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The cyclic Decomposition of SL(2,p^k), where $p^k = 9$, 25 and 27

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Abstract

The problem of finding the cyclic decomposition of the factor group K(G) = cf (G,Z) / R(G) has been considered in this paper for G is the special linear group SL(2,9), SL(2,25) and SL(2,27).

التجزئة الدائرية للزمرة المنتهية الخطية الخاصة SL(2,p^k) عندما $p^k = 9, 25, 27$

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المستخلص

ان مسألة ايجاد التجزئة الدائرية لزمرة القسمة K(G) = cf (G,Z) / R(G) قد اعتبرت في هذا البحث للزمرة المنتهية الخطية الخاصة SL(2,9) ، SL(2,25) و SL(2,27).

Introduction

The set of all Z-valued class functions of a finite group G form an abelian group cf (G,Z) under point wise addition. Inside this group we have a subgroup of Z-valued generalized characters of G denoted by R(G).

The group of invertible $n \times n$ matrices over a field F denoted by GL(n,F). The determinant of these matrices is a homomorphism from GL(n,F) into F-{0} and we denote the kernel of this homomorphism by SL(n,F), the special linear group. Thus SL(n,F) is the subgroup of GL(n,F) which contains all matrices of determinant one over the field F..

§.1 Preliminaries

In this section, we give some basic known results which will be needed later.

Theorem 1.1 : [3]

The order of SL(2,p^k) is $p^k(p^{2k}-1)$.

Theorem 1.2 : [3]

G=SL(2,p^k) has exactly p^k+4 conjugacy classes:

$$1, z, c, d, zc, zd, a, a^2, \ldots, a^{\frac{p^k-3}{2}}, b, b^2, \ldots, b^{\frac{p^k-1}{2}}$$

Let:

- ν be the generator of the cyclic multiplicative group F^*,
- $1 \le \ell \le (p^k-3)/2$,
- $1 \le m \le (p^k-1)/2$.

Thus this conjugacy classes is satisfied

g ∈ G	Notation	C_g	$\lvert C_g \rvert$	$\lvert C_G(g) \rvert$
$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	1	C_1	1	$p^k(p^{2k}-1)$
$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	z	C_z	1	$p^k(p^{2k}-1)$
$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$	c	C_c	$(p^{2k}-1)/2$	$2p^k$
$\begin{pmatrix} 1 & 0 \\ \nu & 1 \end{pmatrix}$	d	C_d	$(p^{2k}-1)/2$	$2p^k$
$\begin{pmatrix} -1 & 0 \\ -1 & -1 \end{pmatrix}$	zc	C_{zc}	$(p^{2k}-1)/2$	$2p^k$
$\begin{pmatrix} -1 & 0 \\ -\nu & -1 \end{pmatrix}$	zd	C_{zd}	$(p^{2k}-1)/2$	$2p^k$
$\begin{pmatrix} \nu^\ell & 0 \\ 0 & \nu^{-\ell} \end{pmatrix}$	a^ℓ	C_{a^ℓ}	$p^k(p^k+1)$	p^k-1
Element of order$(p^k+1)m$	b^m	C_{b^m}	$p^k(p^k-1)$	p^k+1

THEOREM (1.3): [1]

Let G = SL($2,p^k$), then the Schur indices of the irreducible characters of G over the rational numbers Q are as follows:

	$p^k \equiv 1 \bmod 4$	$p^k \equiv 3 \bmod 4$
1_G	1	1
ψ	1	1
χ_i	2 (i odd)	2 (i odd)
	1 (i even)	1 (i even)
θ_j	2 (j odd)	2 (j odd)
	1 (j even)	1 (j even)
ξ_1	1	1
ξ_2	1	1
η_1	2	1
η_2	2	1

LEMMA (1.4): [1]

Let ζ be a primitive n-th root of unity, $i \in$ Z and $d_i = (i,n)$. and let $n \neq d_i, 2d_i$.Then:

$$\sum_{\alpha \in \Gamma_i} (\xi^i + \xi^{-i})^{\alpha} = \mu(\frac{n}{d_i}).$$

where $\Gamma_i = \Gamma(Q(\zeta^i + \zeta^{-i}):Q)$ and μ is the Möbius function.

Recall that μ function defined by:

$$\mu(n)=\begin{cases} 1 & if\ n=1 \\ 0 & if\ a^2 / n\ for some\ a>1 \\ (-1)^K & if\ n=p_1 p_2 \cdots p_K, p_i\ are\ distinct\ primes. \end{cases}$$

See [5]

LEMMA (1.5): [1]

Let ζ be a primitive n-th root of unity, $i \in Z$ and $d_i = (i,n)$. and let $n \neq d_i$, $2d_i$.Let $\Gamma = \Gamma(Q(\zeta + \zeta^{-1}):Q)$. Then

$$\sum_{\alpha \in \Gamma} (\xi^i + \xi^{-i})^\alpha = \frac{\Phi(n)}{\Phi(\frac{n}{d_i})} \mu(\frac{n}{d_i}).$$

Corollary (1.6): [1]

Let ζ be a primitive n-th root of unity. and $1 \le i \le \frac{n}{2}$. Let $\Gamma = \Gamma(Q(\zeta+\zeta^{-1}):Q)$. Then

$$\sum_{\alpha \in \Gamma} (\xi^i + \xi^{-i})^\alpha = \frac{\Phi(n)}{\Phi(\frac{n}{d_i})} \mu(\frac{n}{d_i}).$$

where $d_i = (i,n)$.

LEMMA (1.7): [2]

Let χ be a rational valued character of G and let x, $y \in G$ with $<x> = <y>$. Then $\chi(x) = \chi(y)$.

LEMMA (1.8): [1]

Let G = SL(2,p^k), where p is an odd prime. Then $<c> = <d>$ if and only if k is odd.

NOTATION: [1].

Let G = SL(2,p^k) for some prime $p \neq 2$, e and e' denote divisors of $p^k - 1$ such that $e < \frac{p^k - 1}{2}$ and $e' < \frac{p^k - 1}{2}$, f and f' denote divisors of $p^k + 1$ such that $f < \frac{p^k + 1}{2}$ and $f' < \frac{p^k + 1}{2}$, ρ_e is a primitive $(\frac{p^k - 1}{e})$-th root of unity , σ_f is a primitive $(\frac{p^k + 1}{f})$-th root of unity, 1, z, c, d, a, b are as in theorem (1.2), let

$\varepsilon=(-1)^{(p^k-1)/2}$, let $\rho \in$ C be a (p^k − 1)-th root of unity and $\sigma \in$ C be a (p^k + 1)-th root of unity.

$$B(k)=\begin{cases}1 & \text{if } k \text{ is even}\\ 2 & \text{otherwise}\end{cases} \qquad E(p^k)=\begin{cases}1 & \text{if } p^k \equiv 3 \bmod 4\\ 2 & \text{otherwise}\end{cases}$$

$$A(e)=\frac{1}{2}\Phi\left(\frac{p^k-1}{e}\right) \qquad C(f)=\frac{1}{2}\Phi\left(\frac{p^k+1}{f}\right)$$

$$\tau_1(e,e')=\sum_{\alpha\in\Gamma}(\rho_e^{e'}+\rho_e^{-e'})^{\alpha}=\frac{\Phi\left(\frac{p^k-1}{e}\right)}{\Phi\left(\frac{p^k-1}{ee'}\right)}\mu\left(\frac{p^k-1}{ee'}\right)$$

where $\Gamma=\Gamma(Q(\chi_e):Q)$. [Note that $\Gamma=\Gamma(Q(\rho_e+\rho_e^{-1}):Q)$].

$$\tau_2(f,f')=\sum_{\alpha\in\Gamma_1}(\sigma_f^{f'}+\sigma_f^{-f'})^{\alpha}=\frac{\Phi\left(\frac{p^k+1}{f}\right)}{\Phi\left(\frac{p^k+1}{ff'}\right)}\mu\left(\frac{p^k+1}{ff'}\right)$$

where $\Gamma_1=\Gamma(Q(\theta_j):Q)$. [Note that $\Gamma=\Gamma(Q(\sigma_f+\sigma_f^{-1}):Q)$].

χ_i, θ_j are irreducible characters of G . Then $\sum_{\alpha\in\Gamma}\chi_i^{\alpha}$, where $\Gamma=\Gamma(Q(\chi_i):Q)$, and

$\sum_{\alpha\in\Gamma_1}\theta_j^{\alpha}$, where $\Gamma_1=\Gamma(Q(\theta_j):Q)$, are rational valued characters of G.

$\chi_e=B(e)\sum_{\alpha\in\Gamma}\chi_i^{\alpha}$ where $e=(i,p^k-1)$.

$\theta_j=B(f)\sum_{\alpha\in\Gamma_1}\theta_j^{\alpha}$ where $f=(i,p^k+1)$.

ξ' and η' denote the irreducible characters of the rational representations of G arising from ξ_1 (or ξ_2) and η_1 (or η_2) respectively where k is even.

Also we know that the column for the class zc is obtained from the relation

$\chi(zc)=\frac{\chi(z)}{\chi(1)}\chi(c)$ where χ is an irreducible character of G.

So the character table of rational representations of SL(2,p^k), p an odd prime, k even is described in the following table:

and the character table of rational representations of SL$(2,p^k)$, p an odd prime, k odd is described in the following table:

Cg	1	z	c and d	$a^{e'}$	$b^{f'}$
| Cg |	1	1	$(p^{2k}-1)/2$	$p^k(p^k+1)$	$p^k(p^k-1)$
|CG(g)|	$p^k(p^{2k}-1)$	$p^k(p^{2k}-1)$	$2p^k$	p^k-1	p^k+1
1_G	1	1	1	1	1
ψ	p^k	p^k	0	1	-1
χ_e	$(p^k+1)A(e)B(e)$	$(-1)^e(p^k+1)A(e)B(e)$	$A(e)B(e)$	$A(e)B(e)\tau_1(e,e')$	0
θ_f	$(p^k-1)C(f)B(f)$	$(-1)^f(p^k-1)C(f)B(f)$	$-C(f)B(f)$	0	$-C(f)B(f)\tau_2(f,f')$
$\zeta_1+\zeta_2$	(p^k+1)	$\varepsilon(p^k+1)$	1	$(-1)^{e'}2$	0
$\eta_1+\eta_1$	$(p^k-1)E(p^k)$	$-\varepsilon(p^k-1)E(p^k)$	-1	0	$(-1)^{f'+1}2E(p^k)$

Cg	1	z	c and d	$a^{e'}$	$b^{f'}$
| Cg |	1	1	$(p^{2k}-1)/2$	$p^k(p^k+1)$	$p^k(p^k-1)$
|CG(g)|	$p^k(p^{2k}-1)$	$p^k(p^{2k}-1)$	$2p^k$	p^k-1	p^k+1
1_G	1	1	1	1	1
ψ	p^k	p^k	0	1	-1
χ_e	$(p^k+1)A(e)B(e)$	$(-1)^e(p^k+1)A(e)B(e)$	$A(e)B(e)$	$B(e)\tau_1(e,e')$	0
θ_f	$(p^k-1)C(f)B(f)$	$(-1)^f(p^k-1)C(f)B(f)$	$-C(f)B(f)$	0	$-B(f)\tau_2(f,f')$
$\zeta_1+\zeta_2$	(p^k+1)	$\varepsilon(p^k+1)$	1	$(-1)^{e'}2$	0
$\eta_1+\eta_1$	$(p^k-1)E(p^k)$	$-\varepsilon(p^k-1)E(p^k)$	-1	0	$(-1)^{f'+1}2E(p^k)$

Theorem 1.6 **:** [4]

Let G be a cyclic P–group. Then

$$K(G) = Z_p.$$

Theorem 1.7 **:** [4]

Let G be a cyclic group of order P^n. Then

$$K(G) = \bigoplus_{i=1}^{n} Z_{p^i}$$

§.2 The Cyclic Decomposition of $K(SL(2,p^k))$

In this section we will introduce the diagonalization of the matrix for the rational valued character table by using row and column operations with the condition (when we multiply row or column by a number the number must be integer).

Then we suppose that the diagonalization of this matrix for rational valued character table as

$$\begin{pmatrix} m_1 & 0 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 & 0 \\ 0 & 0 & m_3 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & m_n \end{pmatrix}$$

Then the cyclic decomposition for the group $K(SL(2,p^k))$ is:

$$K(SL(2,p^k)) = Z_{m_1} \oplus Z_{m_2} \oplus Z_{m_3} \oplus \ldots \oplus Z_{m_n} \qquad \ldots(*)$$

As special cases we consider the following:

(1) $p^k = 9 \Rightarrow |SL(2,9)| = 9(81-1) = 9(80) = 720$

e and e' are divisors of 8 which are 1, 2, 4, 8 such that e, $e' < 4$ so $e, e' = 1, 2$, f and f' are divisors of 10 which are 1, 2, 5, 10, such that f, $f' < 5$, so $f = f' = 1, 2$, and $\varepsilon = 1$, from that:

the conjugacy classes are 1, z, c, zc, a, a^2, b, and b^2

the characters are $1_G, \psi, \chi_1, \chi_2, \theta_1, \theta_2, \xi_1 + \xi_2 = \xi'$, and $\eta_1 + \eta_2 = \eta'$

$$A(1) = \frac{1}{2}\Phi(8) = 2,\quad A(2) = \frac{1}{2}\Phi(4) = 1, B(1) = 2,\ B(2) = 1,\quad C(1) = \frac{1}{2}\Phi(10) = 2,$$

$$C(2) = \frac{1}{2}\Phi(5) = 2,\qquad \tau_1(1,1) = \frac{\Phi(8)}{\Phi(8)}\mu(8) = 0,\qquad \tau_1(1,2) = \frac{\Phi(8)}{\Phi(4)}\mu(4) = 0,$$

$\tau_1(2,1) = \dfrac{\Phi(4)}{\Phi(4)}\mu(4) = 0$, by (lemma 1.5) and (corollary 1.6) ($i = 2$, $n = 14$, $d_i = 2$),

so $\tau_1(2,2) = \dfrac{\Phi(14)}{\Phi(7)}\mu(7) = -1, \tau_2(1,1) = \dfrac{\Phi(10)}{\Phi(10)}\mu(10) = 1, \tau_2(1,2) = \dfrac{\Phi(10)}{\Phi(5)}\mu(5) = -1,$

$\tau_2(2,1) = \dfrac{\Phi(5)}{\Phi(5)}\mu(5) = -1$, we find $\tau_2(2,2)$ by (lemma 1.5) and (corollary 1.6) so

$\tau_2(2,2) = \ = \dfrac{\Phi(5)}{\Phi(5)}\mu(5) = -1$, $E(9) = 1$.

By Schur indices (theorem 1.3) we divide χ_1, θ_1, and η' by 2, and others by 1 the character table of rational representation of SL(2,9) is:

C_g	1	z	c	zc	a	a^2	b	b^2
$\lvert C_g \rvert$	1	1	40	40	90	90	72	72
$\lvert C_G(g) \rvert$	720	720	18	18	8	8	10	10
1_G	1	1	1	1	1	1	1	1
ψ	9	9	0	0	1	1	-1	-1
χ_1	20	-20	2	-2	0	0	0	0
χ_2	10	10	1	1	0	-2	0	0
θ_1	16	-16	-2	2	0	0	-1	1
θ_2	16	16	-2	-2	0	0	1	1
ξ'	10	10	1	1	-2	2	0	0
η'	8	-8	-1	1	0	0	2	-2

Then by using elementary row and column operations, we get the following diagonal matrix

$$\begin{pmatrix} 720 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -180 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

Thus by (*), we obtained

$$K(SL(2,9)) = Z_{720} \oplus Z_{180} \oplus Z_1 \oplus Z_2 \oplus Z_1 \oplus Z_2 \oplus Z_1 \oplus Z_2$$

(2) $p^k = 25 \Rightarrow |SL(2,25)| = 25\,(625-1) = 27\,(624) = 15600$

e and e' are divisors of 24 which are 1, 2, 3, 4, 6, 8, 12, 24 such that e, $e' < 12$ so $e, e' = 1, 2, 3, 4, 6, 8$, f and f' are divisors of 26 which are 1, 2, 13, 26, such that $f, f' < 13$, so $f = f' = 1, 2$, $\varepsilon = 1$, from that:

the conjugacy classes are $1, z, c, zc, a, a^2, a^3, a^4, a^6, a^8, b$ and b^2

the characters are $1_G, \psi, \chi_1, \chi_2, \chi_3, \chi_4, \chi_6, \chi_8, \theta_1, \theta_2, \xi_1 + \xi_2 = \xi'$, and $\eta_1 + \eta_2 = \eta'$

$A(1) = \frac{1}{2}\Phi(24) = 4$, $A(2) = \frac{1}{2}\Phi(12) = 2$, $A(3) = \frac{1}{2}\Phi(8) = 2$, $A(4) = \frac{1}{2}\Phi(6) = 1$,

$B(1) = 2$, $B(2) = 1$, $B(3) = 2$, $B(4) = 1$, $B(7) = 2$, $C(1) = \frac{1}{2}\Phi(26) = 6$,

$C(2) = \frac{1}{2}\Phi(13) = 6$, $\tau_1(1,1) = \frac{\Phi(24)}{\Phi(24)}\mu(24) = 0$, $\tau_1(1,2) = \frac{\Phi(24)}{\Phi(12)}\mu(12) = 0$,

$\tau_1(1,3) = \frac{\Phi(24)}{\Phi(8)}\mu(8) = 0$, $\tau_1(1,4) = \frac{\Phi(24)}{\Phi(6)}\mu(6) = 4$, $\tau_1(1,6) = \frac{\Phi(24)}{\Phi(4)}\mu(4) = 0$,

$\tau_1(1,8) = \frac{\Phi(24)}{\Phi(3)}\mu(3) = -4$, $\tau_1(2,1) = \frac{\Phi(12)}{\Phi(12)}\mu(12) = 0$, $\tau_1(2,2) = \frac{\Phi(12)}{\Phi(6)}\mu(6) = 2$,

$\tau_1(2,3) = \frac{\Phi(12)}{\Phi(4)}\mu(4)=0$, $\tau_1(2,4) = \frac{\Phi(12)}{\Phi(3)}\mu(3)=-2$, $\tau_1(2,6) = \frac{\Phi(12)}{\Phi(4)}\mu(4)=0$,

$\tau_1(2,8) = \frac{\Phi(12)}{\Phi(3)}\mu(3)=-2$, $\tau_1(3,1) = \frac{\Phi(8)}{\Phi(8)}\mu(8)=0$, $\tau_1(3,2) = \frac{\Phi(8)}{\Phi(4)}\mu(4)=0$,

by (lemma 1.5) and (corollary 1.6) (i =3, n= 8, d_i = 1), $\tau_1(3,3) = \frac{\Phi(8)}{\Phi(8)}\mu(8)=0$,

$\tau_1(3,4) = \frac{\Phi(8)}{\Phi(2)}\mu(12)=-4$, $\tau_1(3,6) = \frac{\Phi(8)}{\Phi(4)}\mu(4)=0$, $\tau_1(3,8) = \frac{\Phi(8)}{\Phi(3)}\mu(3)=-2$,

$\tau_1(4,1) = \frac{\Phi(6)}{\Phi(6)}\mu(6)=1$, $\tau_1(4,2) = \frac{\Phi(6)}{\Phi(3)}\mu(3)=-1$, $\tau_1(4,3) = \frac{\Phi(6)}{\Phi(2)}\mu(2)=-2$, by (lemma 1.5) and (corollary 1.6) (i = 4, n = 6,d_i = 2) so $\tau_1(4,4) = \frac{\Phi(6)}{\Phi(3)}\mu(3)=-1$,

$\tau_1(4,6) = \frac{\Phi(6)}{\Phi(4)}\mu(4)=0$, $\tau_1(4,8) = \frac{\Phi(6)}{\Phi(3)}\mu(3)=-1$, $\tau_1(6,1) = \frac{\Phi(4)}{\Phi(4)}\mu(4)=0$,

$\tau_1(6,2) = \frac{\Phi(4)}{\Phi(2)}\mu(2)=-2$, by (lemma 1.5) and (corollary 1.6) (i = 6, n = 4,d_i = 2) so $\tau_1(6,3) = \frac{\Phi(4)}{\Phi(2)}\mu(2)=-2$, $\tau_1(6,4) = \frac{\Phi(4)}{\Phi(2)}\mu(2)=-2$, by (lemma 1.5) and (corollary 1.6) (i = 6, n = 4,d_i = 2) so $\tau_1(6,6) = \frac{\Phi(4)}{\Phi(2)}\mu(2)=-2$, by (lemma 1.5) and (corollary 1.6) (i = 6, n = 4,d_i = 2) so $\tau_1(6,8) = \frac{\Phi(4)}{\Phi(2)}\mu(2)=-2$,

$\tau_1(8,1) = \frac{\Phi(3)}{\Phi(3)}\mu(3)=-1$, by (lemma 1.5) and (corollary 1.6) (i = 8,n = 3,d_i = 1) so $\tau_1(8,2) = \frac{\Phi(3)}{\Phi(3)}\mu(3)=-1$, $\tau_1(8,3) = \frac{\Phi(3)}{\Phi(3)}\mu(3)=-1$, by (lemma 1.5) and (corollary 1.6) (i = 8, n = 3,d_i = 1) so $\tau_1(8,4) = \frac{\Phi(3)}{\Phi(3)}\mu(3)=-1$, by (lemma 1.5) and (corollary 1.6) (i = 8, n= 3,d_i = 1) so $\tau_1(8,6) = \frac{\Phi(3)}{\Phi(3)}\mu(3)=-1$, by (lemma 1.5) and (corollary 1.6) (i = 8, n = 3, d_i = 1) so $\tau_1(8,8) = \frac{\Phi(3)}{\Phi(3)}\mu(3) = -1$,

$\tau_2(1,1) = \frac{\Phi(26)}{\Phi(26)}\mu(26) = 1, \tau_2(1,2) = \frac{\Phi(26)}{\Phi(13)}\mu(13) = -1, \tau_2(2,1) = \frac{\Phi(13)}{\Phi(13)}\mu(13) = -1$,

by (lemma 1.5) and (corollary 1.6) (i = 2, n = 13, d_i = 1) so

$\tau_2(2,2) = \frac{\Phi(13)}{\Phi(13)}\mu(13) = -1$, E(25) = 2.

By Schur indices (theorem 1.3) we divide χ_1, χ_3, and θ_1 by 2, and others by 1 the character table of rational representation of SL(2,25) is:

C_g	1	z	c	zc	a	a^2	a^3	a^4	a^6	a^8	b	b^2
$\lvert C_g \rvert$	1	1	312	312	650	650	650	650	650	650	600	600
$\lvert C_G(g) \rvert$	15600	15600	50	50	24	24	24	24	24	24	26	26
1_G	1	1	1	1	1	1	1	1	1	1	1	1
ψ	25	25	0	0	1	1	1	1	1	1	-1	-1
χ_1	104	-104	4	-4	0	0	0	4	0	-4	0	0
χ_2	52	52	2	2	0	2	0	-2	-4	-2	0	0
χ_3	52	-52	2	-2	0	0	0	-4	0	4	0	0
χ_4	26	26	1	1	1	-1	-2	-1	2	-1	0	0
χ_6	26	26	1	1	0	-2	0	2	-2	2	0	0
χ_8	26	26	1	1	-1	-1	2	-1	2	-1	0	0
θ_1	144	-144	-6	6	0	0	0	0	0	0	-1	1
θ_2	144	144	-6	-6	0	0	0	0	0	0	1	1
ξ'	26	26	1	1	-2	2	-2	2	2	2	0	0
η'	24	-24	-1	1	0	0	0	0	0	0	2	-2

Then by using elementary row and column operations, we get the following diagonal matrix

$$\begin{pmatrix} 15600 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -3900 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 \end{pmatrix}$$

Thus by (*), we obtained

$K(SL(2,25)) = Z_{15600} \oplus Z_{3900} \oplus Z_2 \oplus Z_1 \oplus Z_2 \oplus Z_1 \oplus Z_1 \oplus Z_1 \oplus Z_6 \oplus Z_4 \oplus Z_1 \oplus Z_2$

(3) $p^k = 27 \Rightarrow |SL(2,27)| = 27(729-1) = 27(728) = 19656$

e and e' are divisors of 26 which are 1, 2, 13, 26 such that e, $e' < 13$ so $e, e' = 1, 2$, f and f' are divisors of 28 which are 1, 2, 4, 7, 14, 28, such that $f, f' < 14$, so $f = f' = 1, 2, 4, 7$ and $\varepsilon = -1$, from that:

the conjugacy classes are $1, z, c, zc, a, a^2, b, b^2, b^4$ and b^7

the characters are 1_G, ψ, χ_1, χ_2, θ_1, θ_2, θ_4, θ_7, $\xi_1 + \xi_2 = \xi'$, and $\eta_1 + \eta_2 = \eta'$

$A(1) = \frac{1}{2}\Phi(26) = 6$, $A(2) = \frac{1}{2}\Phi(13) = 6$, $B(1) = 2$, $B(2) = 1$, $B(4) = 1$, $B(7) = 2$,

$C(1) = \frac{1}{2}\Phi(28) = 6$, $C(2) = \frac{1}{2}\Phi(14) = 3$, $C(4) = \frac{1}{2}\Phi(7) = 3$, $C(7) = \frac{1}{2}\Phi(4) = 1$

$\tau_1(1,1) = \frac{\Phi(26)}{\Phi(26)}\mu(26) = 1$, $\tau_1(1,2) = \frac{\Phi(26)}{\Phi(13)}\mu(13) = -1$, $\tau_1(2,1) = \frac{\Phi(13)}{\Phi(13)}\mu(13) = -1$,

$\tau_1(2,2) = \frac{\Phi(4)}{\Phi(2)}\mu(2) = -1$, $\tau_2(1,1) = \frac{\Phi(28)}{\Phi(28)}\mu(28) = 0$, $\tau_2(1,2) = \frac{\Phi(28)}{\Phi(14)}\mu(14) = 2$,

$\tau_2(1,4) = \frac{\Phi(28)}{\Phi(7)}\mu(7) = -2$, $\tau_2(1,7) = \frac{\Phi(28)}{\Phi(4)}\mu(14) = 0$, $\tau_2(2,1) = \frac{\Phi(14)}{\Phi(14)}\mu(14) = 1$,

$\tau_2(2,2) = \dfrac{\Phi(14)}{\Phi(7)}\mu(7) = -1$, by (lemma 1.5) and (corollary 1.6) ($i = 2$, $n = 14$, $d_i = 2$

$\tau_2(2,4) = \dfrac{\Phi(14)}{\Phi(7)}\mu(7) = -1$, $\tau_2(2,7) = \dfrac{\Phi(14)}{\Phi(14)}\mu(14) = 1$, $\tau_2(4,1) = \dfrac{\Phi(7)}{\Phi(7)}\mu(7) = -1$,

by (lemma 1.5) and (corollary 1.6) ($i = 4$, $n = 7$, $d_i = 1$) so $\tau_2(4,2)$ $= \dfrac{\Phi(7)}{\Phi(7)}\mu(7) = -1$, by (lemma 1.5) and (corollary 1.6) ($i = 4$, $n = 7$, $d_i = 1$) so

$\tau_2(4,4) = \dfrac{\Phi(7)}{\Phi(7)}\mu(7) = -1$, by (lemma 1.5) and (corollary 1.6) ($i = 4$, $n = 7$, $d_i = 1$)

so $\tau_2(4,7) = \dfrac{\Phi(7)}{\Phi(7)}\mu(7) = -1$, $\tau_2(7,1) = \dfrac{\Phi(4)}{\Phi(4)}\mu(4) = 0$, $\tau_2(7,2) = \dfrac{\Phi(4)}{\Phi(2)}\mu(2) = -2$,

by (lemma 1.5) and (corollary 1.6) ($i = 7$, $n = 4$, $d_i = 1$) so $\tau_2(7,4)$ $= \dfrac{\Phi(4)}{\Phi(4)}\mu(4) = 0$, by (lemma 1.5) and (corollary 1.6) ($i = 7$, $n = 4$, $d_i = 1$) so

$\tau_2(7,7) = \dfrac{\Phi(4)}{\Phi(4)}\mu(4) = 0$, $E(27) = 1$.

By Schur indices (theorem 1.3) we divide χ_1, θ_1, and θ_7 by 2, and others by 1 the character table of rational representation of SL(2,27) is:

C_g	1	z	c	zc	a	a^2	b	b^2	b^4	b^7
$\lvert C_g \rvert$	1	1	364	364	756	756	702	702	702	702
$\lvert C_G(g) \rvert$	19656	19656	54	54	26	26	28	28	28	28
1_G	1	1	1	1	1	1	1	1	1	1
ψ	27	27	0	0	1	1	-1	-1	-1	-1
χ_1	168	-168	6	-6	1	-1	0	0	0	0
χ_2	168	168	6	6	-1	-1	0	0	0	0
θ_1	156	-156	-6	6	0	0	0	-2	2	0
θ_2	78	78	-3	-3	0	0	-1	1	1	-1
θ_4	78	78	-3	-3	0	0	1	1	1	1
θ_7	26	-26	-1	1	0	0	0	2	0	0
ξ'	28	-28	1	-1	-2	2	0	0	0	0
η	26	26	-1	-1	0	0	2	-2	2	-2

Then by using elementary row and column operations, we get the following diagonal matrix

$$\begin{pmatrix} 19656 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -4914 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4 \end{pmatrix}$$

Thus by (*), we obtained

$$K(SL(2,27)) = Z_{19656} \oplus Z_{4914} \oplus Z_1 \oplus Z_7 \oplus Z_1 \oplus Z_2 \oplus Z_2 \oplus Z_1 \oplus Z_2 \oplus Z_4$$

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الفهرست

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