\in J}\rho_j)(y)>l(y)$, where $\{\rho_j\}_{j\in J}$ is a family of fuzzy closed subsets of X and J_o is finite subfamily of

J.

Proof Let (X,τ) be a F-L-compact space, $(\bigwedge_{j\in J_o}\rho_j^o)(y)>l(y)$ and let $(\bigwedge_{j\in J}\rho_j)(y)\leqslant l(y)$. Now we have $(\bigwedge_{j\in J}\rho_j)(y)\leqslant l(y)\Leftrightarrow(\bigvee_{j\in J}\rho_j^c)(y)>1_X-l(y)$, but (X,τ) is F-L-compact space, then which is contradiction. For the converse we have $(\bigwedge_{j\in J_o}\rho_j^o)(y)>l(y)\Rightarrow(\bigwedge_{j\in J_o}\rho_j)(y)>l(y)\Leftrightarrow(\bigwedge_{j\in J_o}\rho_j)(y)>l(y)$. Thus by Theorem 2.6 and Theorem 2.7 (X,τ) is fuzzy L-compact.

Now, from the above discussions and some known types of fuzzy compactness we have the following diagram:

But the converse may not be true by examples in [11] and the following example.

Example 2.1 Let (X,τ) be a fts where $\tau=\{1_X,0_X,\mu_1,\mu_2,\mu_3\}$ where $\mu_1(x)=0.3$, $\mu_2(x)=0.6$ and $\mu_3(x)=0.8$ with fuzzy ideal $L=\{0_X,\eta\}\vee\{x_\varepsilon:\varepsilon\leqslant 0.2\}$, $\eta(x)=0.2$ if $\rho(x)=0.3$ then ρ is fuzzy L-compact but ρ is not fuzzy compact.

Theorem 2.9 Given a fts (X,τ) and $0_X\neq\mu\in I^X$, then $\tau(\mu)$ is a fuzzy topology on X given by $\tau(\mu)=\{1_X,0_X\}\cup\{\mu\vee\eta:\eta\in\tau\}$.

Proof Obvious.

Theorem 2.10 For a fts (X,τ) with fuzzy ideal L the following statements are equivalent:

i. (X,τ) is fuzzy L-compact.

ii. For each non-empty fuzzy set $\eta\in\tau$, then the fuzzy topology $\tau(\eta)$ is fuzzy L-compact.

iii. Each fuzzy closed subset of (X,τ) is fuzzy L-compact.

Proof i→ii Let η be any fuzzy non-empty τ-open subset and μ be a $\tau(\eta)$-open cover of X, then $\mu=\{(\eta\vee\mu_j)(y):j\in J,\mu_j\in\tau\}$ is open cover of (X,τ). Since (X,τ) is fuzzy L-compact, then there exists a finite fuzzy subset J_o of J such that $1_X-\bigvee\{\eta,\mu_j:j\in J_o\}(y)\leqslant l(y)$, so that $\tau(\eta)$ is fuzzy L-compact.

ii→iii Let ρ be any closed subset of (X,τ), then $\eta(y)=1_X-\rho(y)$ is τ-open fuzzy subset. Let $\{\mu_j\}_{j\in J}$ be a fuzzy τ-open of ρ. Then $\{\eta\vee\mu_j:j\in J\}$ is $\tau(\eta)$-open cover of (X,τ), by (ii) since $\tau(\eta)$ is L-compact, there exists a finite subset J_o of J such that $1_X-\bigvee\{\eta,\mu_j:j\in J_o\}(y)\leqslant l(y)$. Thus

$1_X - \vee \bigvee_{j\in J_o} \{1_X-\rho, \mu_j\}(y) \leqslant l(y) = ((1_X-(1_X-\rho(y))) \wedge (1_X - \bigvee_{j\in J_o}(\mu_j)(y))) = \eta(y) \wedge (1_X - \bigvee_{j\in J_o}\mu_j(y)) = (\eta(y) \wedge 1_X - \eta \wedge (\bigvee_{j\in J_o}\mu_j)(y)) = \eta(y) - \eta(y) \bigvee_{j\in J_o}(\mu_j)(y) \leqslant l(y)$. By heredity, then $\eta(y) - \bigvee_{j\in J_o}(\mu_j)(y) \leqslant l(y)$. Therefore η is fuzzy L-compact.

We shall prove that the image of a fuzzy L-compact under the fuzzy continuous function is fuzzy $f(L)$-compact and this result can be generalized as follows:

Theorem 2.11 Given $f:(X,\tau)\rightarrow(Y,\sigma)$ is fuzzy continuous function with fuzzy ideal L on X and $\mu \in I^X$ is fuzzy L-compact subset of X. then $f(\mu)$ is $f(L)$-compact subset of Y.

Proof Let μ be a fuzzy L-compact subset of X and $\{\rho_j\}_{j\in J}$ be a fuzzy open cover $f(\mu)$ in Y, then $\{f^{-1}(\rho_j)\}_{j\in J}$ is a fuzzy open cover of μ (because is fuzzy continuous function) so there exists a finite subset j_o of J such that $(\mu - \vee\{f^{-1}(\rho_j): j\in J_o\})(y) \leqslant l(y)$ forevery $y\in Y$.

Therefore $f(\mu)(y) - (\bigvee_{j\in J_o}\mu_j)(y) \leqslant f(\mu)(y) - (\bigvee_{j\in J_o} ff^{-1}(\mu_j))(y) \leqslant f(\mu)(y)$ By heredity of $f(l)$ Then $f(\mu)$is fuzzy-$f(\mu)$ compact subset of Y.

Corollary 2.1 Given a fuzzy function $f:(X,\tau)\rightarrow(Y,\sigma)$ is fuzzy continuous with fuzzy ideal L on Y is a surjective of a fuzzy L-compact space X into Y. Then Y is $f(\mu)$-Compact.

Theorem 2.12 Given a fuzzy open bijection function $f:(X,\tau)\rightarrow(Y,\sigma)$ with fuzzy ideal J on Y. If (Y,σ) is a fuzzy J-compact space. Then (X,τ) is fuzzy-$f^{-1}(J)$ compact space.

Proof Obvious by Theorem 2.4 and Example 2.1.

Theorem 2.13 Given a fts (X,τ) with fuzzy ideal L. If $\mu\in I^X$, then

i. Fuzzy L-compact subset of (X,τ) if μ is fuzzy L-compact subset of $(X,\tau^*(L))$.

ii. (X,τ) is fuzzy compact iff (X,τ) is fuzzy L_f-compact.

Proof Follows from the fact that $\tau\leqslant\tau^*(L)$.

3 Fuzzy L^*-compact spaces

In what follows we give some properties and characterizations of fuzzy L^*-compactness via fuzzy ideals by using fuzzy L-open[1].

Definition 3.1 A fuzzy ideal L in a fts (X,τ) is τ-fuzzy condense if $L\wedge\tau=\{0_X\}$.

Theorem 3.1 If L is τ-condense in a fts (X,τ). Then (X,τ) is fuzzy extremely-disconnected if and only if $(X,\tau^*(L))$ is fuzzy extremely-disconnected.

Proof $\Rightarrow$ Let (X,τ) be a fuzzy extremely-disconnected and $\mu$is τ^*-open. Then $\mu=\rho-l$ where $\rho\in\tau$ and $l\in L$, and hence $\tau^*\text{-}cl(\mu)=cl(\mu)=cl(\rho)$. which means $\tau^*\text{-}cl(\mu)$ is τ^*-open. Therefore $(X,\tau^*(L))$ is fuzzy extremely-disconnected.

$\Leftarrow$ Assume that $(X,\tau^*(L))$ is fuzzy extremely-disconnected and ρ_1,ρ_2 are open sets such that $cl(\rho_1)$, $cl(\rho_2)\neq 0_X$. Also, $cl(\rho_1)=\tau^* cl(\rho_1)$, $cl(\rho_2)=\tau^*-cl(\rho_2)$ which gives $\tau^*-cl(\rho_1)$, $\tau^*-\mathrm{cl}(\rho_2)\neq 0_X$ this implies $\rho_1,\rho_2\neq 0_X$

Definition 3.2 Given a fts (X,τ) with fuzzy ideal L on X and $0_X\neq\mu\in I^X$ such that L is τ-fuzzy condense then μ is said to be a fuzzy L^*-compact subset of X via L iff every fuzzy L-open cover $\{\mu_j\}_{j\in J}$ of μ in X has a finite sub caver.

Remark 3.1 One can deduce that FL-compact$\rightarrow$FL*-compact.

Theorem 3.2 If a fts (X,τ) with fuzzy ideal L is fuzzy L^*-compact space with respect to L_1, and L_2 is fuzzy ideal on X such that $L_2\leqslant L_1$. Then (X,τ) is a L_1^*-compact space with respect to L_2.

Proof Obvious.

Theorem 3.3 A fts (X,τ) with fuzzy ideal L on X is fuzzy L^*-compact iff every fuzzy L-closed subset of X is fuzzy L^*-compact and $L\wedge\tau=\{0_X\}$.

Proof Let $\{\mu_j\}_{j\in J}$ be a fuzzy L-open cover of X, and $L\wedge\tau=\{0_X\}$, choose $i,k\in J$ such that $0_X\neq\mu_k\neq 1_X$, μ_k be a fuzzy L-open subset, then $\{\mu_j\}_{j\in J-\{k\}}$ is L-open cover of $(\mu_k)^c$. Since $(\mu_k)^c$ is L-closed subset of X, there exists a fuzzy finite subset J_o of $J-\{k\}$ such that $(\mu_k)^c<\bigvee_{j\in J_o}(\mu_j)$. Hence $1_X=\mu_K\vee\{\mu_j\}_{j\in J_o}$ which implies $1_X=\vee\{\mu_j\}_{j\in J_o}$. Therefore, (X,τ) is fuzzy L^*-compact space.

On other hand, let ρ be a fuzzy L-closed subset of a fuzzy L^*-compact and let $\{\mu_j\}_{j\in J}$ be a fuzzy L-open cover of ρ in X. Then $\{\rho^c,\mu_j\}_{j\in J}$ is fuzzy L-open cover of X. Hence there exists a finite subset J_o of J such that $1_X=\bigvee_{j\in J_o}\{\rho^c,\mu_j\}$ and so $\rho<\bigvee_{j\in J_o}\mu_j$. therefore, ρ is fuzzy L^*-compact subset of X.

Remark 3.1 One can shows that the intersection of two fuzzy L^*-compact subsets of a fts (X,τ) is fuzzy L^*-compact subset of X.

Lemma 3.1 Every fuzzy regular open set is a fuzzy L_n-open set.

Proof Let μ be a fuzzy regular open set in fts (X,τ) with fuzzy ideal L_n then $\mu(x)=\mu^{-o}(x)=\mu^{-o-o}(x)=(\mu^*(L_n,\tau))^o$.

Theorem 3.4 A fts (X,τ) with fuzzy ideal L_n on X is fuzzy L_n^*-compact. Then (X,τ) is fuzzy nearly compact.

Proof Obvious by using Lemma 3.1.

It is clear that the family of fuzzy L_n^*-compact spaces contains the fuzzy of nearly compact spaces. Many results concerning with fuzzy nearly compact can be derived easily if we take $L=L_n$ in our notion.

IN conclusion, we may stress once more the importance of fuzzy topology as a nontrivial extension of fuzzy sets and fuzzy logic[13] and the possible application in quantum physics[11-12]. We can use this new results of this paper in fuzzy bitopological spaces and expert systems and fuzzy control.

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