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# Design mathematical model for studying properties thin layers

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**Abstract.** In this paper, highlighted the study of properties thin films of squeeze film characteristics The Reynolds equation a commutating for speed molecules and squeeze action is mathematically derived. The Reynolds equation is disband for the hydrodynamic pressure and the bearing characteristics, such as load carrying capacity and time approach. It is found increases speed molecules for first layers lead to decrease hydrodynamic pressure and load compared to final layers case.

## Nomenclature

H	Film thickness	x, y	Coordinates along the direction
B	Porosity of thin film	P	Pressure in thin film
U	Sliding motion	$\rho$	Density
V	Squeeze action	$\mu$	Viscosity of thin film
$U_s$	Infusion of molecules	$f_h$	Flexibility of the thin film
$h_m$	Minimum film thickness	R	Radius
W	Load carrying capacity	T	Time flow

## 1. Introduction

Thin films are described by thin layers of materials whose dimensions range from nanometres to several micrometres in thickness. They are added to the surface of the material in order to add properties that were not there before, such as durability, various loads, reducing friction and corrosion. This technology is used in the manufacture of semiconductors and in coatings [1]. They are exercised to memorize surfaces from wear, recover lubricity, improve attrition and chemical resistance, modify optical and electrical properties. Thin films deposition technology and the science have progressed rapidly in the direction of engineered thin film coatings have surface engineering [2]. Plasmas are used more extensively. Accordingly, advanced thin film deposition processes have been developed and new technologies have been adapted to conventional deposition processes. The market and applications for thin film coating have also increased astronomically, particularly in the biomedical, display and energy fields. Thin films have distinct advantages on porosity materials [3].



A thin film is a structure whose dimensions are such that it has a substantially large surface to volume ratio. For example, while the structure may be macroscopically large in length and width, it may have a thickness that is only on the order of a micron or less. Thin films do not have to be planar. The properties of such thin film structures are strongly influenced by the surface properties and may be very different from that of the same material in bulk form. The thin films may consist of a pure material, or a composite or a layered structure, and several of thin films may be present in a more complex device. Each thin film feature is dependent on the porosity process and can be modified and not all processes produce materials with the same porosity and permeability of thin layers. Therefore, we found each thin layers have different load carry capacity, time approach that transfer film thickness to minimum film thickness [4].

## 2. Analysis

Based on Reynolds' equation, we can study the effect of some factors on the work of thin films, our study is on thin films in a case poiseuille flow when the surface fixed, now the Reynolds' equation is

$$\frac{\partial}{\partial x} \left( \frac{ph^3}{12\mu} \frac{dp}{dx} \right) + \frac{\partial}{\partial y} \left( \frac{ph^3}{12\mu} \frac{\partial p}{\partial y} \right) = \rho \frac{(U_a + U_b)}{2} \frac{\partial h}{\partial x} + (V_2 - V_1) + h \frac{\partial p}{\partial z} + F_h \quad (1)$$

The first and two terms of the equation (1) are the poiseuille flow. The three is hydrodynamic and four terms squeeze film. The five is local expansion. In addition, the six is flexibility of the thin film, we need to satisfy the following boundary condition, at the inlet  $P=0$  & At the outlet  $P = \frac{dp}{dx} = 0$ , the motion assumed as pure sliding and squeeze term and applying the last assumptions equation (2.1) becomes

$$\frac{d}{dx} \left( \frac{h^3}{\mu} \frac{dp}{dx} \right) = 6(U) \frac{dh}{dx} + 12(V) + 12F_h \quad (2)$$

Such that  $U=(U_a + U_b)$  &  $V=(V_2 - V_1)$ ,

Equation above can be integrated with consideration to  $x$  to give

$$\frac{h^3}{\mu} \frac{dp}{dx} = 6Uh + 12Vx + 12F_h x + A \quad (3)$$

Such that  $A$  is constant. So as to solve the Reynolds equation the integral  $A$  should be determined depended on boundary condition  $\frac{dp}{dx} = 0$  at  $h = h_0$ ,  $x = 0$ . Hence  $A = -6Uh_0$  replace this into equation (3) gives

$$\frac{dp}{dx} = 6U\mu \frac{(h-h_0)}{h^3} + \frac{12V\mu x}{h^3} + 12 \frac{F_h\mu}{h^3} x \quad (4)$$

$$h = h_0 + \frac{x^2}{2R} \quad (5)$$

Where  $h_0$  is film thickness &  $x$  is: coordinate

Now replace equation (5) in equation (4)

$$\frac{dp}{dx} = 6U\mu \frac{\left( \left( h_0 + \frac{x^2}{2R} \right) - \left( h_0 + \frac{x^2}{2R} \right) \right)}{\left( h_0 + \frac{x^2}{2R} \right)^3} + \frac{12V\mu}{\left( h_0 + \frac{x^2}{2R} \right)^3} x + \frac{12F_h\mu}{\left( h_0 + \frac{x^2}{2R} \right)^3} x \quad (6)$$

$$\tan \beta = \frac{x}{\sqrt{2Rh_0}} \quad (7)$$

Replace equation (7) in equation (5) yields to

$$h = h_0(1 + \tan^2 \beta) \quad (8)$$

$$h = h_0 \sec^2 \beta \quad (9)$$

Now distinguishing  $x$  and  $p$  with respect to  $\beta$  for equation (7)

$$\frac{\partial x}{\partial \beta} = \sqrt{2Rh_0} \sec^2 \beta \quad (10)$$

$$\partial p = \left[ 6U\mu \frac{(h_0 \sec^2 \beta - h_0 \sec^2 \bar{\beta})}{h_0^3 \sec^6 \beta} + \frac{12V\mu}{h_0^3 \sec^6 \beta} x + \frac{12F_h\mu}{h_0^3 \sec^6 \beta} x \right] * \sqrt{2Rh_0} \sec^2 \beta \partial \beta \quad (11)$$

### 2.1. Dimensionless squeeze film pressure

To write modified Reynolds equation governing in the film pressure introducing the following dimensionless variables

$$P^* = \frac{PH_0^2}{\mu U_0 R}, h_0^* = \frac{h_0}{H_0}, U^* = \frac{U}{U_0}, V = \frac{V}{U_0}$$

Stratify the above dimensionless formula in equation (11). Thus, the conclusive dimensionless form of the modified Reynolds becomes:

$$\partial P^* = \frac{6\sqrt{2}}{(h_0^*)^{\frac{3}{2}}} U^* \left[ \frac{\partial \beta}{\sec^2 \beta} - \frac{\sec^2 \bar{\beta}}{\sec^4 \beta} \right] \partial \beta + \frac{12\sqrt{2}}{(h_0^*)^{\frac{3}{2}}} x^* (V^* - U_s^*) \frac{d\beta}{\sec^4 \beta} \quad (12)$$

The dimensionless. Pressure equation become

$$P^* = \frac{6\sqrt{2}U^*}{(h_0^*)^{\frac{3}{2}}} \left[ \frac{13}{2} + \frac{\sin 2\beta}{4} \right] - \frac{6\sqrt{2}U^*}{(h_0^*)^{\frac{3}{2}}} \left[ \sec^2 \bar{\beta} \left[ \frac{3}{8}\beta + \frac{\sin 2\beta}{4} + \frac{\sin 4\beta}{32} \right] + \frac{12V_2}{(h_0^*)^{\frac{5}{2}}} x^* (V^* - U_s^*) \left[ \frac{3}{8}\beta + \frac{1}{4}\sin 2\beta + \frac{1}{32}\sin 4\beta \right] \right] + k \quad (13)$$

Applying the boundary conditions  $P^* = 0, \beta = 0$  on equations (13) hence, it was gained the integration fixed  $k = 0$

$$P^* = \frac{6\sqrt{2}U^*}{(h_0^*)^{\frac{3}{2}}} \left[ \frac{13}{2} + \frac{\sin 2\beta}{4} \right] - \frac{6\sqrt{2}U^*}{(h_0^*)^{\frac{3}{2}}} \left[ \sec^2 \bar{\beta} \left[ \frac{3}{8}\beta + \frac{\sin 2\beta}{4} + \frac{\sin 4\beta}{32} \right] + \frac{12V_2}{(h_0^*)^{\frac{5}{2}}} x^* (V^* - U_s^*) \left[ \frac{3}{8}\beta + \frac{1}{4}\sin 2\beta + \frac{1}{32}\sin 4\beta \right] \right] \quad (14)$$

### 2.2. Dimensionless load carrying capacity

In general, the load carrying capacity is obtained by integration of the pressure distribution

$$W = 2\pi \int_0^L H_0 P \, dx \quad (15)$$

Introduce the non-dimensionless load carrying capacity in consideration

$$W^* = \frac{W}{\mu U_0 R} \quad (16)$$

Replace equation (16) inside (15) and we integrate dimensionless pressure with assessment to the dimensionless  $x$  and thus we get

$$W^* = 2\pi \int_0^L P^* \, dx^* \quad (17)$$

$$W^* = \frac{12\pi\sqrt{2}U^*L}{(h_0^*)^{\frac{3}{2}}} \left( \frac{\beta}{2} + \frac{\sin 2\beta}{4} \right) + \frac{12\pi\sqrt{2}U^*L}{(h_0^*)^{\frac{3}{2}}} \sec^2 \bar{\beta} \left( \frac{3\beta}{8} + \frac{\sin 2\beta}{4} + \frac{\sin 4\beta}{23} \right) + \frac{12\sqrt{2}L^2}{(h_0^*)^{\frac{5}{2}}} (V^* - U_s^*) \left( \frac{3\beta}{8} + \frac{\sin 2\beta}{4} + \frac{\sin 4\beta}{32} \right) \quad (18)$$

### 2.3. Dimensionless time

By integrated of load carrying capacity, we get on time

$$T^* = \int_{h_m^*}^3 W^* dh_m^* \tag{19}$$

$$T^* = \frac{36\pi\sqrt{2}LU^*}{(h_0^*)^{\frac{3}{2}}} \left(\frac{\beta}{2} + \frac{\sin 2\beta}{4}\right) - \frac{36\pi\sqrt{2}LU^*}{(h_0^*)^{\frac{3}{2}}} \sec^2 \bar{\beta} \left(\frac{3\beta}{8} + \frac{\sin 2\beta}{4} + \frac{\sin 4\beta}{32}\right) + \frac{36\pi\sqrt{2}L^2}{(h_0^*)^{\frac{5}{2}}} (V^* - U_s^*) \left(\frac{3\beta}{8} + \frac{\sin 2\beta}{4} + \frac{\sin 4\beta}{32}\right) - \frac{12\pi\sqrt{2}}{(h_0^*)^{\frac{3}{2}}} UL^* h_m^* \left(\frac{\beta}{2} + \frac{\sin 2\beta}{4}\right) + \frac{12\pi\sqrt{2}}{(h_0^*)^{\frac{3}{2}}} LU^* h_m^* \sec^2 \bar{\beta} \left(\frac{3\beta}{8} + \frac{\sin 2\beta}{4} + \frac{\sin 4\beta}{32}\right) - \frac{12\pi\sqrt{2}}{(h_0^*)^{\frac{5}{2}}} L^2 h_m^* (V^* - U_s^*) \left(\frac{3\beta}{8} + \frac{\sin 2\beta}{4} + \frac{\sin 4\beta}{32}\right) \tag{20}$$

### 3. Result and discussion

#### 3.1. Hydrodynamic pressure

Dimensionless the hydrodynamic pressure on the thin of layers consisting of atoms and molecules. The various dimensionless hydrodynamic pressure with the distance for various value speed of molecular ( $U_s$ ) is seen in figure (1). It is found that pressure fluid is caused by the random motion of fluid molecules when the speed of molecular ( $U_s$ ) up, some of energy from that random motion in the fluid direction of motion the results in a lower pressure. In first layers of membranes pressure increases and the movement of molecules decreases, while the lower layers of membrane movement of molecules increases that lead to decreasing pressure. The various dimensionless hydrodynamic pressure with the distance for various value film thickness of layers ( $h^*$ ) is seen in figure (2) each layer has film ranges between (thin- thickness) according to the pressure applied, one- layer has very low random motion of particles due to compression, so energy absorption decreases and hydrodynamic pressure increases while final- layer has high thin film and low pressure due to increasing random motion of particles or atoms. The various dimensionless pressure distribution with the distance for various value various squeeze action is seen figure (3). Squeeze action strength generated hydrodynamic pressure between layers of any material so found squeeze action various of each layer explain that motion of atoms and particles vary between (random- laminar) when motion of atoms and particles be laminar then squeeze action few as a result hydrodynamic pressure be low this is consistent whit final - layer .when squeeze action be high and effect on random motion thus hydrodynamic pressure high. The various dimensionless hydrodynamic pressure with the distance for various value porosity is seen figure (4). Porosity of member various from layer to another of any material where first layer radius of porosity become micro Porosity and speed of molecules lower that lead to the appearance of the highest porous pressure curves while final layer radius of porosity become macro porosity and this connect with speed of molecules that increasing it causes drop porous pressure curves. The various dimensionless hydrodynamic pressure with the distance for various value sliding motion is seen figure (1) in sliding motion occurs interaction between particulars fluid and members that it generated couple stress that results in it-curved pores are high.

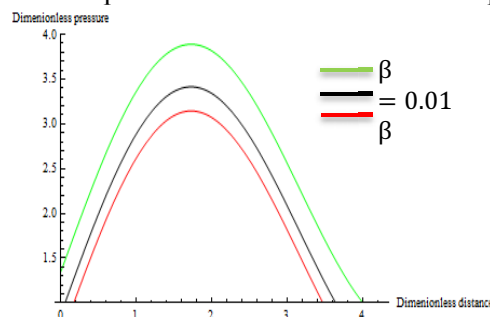


Figure (1): sliding motion occurs interaction between particulars fluid and members that it generated couple stress

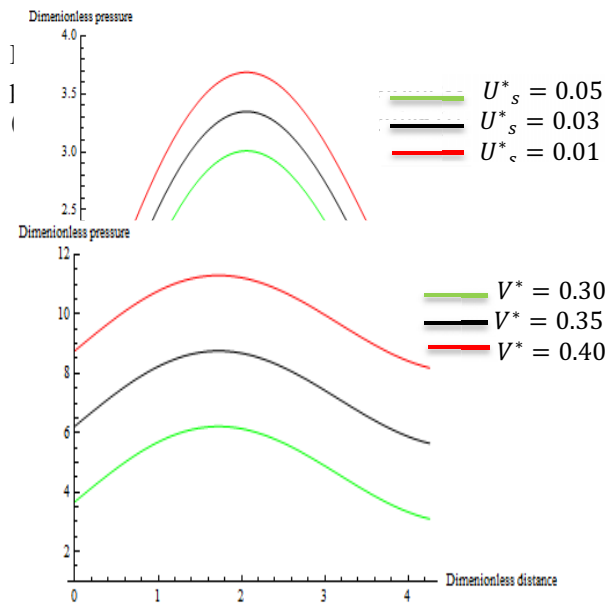


Figure (3): The variation of dimensionless pressure ( $p^*$ ) of with a dimensionless distance ( $x^*$ ) for various squeeze action ( $V^*$ ).

Figure (2): The variation of dimensionless pressure ( $p^*$ ) of with a dimensionless distance ( $x^*$ ) for various film thickness ( $h^*_0$ ).

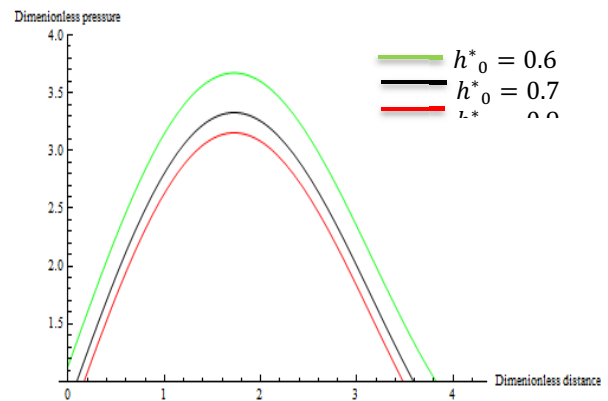
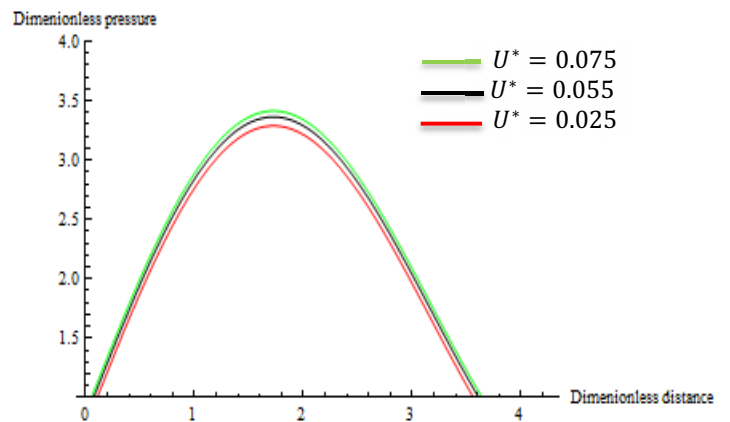


Figure (4): The variation of dimensionless pressure ( $p^*$ ) of with a dimensionless distance ( $x^*$ ) for various porosity ( $\beta$ ).

Figure (5): The variation of dimensionless pressure ( $p^*$ ) of with a dimensionless distance ( $x^*$ ) for various sliding motion ( $U^*$ ).



### 3.2. Load carrying capacity

The dimensionless load carrying capacity ( $w^*$ ) with dimensionless length ( $L$ ) for various values of the speed of leaks to molecules parameters ( $U^*_s$ ) is seen in figure (6). After stratify equation (18) in the computer program Mathematica 11 , it's seen that the load carry capacity of thin films increases with high values of ( $U^*_s$ ). The impact of the sliding motion parameter ( $U^*$ ) on the different of ( $w^*$ ) with ( $L$ ) is seen in figure (7). It's presented the load carry capacity of high remarkably with high values of ( $U^*$ ). The effect of the squeeze action parameter ( $V^*$ ) on the variation of ( $w^*$ ) with ( $L$ ) is shown in figure (8). It's shown that the load carry capacity of decreases with increasing values of ( $V^*$ ). The effect of the pore size of film thickness parameter ( $\beta$ ) on the variation of ( $w^*$ ) with ( $L$ ) is shown in figure (8). It's shown that

the load carry capacity of increases with increasing values of ( $\beta$ ). The effect of the film thickness parameter ( $h^*_0$ ) on the variation of ( $w^*$ ) with ( $L$ ) is shown in figure (10). It's shown that the load carry capacity of increases with increasing values of ( $h^*_0$ ). The effect of the length parameter ( $L$ ) on the variation of ( $w^*$ ) with pore size ( $\beta$ ) is shown in figure (9). It's shown that the load carry capacity of increases with increasing values of ( $L$ ).

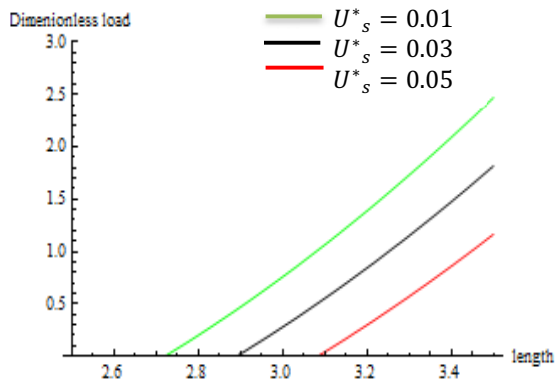


Figure (6): The variation of dimensionless load carry capacity ( $W^*$ ) of with length ( $L$ ) for different the speed of molecules parameters( $U^*_s$ ).

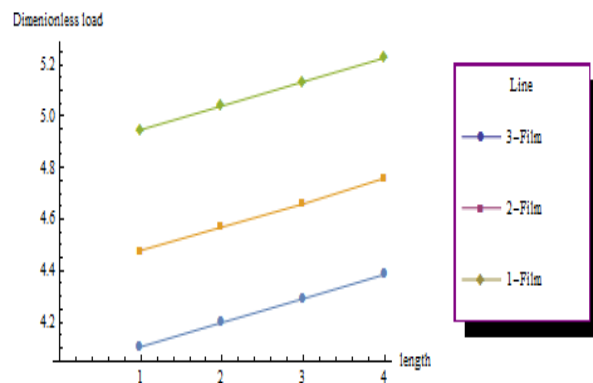


Figure (7): The variation of dimensionless load carry capacity ( $W^*$ ) of with length ( $L$ ) for different the sliding motion parameter( $U^*$ ).

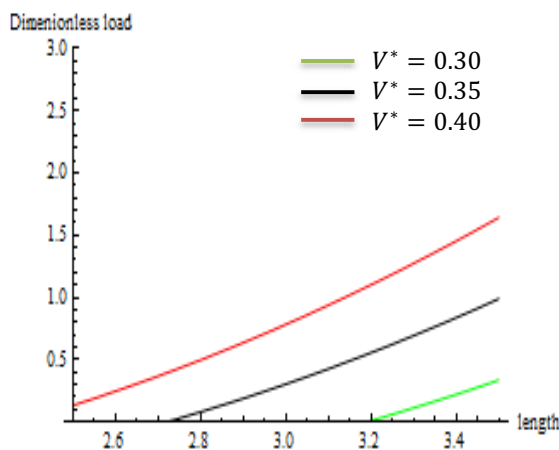


Figure (8): The variation of dimensionless load carry capacity ( $w^*$ ) of with length ( $L$ ) for various the thin film parameter( $h^*_0$ ).

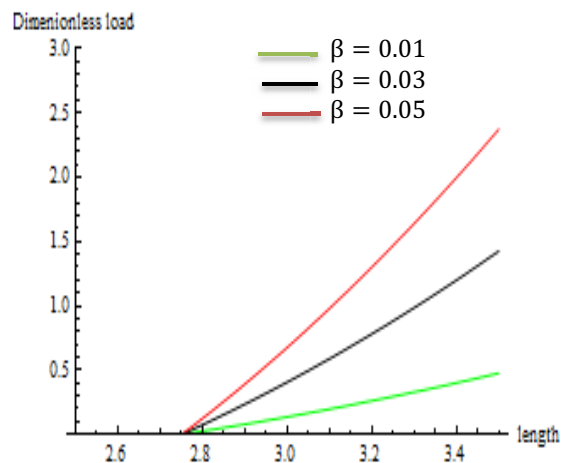


Figure (9): The variation of dimensionless load carry capacity ( $w^*$ ) of with pore size ( $\beta$ ) for various the length parameter( $L$ ).

**3.3. Dimensionless time**

The dimensionless time ( $t^*$ ) with dimensionless, the speed of leaks to molecules parameters ( $U^*_s$ ) for various values of the sliding motion parameter ( $U^*$ ) is shown in figure (10). After applying equation (20) in the computer program, it's then observed that the response time of the squeeze membrane layers and transfer film thickness to minimum film in each layer effective greatly with sliding motion and speed of molecules, when increasing speed of molecules it is absorbs the energy portion that lead to the pressure is reduced, and thus is transformed film thickness to minimum film with less time while increasing sliding motion between membrane layers increasing film thickness therefore required long time to transfer film thickness to minimum film. The effect of the squeeze action parameter ( $V^*$ ) on the variation of ( $t^*$ ) with ( $U^*_s$ ) is shown in figure (11). It's then observed that the time of increases with increasing values of ( $V^*$ ).The effect of the pore size of film thickness parameter ( $\beta$ ) on the variation of ( $t^*$ ) with ( $U^*_s$ ) is shown in figure (12). It's then observed that the time of increases with increasing values of ( $\beta$ ). The effect of the length parameter ( $L$ ) on the variation of ( $t^*$ ) with ( $U^*_s$ ) is shown in figure (15). It's then observed that the time of increases with increasing values of ( $V^*$ ).The effect of the pore size of film thickness parameter ( $\beta$ ) on the variation of ( $t^*$ ) with ( $U^*_s$ ) is shown in figure (14). It's then observed that the time of increases with increasing values of ( $\beta$ ). The effect of the length parameter ( $L$ ) on the variation of ( $t^*$ ) with ( $U^*_s$ ) is shown in figure (13). It's then observed that the time of increases with increasing values of ( $L$ ).The effect of the film thickness parameter ( $h^*_0$ ) on the variation of ( $t^*$ ) with ( $U^*_s$ ) is shown in figure (14). It's then observed that the time of decreases with increasing values of ( $h^*_0$ ). The effect of the minimum film thickness parameters ( $h^*_m$ ) on the variation of ( $t^*$ ) with ( $U^*_s$ ) is shown in figure (15). It's then observed that the time of decreases with increasing values of ( $h^*_m$ ). The effect of the speed of leaks to molecules parameters ( $U^*_s$ ) on the variation of ( $t^*$ ) with length ( $L$ ) is shown in figure (16). It's then observed that the time of decreases with increasing values of ( $U^*_s$ ).

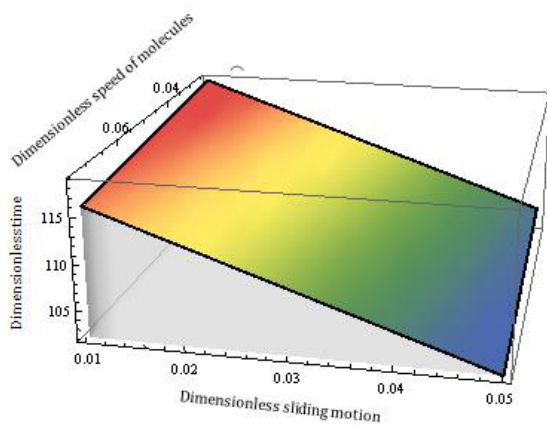


Figure (10): The variation of dimensionless time ( $t^*$ ) of with leaks to molecules ( $U^*_s$ ) for various the sliding motion parameter( $U^*$ ).

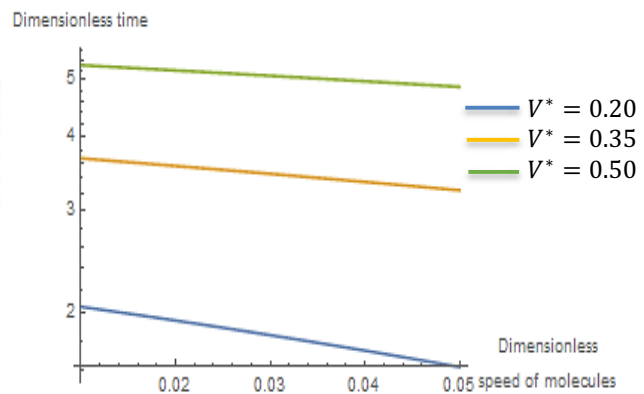


Figure (11): The variation of dimensionless time ( $t^*$ ) of with leaks to molecules ( $U^*_s$ ) for various the squeeze action parameter ( $V^*$ ).



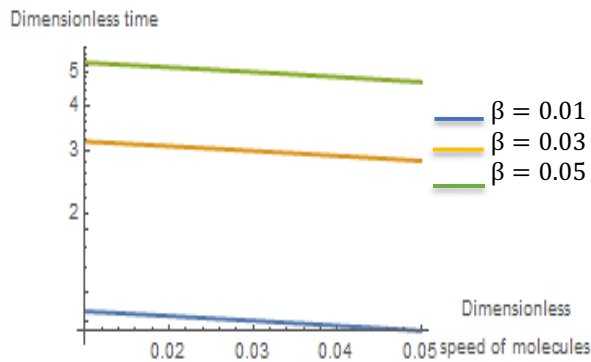


Figure (12): The variation of dimensionless time ( $t^*$ ) of with leaks to molecules ( $U_s^*$ ) for various the pore size of film thickness parameter.

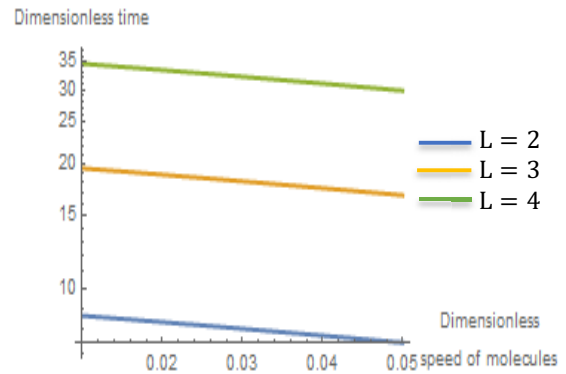


Figure (13): The variation of dimensionless time ( $t^*$ ) of with leaks to molecules ( $U_s^*$ ) for various the length parameter ( $L$ ).

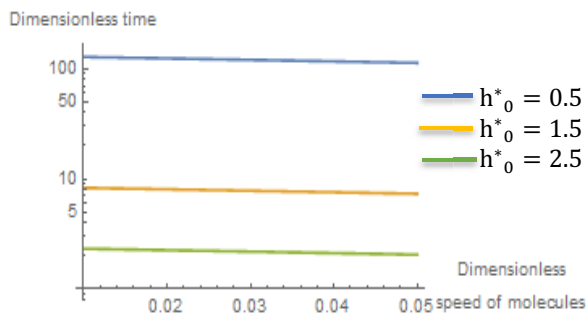


Figure (14): The variation of dimensionless time ( $t^*$ ) of with leaks to molecules ( $U_s^*$ ) for various the film thickness parameter ( $h_0^*$ ).

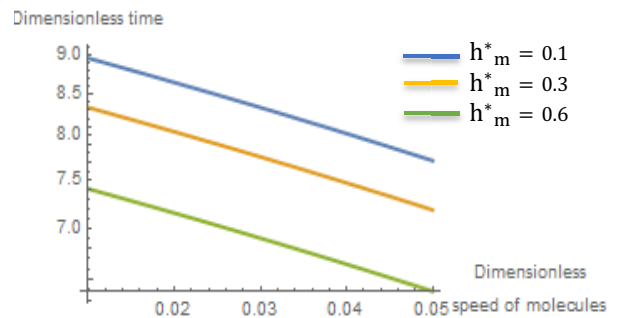
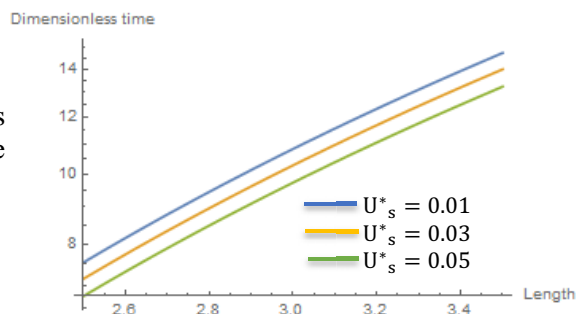


Figure (15): The variation of dimensionless time ( $t^*$ ) of with leaks to molecules ( $U_s^*$ ) for various the minimum film thickness parameters ( $h_m^*$ ).

Figure (16): The variation of dimensionless time ( $t^*$ ) of with the length ( $L$ ) for various the leaks to molecules Parameters ( $U_s^*$ )



#### 4. Conclusion

1. When an external force is applied on surface of thin ,it is generated by the interaction of the molecules hydrodynamic pressure that various between thin film where in first thin be a low while in final thin hydrodynamic pressure is high.
2. Hydrodynamic pressure between thin film (first –support -final) significantly increases with increase sliding motion and squeeze action while increase porosity of thin film ,film thickness and speed molecules  
Lead to decrease in pressure between thin films.
3. load carrying capacity was divided on (first-support-final) thins ,it was found in first thin very high tolerability which is an important advantage of these films
4. When squeeze action between layers then load carrying capacity increasing same result applies to length and porosity of thin film.
5. Decreasing Speed molecules reduces energy consumption, and this reflects positively on increased endurance same result applies to film thickness.
6. Increasing squeeze action that transfer film thickness to minimum film thickness this result according with increasing length while decrease porosity on film layers lead to film thickness become minimum with few time.

#### 5. Reference

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