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The majorant function modelling to solve nonlinear algebraic system

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Abstract

The majorize modelling of the modified Newton method (MNM) is an effective tool for concluding the existence and uniqueness of the solution of nonlinear operator equations. In this paper, we consider MNM together with a new majorant function (MF) that needed weak conditions to solve a nonlinear algebraic system (NLAS). The main advantage of this method is that you only require to establish the inverse of the Jacobian matrix (JM) at the initial guess (IG) and there is no need to compute it at the other iterations. The proposed strategy in this paper is logical and easy to execute. The theorems of MF and convergence are established for the method. Numerical results show that the strategy is efficient and promising.

Subject Classification: *(2020) 47A08, 08-08, 41A99, 93A10, 65H10.*

Keywords: *Majorant function, Modified Newton method, Nonlinear algebraic system, Nonlinear operator.*

1. Introduction

Certainly, solving nonlinear system of equations has gained the attention of numerous scientific and engineering researchers throughout

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the history of mathematics. Attempting to solve NLAS is one of the most challenging issues that researchers encounter because NLAS occurs in several disciplines such as mechanics, chemistry, engineering, physics, economics, and applied mathematics, see [1–5] for instance. NLAS normally, cannot be solved in analytical ways but it needs numerical approaches to approximate the solutions. The authors in [6–11] provided the concept of MF and MNM to solve several nonlinear integral equations. In this paper, we consider the NLAS of the form

$$F(X) = (f_1(X), f_2(X), \dots, f_n(X))^t = 0, \tag{1}$$

where $F : \Omega \to \Omega$, $\Omega = \Pi_{i=1}^{n}[a_i, b_i]$, $a_i, b_i \in R$, $i = 1, 2, \dots, n$ and each of $f_i, i = 1, 2, \dots, n$ is a continuous and differentiable nonlinear function that maps a vector $X = (x_1, x_2, \dots, x_n)$ from Ω to itself.

We introduce our method which depends on MNM with the concept of a new MF to solve Eq. (1) that requires the derivatives of each function $f_i, i = 1, 2, \dots, n$ at the initial guess only as well as satisfy the weak MCs.

2. Description of the method

The operator form of Eq. (1) is

$$P(F) = F(X) = 0, \quad F = (f_1(X), f_2(X), \dots, f_3(X))^t \tag{2}$$

Utilize the first iteration of MNM

$$P'(F_0)(F - F_0) + P(F_0) = 0 \tag{3}$$

to Eq. (1), where $F_0 = (f_1(X_0), f_2(X_0), \dots, f_n(X_0))^t$ to be any nonlinear continuous functions, such that $X_0 = (x_1^0, x_2^0, \dots, x_n^0)$ is the IG. The derivative of $P(F(X))$ at the initial guess $F_0(X)$ is appointed as

$$P'(F_0)F = \frac{dP(F_0)}{dF}F \tag{4}$$

From Eq. (3) and (4) we obtain

$$\left.\frac{dP}{dF}\right|_{F_0} \left(\Delta F(X)\right) = -P\left(F_0(X)\right), \tag{5}$$

where $\Delta F(X) = F_1(X) - F_0(X)$, is the IG.

Is recognized as Frechet derivative of the operator P or the JM at F_0.

Establishing the solution of Eq. (5) for $\Delta F(X)$, we get

$$\Delta F\left(X\right)=\left(\left.\frac{dP}{dF}\right|_{F_0}\right)^{-1}\left(-P(F_0(X))\right), \tag{6}$$

then $F_1(X)=\Delta F(X)+F_0(X)$. Subsequently, continuing this procedure, we get a sequence of approximate solution $F_m(X)$, $(m=2,3,\ldots)$ from the equation

$$\left.\frac{dP}{dF}\right|_{F_0}\left(\Delta F_m(X)\right)=-P(F_{m-1}(X)),\ (m=2,3,\ldots), \tag{7}$$

where $\Delta F_m(X)=F_m(X)-F_{m-1}(X),\ m=2,3,\ldots$

3. The Numerical Result and Discussion

In this section, numerical results are presented to demonstrate the effectiveness of the proposed method. The codes are written in Matlab *R*2014*a* and run on a personal computer with a 2.4 GHz CPU processor. For comparison, we give the numerical results of improved Hager-Zhang projection method (IHZPM) by Waziri et al. [12].

Example 1 : The mapping $F(X)=(f_1(X),f_2(X),\ldots,f_n(X))^t=0,$ where

$$f_i(X)=\log(x_i+1)-\frac{x_i}{n},\quad i=1,2,\ldots,n. \tag{8}$$

was discussed by Waziri et al. [12].

Example 2 : Consider the problem of $F(X)=(f_1(X),f_2(X),\ldots,f_n(X))^t=0,$ as in Zhou and Li [13], where

$$f_i(X)=2x_i-\sin\left|x_i\right|,\quad i=1,2,\ldots,n. \tag{9}$$

Table 1. reveals a list of all numerical consequences obtained from (MNM and IHZPM). The examples with their dimensions are listed, initial guess, the number of iterations (Iter), CPU time (CPU), and stopping criterion $\left\|F(X_k)\right\|$.

5. Conclusion

In this article, modelling of the majorant function and MNM are displayed to establish the solution of the NLAS. One advantage of this method is that there is no need for the inverse of the Jacobean matrix to be

Table 1

Test results of MNM and IHZPM methods for examples (1-2).

Example	Initial guess	n		MNM			IHZPM	
	n times		Iter	CPU	$\|F(X_k)\|$	Iter	CPU	$\|F(X_k)\|$
1	00.10	10	12	0.0437	1.99E-12	8	0.0645	3.89E-12
	-0.05	10	9	0.0429	2.06E-12	9	0.0770	4.09E-12
	00.01	10	6	0.0433	8.61E-13	7	0.0712	1.03E-11
	-0.10	100	11	0.3510	4.96E-12	10	1.0379	2.00E-12
	00.05	100	9	0.3648	1.11e-12	8	0.9238	4.70E-12
	-0.01	100	6	0.3523	5.21E-13	9	1.2106	2.70E-12
2	01.00	10	23	0.0431	4.03E-12	10	0.0835	5.11E-11
	10.00	10	59	0.0447	8.06E-12	13	0.1224	1.69E-11
	00.10	100	6	0.3581	2.01E-13	9	0.9030	1.11E-12
	-10.0	100	21	0.3774	2.23E-12	24	1.8558	3.25E-11

computed at each iteration, except the first iteration. Moreover, satisfactory results of illustrative examples relative to the one another method were used to explain the application of this method.

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