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DYNAMICAL STABILITY OF CHARGED ISENTROPIC SUPERDENSE STAR MODEL

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Abstract: The objective of this paper is to study the stability of charged isentropic superdense star models. A limitation of the density variation for different models guided by a specific choice of measure of departure from physical geometry of the physical space to ensure the physical acceptability has been obtained and analyzed for the stability performance. The solution so obtained has investigates large density and pressure at the center of the super dense star model; however the energy conditions are seen to be satisfied throughout certain spherical regions. In addition to that, the analysis yields a strong indication that the model is stable with respect to infinitesimal redial pulsation. We also found that the adiabatic speed of sound is smaller than unity inside the fluid sphere if and only if the radius of the sphere is larger than 1.46 times of its Schwarzschild radius. Furthermore the solution for *K= -11* (case study), has been tested for the stability and found it is stable for least admissible value $\lambda(r) = \dfrac{\rho_a}{\rho_0} = 0.3$.

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1 Introduction

In general relativity and allied theories, the distribution of the mass, momentum, and stress due to matter and to any non-gravitational fields is described by the energy-momentum-tensor (or *matter tensor*) T^{ab}. However, the Einstein field equation is not very choosy about what kinds of states of matter or non-gravitational fields are admissible in a space-time model. It is well known that the static, spherically symmetric, uncharged fluids cannot be held in equilibrium below a certain radius without developing singularities inside [6]. The possibility of holding a non-singular object in stable equilibrium but not compact enough to be close to a black hole state, is of great interest not only to judge the state of matter in this condition, that is being about to turn into black hole, but also to yield a classic model of charged massive particles which might have astrophysical and cosmological implications.

For the last four decades researchers have been busy in deriving solutions for charged fluid spheres to provide source of Reissener (1916) and Nordstrom (1918) solutions [7]. Such fluid models are not likely to undergo gravitational collapse to reduce into a singularity point, in presence of charges. The gravitational attraction may be nullified by electrostatic repulsion and pressure gradient. Several research papers have studied charged fluids spheres in different contexts such as Tikekar (1990), Gupta et al (1986), Ray et al (2003). Moodely et al (2003) found a class of accelerating, expanding and shearing solutions which is characterized geometrically by conformal Killing vector. Gupta, et al (2005, 2011), considered the charged case of Vaidya-Tikekar type solutions, then followed have charged Buchdahl's fluid spheres.

In the present article, the stability of charged superdense star has been investigated and analyzed with respect to the reality conditions. Runge-Kutta method has been implemented for our proposed model and found to satisfy various physical conditions.

2 Basic Equations

In standard coordinates $x^i = (t, r, \theta, \phi)$, the general line element for a charged fluid sphere model takes the form

$$ds^2 = -e^{\lambda(r)} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2\right) + e^{\nu(r)} dt^2 , \tag{1}$$

where

$$\lambda(r) = \ln \left(\left(1 - K \frac{r^2}{R^2}\right) \Big/ \left(1 - \frac{r^2}{R^2}\right) \right) \tag{1a}$$

and $\nu(r) = \ln y^2 = \ln\left(\left(1 - K \frac{r^2}{R^2}\right)(1-K)\right)^3$ (1b)

The static spherical symmetric space-time with t=constant hypersurfaces as spheroid can be written as:

$$ds^2 = -\frac{1 - K(\frac{r^2}{R^2})}{1 - \frac{r^2}{R^2}} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2\right) + y^2(r) dt^2 \tag{2}$$

Equation (2) is regular and positive definite at all points r<R.

If Eq. (2) describes charged fluid distribution then the space-time satisfied by equation (1) has to satisfy the following Einstein –Maxwell equation:

$$R^i_j - \frac{1}{2} R \delta^i_j = -8\pi \frac{G}{C^4} T^i_j$$

where C=1, G=1 and $T^i_j = M^i_j + E^i_j$ (3)

In the interior M_j^i can be described in terms of isentropic pressure P and the mass density ρ ; it takes the form:

$$M_j^i = (P+\rho)u_i u^i - P\delta_j^i \tag{4}$$

where $u^i = \left(0,0,0,e^{-\nu/2}\right)$ (5)

while, E_j^i ,is the electromagnetic contribution to the stress energy tensor; it can be written as:

$$E_j^i = -\frac{1}{4\pi}\left(F^{im}F_{jm} - \frac{1}{4}\delta_j^i F_{mn}F^{mn}\right) \tag{6}$$

F_{ik} , being the skew symmetric electromagnetic field tensor satisfying the following Maxwell equations

$$F_{ik,j} + F_{kj,i} + F_{ji,k} = 0 \tag{7}$$

$$\frac{\partial}{\partial x^k}\left(\sqrt{-g}F^{ik}\right) = -4\pi\sqrt{-g}\, j^i \tag{8}$$

where $j^i = \sigma v^i$ represents the four-current vector of charged fluid with σ as the charged density.

In view of Equations (1), (2) and (3), the field equation can be furnished as:

$$8\pi\rho = \frac{\lambda'}{r}e^{-\lambda} + \frac{\left(1-e^{-\lambda}\right)}{r^2} - 8\pi\left(E_j^i\right)^2 \tag{9}$$

$$8\pi P = \frac{\nu'}{r}e^{-\lambda} - \frac{\left(1-e^{-\lambda}\right)}{r^2} + 8\pi\left(E_j^i\right)^2 \tag{10}$$

$$8\pi P = \left[\frac{v''}{2} - \frac{\lambda' v'}{4} + \frac{v'^2}{4} + \frac{v' - \lambda'}{2r} \right] e^{-\lambda} - 8\pi \left(E^i_j \right)^2 \tag{11}$$

where, prime denotes the differentiation with respect to r and [10]

$$\sigma^i_j = 4\pi \int_0^r \sigma r^2 e^{\lambda/2} dr = r^2 \sqrt{-F_{14} F^{14}} = r^2 F^{41} e^{(\lambda+\nu)/2} \tag{12}$$

which represents the total charge contained within the sphere of radius $r=a$.

We have proposed a charged fluid distribution by considering the electric field intensity (Gupta, et al 2005)

$$\frac{q^2}{r^4} = \frac{K^2 r^2 \gamma^2}{2R^2 \left(K + R^2 \right)} \tag{13}$$

where $K \,\&\, \gamma$ are constants to be determined.

The consistency of the field Equations (9)-(11) using (1a) & (1b) yields the following hypergeometric equation

$$(1 - X^2) \frac{d^2 y}{dX^2} + X \frac{dy}{dX} + \left(1 - K + K\beta^2 \right) y = 0 \tag{14}$$

where $X = \sqrt{\frac{K}{K-1}} \sqrt{1 + \frac{Cr^2}{K}}$, $K < 0$ *or* $K > 1$ and $y = \sqrt{e^{\nu}}$

Equation (14) can be solved exactly for two cases:

Case I: Null charged by putting $\beta = 0$ (Gupta-Jasim, 2000 and 2003) [3, 4]

Case II: for charged case $\beta \neq 0$, the case was discussed by Athraa-Jasim (2004), and latter on by Gupta-Mukesh (2005, 2011), whom discussed some physical properties of such case.

3 Dynamical Stability of Charged Fluid Sphere

The basic method for examining whether a relativistic charged fluid sphere is stable with respect to infinitesimal radial adiabatic pulsations has been developed by Chandrasekhar (1964).

Athraa-Jasim (2004) obtained their solution by demanding that the energy-momentum tensor was that of a perfect fluid. However, the same static solution is in fact obtained assuming only that the static energy-momentum tensor is given by:

$$T_i^{\,j} = (-P, -P, -P, \rho) \tag{15}$$

Now, the investigating of stability requires studying a dynamical object, and to describe its behavior needs to know the non-static energy momentum tensor. We restrict our analysis to the case where the energy-momentum tensor is given by that of a perfect fluid, i.e.

$$T_j^i = (P+\rho)u_i u^i - P g_j^i \tag{16}$$

To perform the stability analysis, we restrict our examination to the case where the fluid is isentropic under static conditions. This restriction has also, in fact, been done by Vaidya and Tikekar (1982). Since the speed of sound given by $\frac{dP}{d\rho}$ should be less than the velocity of light inside or on charged fluid spheres; the fluid is isentropic, i.e., if the entropy per baryon is constant everywhere, since we generally have:

$$v^2{}_{sound} = \left(\frac{dP}{d\rho}\right)_s , \tag{17}$$

where s denotes the entropy per baryon. The fluid flow is isentropic for a perfect fluid, it is thus constant everywhere as in the static case. Our analysis is thus valid at absolute zero (white dwarfs, neutron stars) or a star in convective equilibrium (super massive star).

Barden, et al (1966) used the pulsation equation for the line element of Chandrasekhar's (1964) as:

$$\sigma^2 \int_0^a e^{(3\lambda+v)/2} \frac{(\rho+p)u^2}{r^2}\, dr =$$
$$\int_0^a e^{\frac{(3\lambda+v)}{2}} \frac{(\rho+p)}{r^2} * \left\{ \left[-\frac{2}{r}v' - \frac{1}{4}v'^2 + 8\pi p e^{\lambda} \right] u^2 + \frac{dp}{d\rho} u'^2 \right\} dr \tag{18}$$

where $u = \xi(r) r^2 e^{-v/2}$

The relativistic adiabatic index γ is given by

$$\gamma = \frac{p+\rho}{p} \frac{dp}{d\rho} \tag{19}$$

The adiabatic index γ should be larger than unity for temperature away from the center, or to be larger than 4/3 which is the necessary but not sufficient condition to prevent the instability under the radial perturbation.

The pulsation equation for Athraa-Jasim model (2004) (tested case *K= -11*) provides the following data for the adiabatic index γ inside the star at $\lambda = 0.3$ for charged γ_c and null charged γ_{nc} spheres.

$x = \frac{a^2}{R^2}$	$\gamma_c = \frac{p+\rho}{p} \frac{dp}{d\rho}$	$\gamma_{nc} = \frac{p+\rho}{p} \frac{dp}{d\rho}$
1	∞	∞
0.9	8.4726	7.3615
0.8	5.9932	4.2021
0.7	5.1264	3.1425
0.6	4.0262	2.6064
0.5	3.8234	2.2811
0.4	3.4244	2.0653
0.3	3.3.121	1.9179

0.2	3.1798	1.8205
0.1	3.1674	1.7646
0	3.1103	1.7463

Equation (18) has been integrated numerically for different values of $x = \frac{a^2}{R^2} \leq 0.5$ to fulfill the physical properties as follows:

$x = \frac{a^2}{R^2}$	**Value of integral**
0.5	0.1871×10^{-1}
0.4	0.2372×10^{-1}
0.3	0.2845×10^{-1}
0.2	0.2841×10^{-1}
0.1	0.2017×10^{-1}

Summing up, to ensure the physical properties the following figures show the behaviour of standard physical quantities inside the Star for the least admissible values of λ(denoted as *L* in graphs).

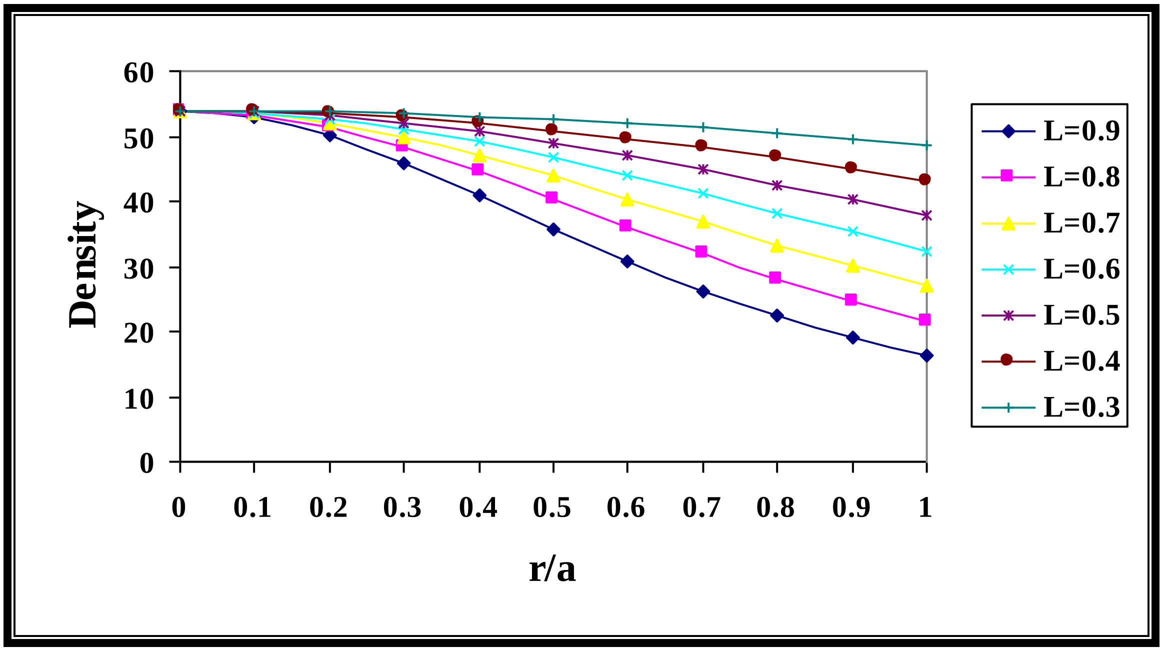

Fig.(1) Shows the Behavior of density inside the fluid sphere at (K = –11).

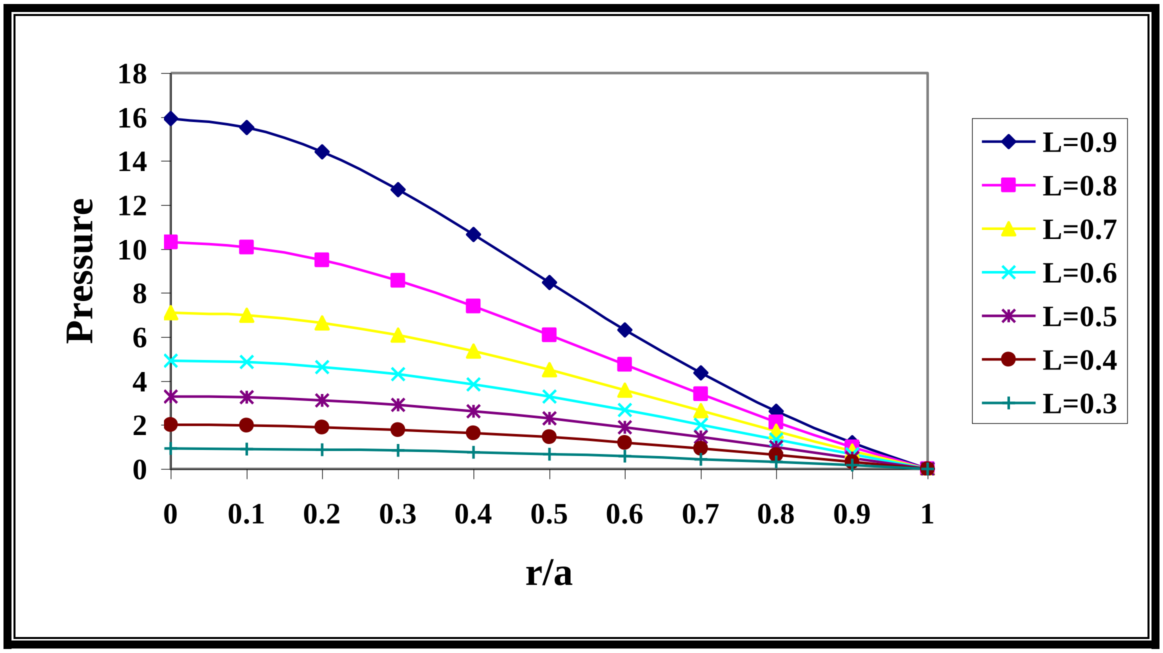

Fig.(2) Shows the Behaviour of pressure inside the fluid sphere at (K = –11).

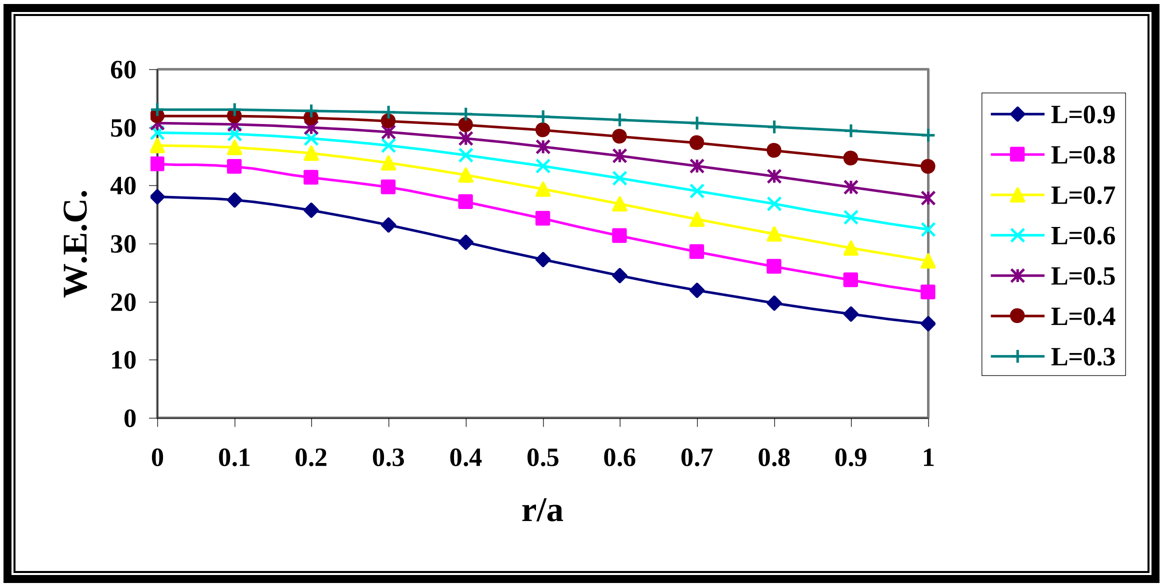

Fig.(3) Shows the Behaviour of W.E.C. inside the fluid sphere at (K=–11).

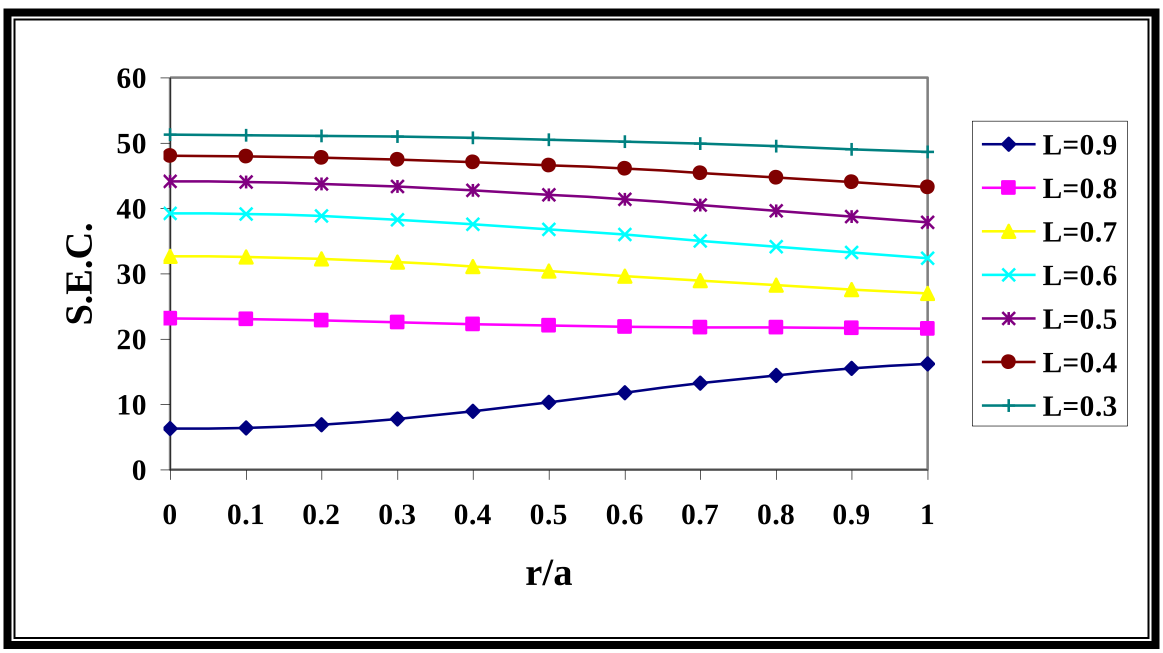

Fig..4) Shows the Behaviour of S.E.C. inside the fluid sphere at (K=–11).

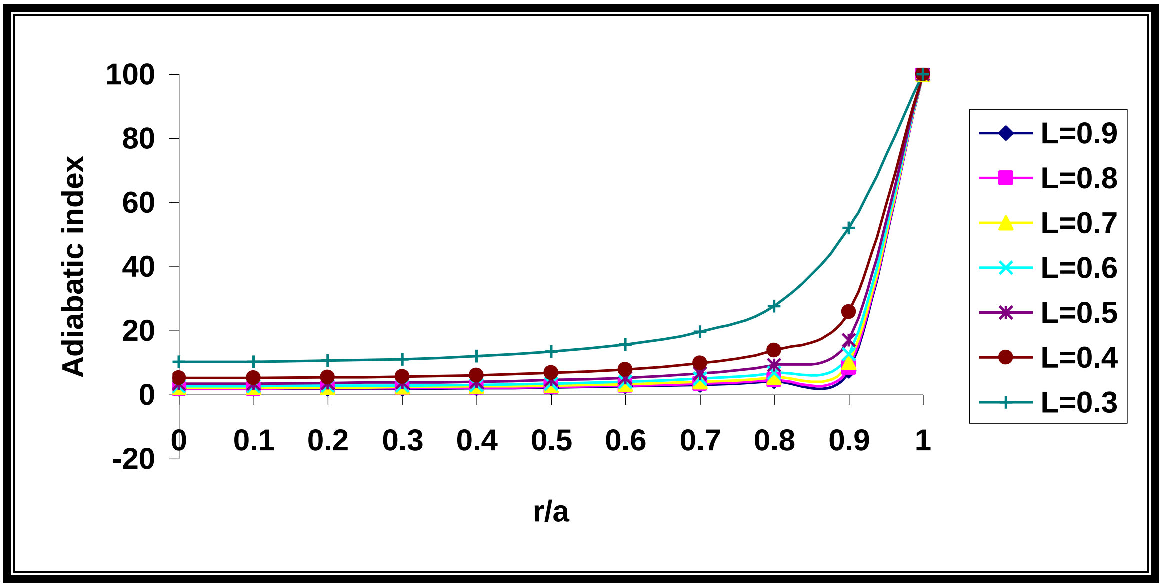

Fig.(5) Shows the Behaviour of adiabatic index inside the fluid sphere at (K= –11).

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