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Measuring the Efficiency of Hospital's Cardiology Wards Using the Free Disposal Hull Approach

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Abstract The assessment of efficiency of public hospitals in Tunisia is almost missing. Actually, the efficient utilization of existing resources becomes crucial for strengthening the healthcare delivery. The objective of this study was to measure technical efficiency of five cardiology wards, using an innovative nonparametric approach through an aggregated efficiency at patient level. It can assist practitioners to understand the underlying causes of clinical practice inefficiency. Linearized Free Disposal Hull using the non-radial input directional distance function provide a efficiency scores at the patient level and aggregate scores at ward's level. The cardiology wards operate at high inefficiency. Through the 217-treated diagnosis' disease, 50 are the greatest sources of inefficiency. Each ward could save more than 50% of inputs used. The decision makers can ensure the optimum utilization of the available resources through a new design of the management and clinical practices of these wards. High inefficiency is due to the lack evaluation, accountability and effective management of public hospitals.

Keywords: *efficiency, directional distance function, free disposal hull, cardiology wards, Tunisian's teaching hospitals*

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1. Introduction

Understanding and improving efficiency is an important step in evaluating the individual performance of hospital wards and establishing healthcare policy in the country. Ward's efficiency involves the rational frameworks for the distribution of resources between and within the hospitals [1]. This paper investigates the relative efficiency of cardiovascular wards of five teaching hospitals in Tunisia using a non-parametric approach and patients' data as the base decision-making tool. The research question pursued is: did hospital ward efficiently use available resources and how much inputs can be saved for a given level of outputs?

Hospitals are key elements in the Tunisian health care system consuming a major share of health care resources. Capturing and monitoring their inefficiencies has become critical. Reforms have aimed to improve performance by increasing managerial autonomy as it may lead to gains in both technical and allocative efficiency. Management tools have been introduced such as accounting and financial management systems and a computerized management system. A new billing system for inpatient and outpatient services has been developed through a simplified version of diagnostic related groups. This newly piloted payment mechanism and billing system has enabled hospitals to increase their revenues. Experience to date demonstrates a marked improvement in the teaching hospitals performance: activities have increased by 15-20%, particularly in the outpatient wards without an increase in personnel, and the average length of stay has dropped from 8.5 to 7.5 days [2]. The 2004 reform of the mandatory social health insurance has generated more funding for teaching hospitals.

In this context, the objective of this study is to estimate technical efficiency of the cardiovascular wards of five teaching hospitals in the Great Tunis (capital city plus three nearby "governorates").

The study aims to estimate the input-oriented inefficiency of five cardiovascular wards following three steps: (i) diagnosis of Tunisian's hospital inefficiency using non-parametric approaches followed by (ii) an extension of the non-parametric non-convex approach to measure ward inefficiency at patient level and then aggregated at ward level followed by (iii) an empirical illustration for the five cardiovascular wards. Free

Disposal Hull (FDH) and Data envelopment analysis (DEA) have emerged as an effective and popular method for evaluating production units' efficiency in different sectors including the health sector. Special emphasis is given to the development of the appropriate model using a non-convex FDH approach to estimate technical efficiency, designated as clinical efficiency [3]. The results of this study will be useful for decision makers in reviewing and tacking inefficiencies and resource utilization of the hospital wards.

Traditionally, hospital inefficiency has been evaluated using ratios of activity such as bed occupancy rate, bed turnover rate, cost per day, cost per patient, etc., and through econometric methods estimating a production function or a cost function [4]. DEA was initially introduced by [5] and applied by [6] to rank relative hospital inefficiency based on combinations of inputs and outputs. DEA methods are superior to the simple application of ratios (one output to one input); mainly because the model considers interaction between the number of hospital inputs and outputs. The relevance of econometric methods such as Stochastic Frontier Analysis, SFA, is limited to those situations in which a single overall output is used to estimate the production function and relatively complete price data is available to estimate the cost function. DEA, is preferred for studying the hospital industry because it does not impose a functional form and need for market price of inputs and outputs [7].

Several extensions of DEA methodology have been developed to compute efficiency scores of health entities [3,9] for general surveys, and [10,11,12,13,14], for empirical studies. DEA is a linear programming technique used for measuring the relative efficiency of health entities (hospital, wards, etc.). This non-parametric technique exhibits a well-defined production set, being a formal relationship between inputs and outputs. DEA uses the efficient frontier to calculate the relative inefficiencies for entities falling outside the efficient frontier and provides information on inefficient units [13]. These individual units are often, called production unit (PU) or decision-making unit (DMU). PUs can be whole facility such as hospital or units within hospitals such as separate wards.

Hospital wards can be considered as production units (PUs), but the analysis becomes complex when we want to specify ward production technology and to separate its multiple inputs and outputs from other wards. To solve this, it was decided to define the production technology as a process of health care provided to patients. DEA methodology, however, cannot be used at patient level data because DEA is a method of constructing a 'piece-wise linear' approximation of the production technology. DEA is based on a convexity axiom, which cannot be conceived for patients.

An alternative to DEA is the FDH technology [16,17] and [4]. It is a complete representation of the production technology without convexity assumption. The PUs are initially, defined as the patient, so efficiency will be measured with reference to the production technology specified from the process of patient treatment. From the patient level, ward efficiency can be gauged by aggregating patient scores within the ward. Important characteristics of the data are revealed by FDH that are not grasped by DEA, due to a better data fit [15]. The present paper intends to take stock of the significance of this methodology, and to illustrate it further with a representative case study and hope contribute to the existing literature on hospital performance by using the innovative FDH approaches.

The measurement of input-orientated efficiency can be provided by the input directional distance function, DDF [16]. Under standard assumptions, DDF provides a complete representation of technology [19,20]. Applying the input DDF, efficiency is measured relative to the production frontier using a non-radial measure as suggested by [21]. We estimate the input DDF using a linear FDH model. In fact, FDH is initially a mixed integer linear program, precluding any dual interpretation. Agrell [22] and Leleu [18] offer a general linear programming framework to define primal and dual FDH models.

The remainder of the paper unfolds as follows. In section two, we develop the model of the non-radial technical efficiency measure using FDH linear programming techniques. In the third section, we present the data used. Section four, shows FDH results followed by a discussion of the empirical application in section five. In the last section, we summarize our conclusions.

2. Methodology

Like DEA, FDH is a frontier efficiency estimation technique that computes technical efficiency for each individual production unit relative to others in the sample. The method provides solutions for an artificial frontier comprising a non-linear combination of the most technically efficient units [16]. Here, we use the linear form of the FDH model [17] to estimate the technical efficiency at the patient level without the convexity assumption. To have relevant aggregation properties at ward level, we define a weighted Färe-Lovell input DDF.

The production technology can be defined from a set of K observed PUs $\{(x_k, y_k),\ k = 1,...,K\}$. Each PU uses a non-negative vector of inputs ($x = (x_1,...,x_N) \in \Re_+^N$) to produce a non-negative vector of output ($y = (y_1,...,y_M) \in \Re_+^M$). The production technology can be also given by its input set $L(y) = \{x \in \Re_+^N \,|\, x \in L(y)\}$. $L(y)$ describes the set of input vectors that are feasible for each output vector., The input set is assumed to satisfy the axioms defined by [23,24] and extended by [25].

Out of this defined set, a technology can be conveniently represented by the input DDF as initially given by [18] and taken back by [25]. The input oriented technical efficiency is measured using this function, defined as:

$$\vec{D}_i\left(x, y;\ g_x\right) = \max_{\theta}\left\{\theta \left| \left(x - \theta g_x\right) \in L(y)\right.\right\} \qquad (1)$$

Next, we will show that a natural definition of the FDH technology is given by the individual production requirement sets, obtained from each PU $\{(x_k, y_k),\ k = 1,...,K\}$:

$$L^k(y_k) = \{x \mid x \geq x_k\} \tag{2}$$

The non-convex technology is the union of these individual production setsand it is defined as:

$$L_{FDH} = \bigcup_{k=1}^{K} L^k(y_k). \tag{3}$$

From the definition of the K subsets, the operational definition of the FDH technology is derived [21]:

$$T_{FDH}(x_k, y_k) = \left\{ (x,\ y) \left| \begin{array}{l} \exists z_k \geq 0, \sum_{k=1}^{K} z_k = 1, z_k x \geq z_k x_k, \\ z_k y \leq z_k y_k, \forall k = 1, ..., K \end{array} \right. \right\} \tag{4}$$

where z is a vector of activity. Regarding the convex technology, the non-convex one is the smallest approximation of the true production frontier [27].

By using the FDH "non-convex technology" and DDF, we benefit from two relevant properties in our framework at the patient level. First, the convexity as sumption is not appropriate valid at individual patient level. Second, we use the input DDF instead of the traditional Shephard's radial distance function since the former allows comparing and aggregating individual patient's inefficiencies to an overall inefficiency score by ward. This approach is different from the traditional efficiency DEA since the scores are computed relative to the particular input vector of the evaluated observation. Hence $g = x_0$ and the direction are observation specific. If projections of observations are not pointing in the same direction, they cannot be aggregated and a total ward score cannot be attained.

The DDF can also be used to explicitly express all the inefficiencies relative to the same standard moving in the same direction. For our purposes, the direction "g" will be the same for all observations of a given ward but can differ among wards.

However, Input DDF has a major drawback, especially for the measurement of input-efficiency at patient level. This drawback is related to zero input values. In the patient data set, many patients do not use some of the inputs every day during his/her hospitalization, meaning these patients will always be efficient in respect to the input DDF. This case can be deduced from the function definition: $\vec{D}_i(x, y;\ g_x) \geq 0$ and if the input direction is non-negative ($g_x > 0$) then $\vec{D}_i(x,\ y;\ g_x) = 0$ for zero input value. To avoid this problem and to keep the appealing aggregation property of DDF, a Färe-Lovell DDF is introduced. This function provides a separated reduction of each input (θ_n) instead of a common θ as in definition (1). Following the Färe-Lovell [21] the equation can be defined as follows:

$$FL(x, y) = \min \frac{\sum_{n=1}^{N} \theta_n}{\sum_{n=1}^{N} \delta(x_{ni})} \left| (\theta_1 x_1, ..., \theta_n x_n, ..., \theta_N x_N) \in L(y), \right.$$

$$\theta_n \in (0,1], \forall n = 1, ..., N \tag{5}$$

where $\delta(x_n) = 1$ if $x_n > 0$

$\delta(x_n) = 0,$ otherwise

The $FL(x, y)$ measure minimizes the arithmetic mean of the scalar θ_n as a proportional reduction for each input, separately. For an observation (x_k, y_k), the projection point (x_k^*, y_k^*) is determined by scaling down each input by the corresponding element of the efficient measure $(x_k^*,\ y_k^*) = (\theta_1 x_{k1}, ..., \theta_N x_{kN}\ ,\ y_{kN})$ and will always belong to the Pareto-Koopmans efficient subset of $L(y_k)$ as defined by [8].

Each input is then, contracted in a non-radial manner according to its own direction. Hence, we must adapt this measure in order to aggregate. The Färe-Lovell input DDF is defined as:

$$\vec{D}_i(x, y; g_x) = \min \left\{ \frac{\sum_{n=1}^{N} \theta_n}{\sum_{n=1}^{N} \delta(x_n)} \left| \begin{array}{l} \begin{pmatrix} x_1 - \theta_1 g_1, ..., x_n - \theta_n g_n, \\ ...., x_N - \theta_N g_N \end{pmatrix} \in L(y), \\ \theta_n \in (0,1], \forall n = 1, ..., N \end{array} \right. \right\} \tag{6}$$

where $\delta(x_n) = 1$ if $x_n > 0,\ \delta(x_n) = 0,$ otherwise

The main problem of using the DDF is interpreting the economic meaning of the direction, g_x, which must be the same for all evaluated PUs. We solve this problem by using the total resources used in the ward by all patients. The inefficiency measure has thus a clear economic interpretation. We compute the proportion of each resource used in award that could be saved if the patient was treated efficiently. Obviously, these proportions for each evaluated patient can be added to obtain the total inefficiency of the ward.

In its original version, the directional Färe-Lovell function (5) uses the arithmetic mean to compute the minimal contraction on each input and the same is used here with a minimum average of inefficiencies on inputs. A potential problem of the $FL(x, y)$ measure is the implicit assumption that all inputs have equal weights. Here, the share of the input costs on the amount of total expenditures is used as the weighting element (v_n). Finally, we get the following DDF function:

$$\vec{D}_i(x, y; g_x) = \min \left\{ \frac{\sum_{n=1}^{N} v_n \theta_n}{\sum_{n=1}^{N} \delta(x_n)} \left| \begin{array}{l} \begin{pmatrix} x_1 - \theta_1 g_1, ..., x_n - \theta_n g_n, \\ ...., x_N - \theta_N g_N \end{pmatrix} \in L(y), \\ \theta_n \in (0,1], \forall n = 1, ..., N \end{array} \right. \right\} \tag{7}$$

where $\delta(x_n) = 1$ if $x_n > 0,\ \delta(x_n) = 0,$ otherwise

the weighted elements sum up to unity ($\sum_{n=1}^{N} v_n = 1$).

The last step consists of the linear program, which allows estimating this weighted Färe-Lovell input DDF for a FDH technology. Following the Leleu development of [17], the FDH technology can be written using a linear program. This linearization allows applying duality and finding an economic interpretation of the FDH technology in terms of shadow prices.

Using FDH technology (4) and the weighted Färe-Lovell input DDF (7), the linear FDH model to gauge technical efficiency of DMU $(x_{k'}, y_{k'})$ is defined by:

$$\vec{D}_i^{FDH}(x_{k'}, y_{k'}; g_x) = \max_{\theta_{kn}, z_k} \sum_{n=1}^{N}\sum_{k=1}^{K} v_n \theta_{kn}$$

$$\begin{aligned} \text{s.t.}: \; & z_k (x_{kn} - x_{k'n}) \le -\theta_{kn} g_x, \; n = 1, \cdots, N \\ & z_k (y_{km} - y_{k'm}) \ge 0, \; m = 1, \cdots, M, \\ & \sum_{k=1}^{K} z_k = 1, \; z_k \ge 0, \; k = 1, \cdots, K \\ & \theta_{kn} \ge 0, \; n = 1, \cdots, N, \; k = 1, \cdots, K \end{aligned} \quad (8)$$

3. Data Sources

The data should describe the production technology that deals with the process of patient treatment, which unfortunately, is not directly available in the health information system of Tunisian hospitals. For this study, two database sources were combined to build a patient database for the year 2004. Data were retrieved from the hospital morbidity and mortality survey [27] and completed by additional information obtained from the patient's bill. The survey data include patient's socio-economic characteristics (age, sex, residence, etc.) and medical parameters (main and secondary diagnosis coded in ICD-10, discharge health status- dead or alive). The patient billing system provides information on the quantity of medical acts, length of stay, as well as their corresponding expenditure. The database was created, as it is not available in routine hospital information system in most developing countries, which is also the case in Tunisia.

As stated above, three inputs and one output are used to describe the production technology. The output is a binary variable indicating the patient's health status at discharge (alive or dead). The status alive shows that health status improved following treatment received at the ward. The three inputs are biological assessments (B), specialized medical care (KE), and inpatient days (ID). Information on these inputs is available in quantity and monetary units, as defined in the Tunisian official nomenclature book [28].

To ensure appropriate bench marking, monitoring variables (major diagnoses, cardiovascular surgery acts and age) are used, guaranteeing that the calculation of the efficiency scores at patient level belongs to groups having similar diagnosis, as well as age of the patient.

The database shows that for 2004, the five wards (denoted W1 to W5) have treated 4878 patients having 217 ICD-10 diagnoses. It shows that these wards have carried out 58 9178 B acts, 36087 KE acts and produced 7224 ID (Table 1).

Table 1. Description statistics of the three inputs (unit: TND)

Ward	Input	Sum	Mean	Min	Max
W1	Biological assessment, B	157 394.160	239.930	0	2 416.8
	Specialized medical care, KE	54 457.600	83.015	0	2 074.8
	Inpatient days, ID	256 807.504	391.475	0	2 520.0
W2	B	56 759.400	55.650	0	752.0
	KE	52 938.200	51.900	0	595.0
	ID	376 895.834	369.506	0	9 590.0
W3	B	70 551.520	81.374	0	572.8
	KE	85 036.650	98.082	0	520.8
	ID	301 605.317	347.872	11.7	4 305.0
W4	B	34 157.000	21.402	0	349.6
	KE	230 389.600	144.354	0	6 067.2
	ID	73 770.864	462.275	0	17 200.0
W5	B	94 268.480	127.735	0	1 842.4
	KE	433 044.400	58.678	0	5 487.6
	ID	252 381.251	341.980	0	3 325.0

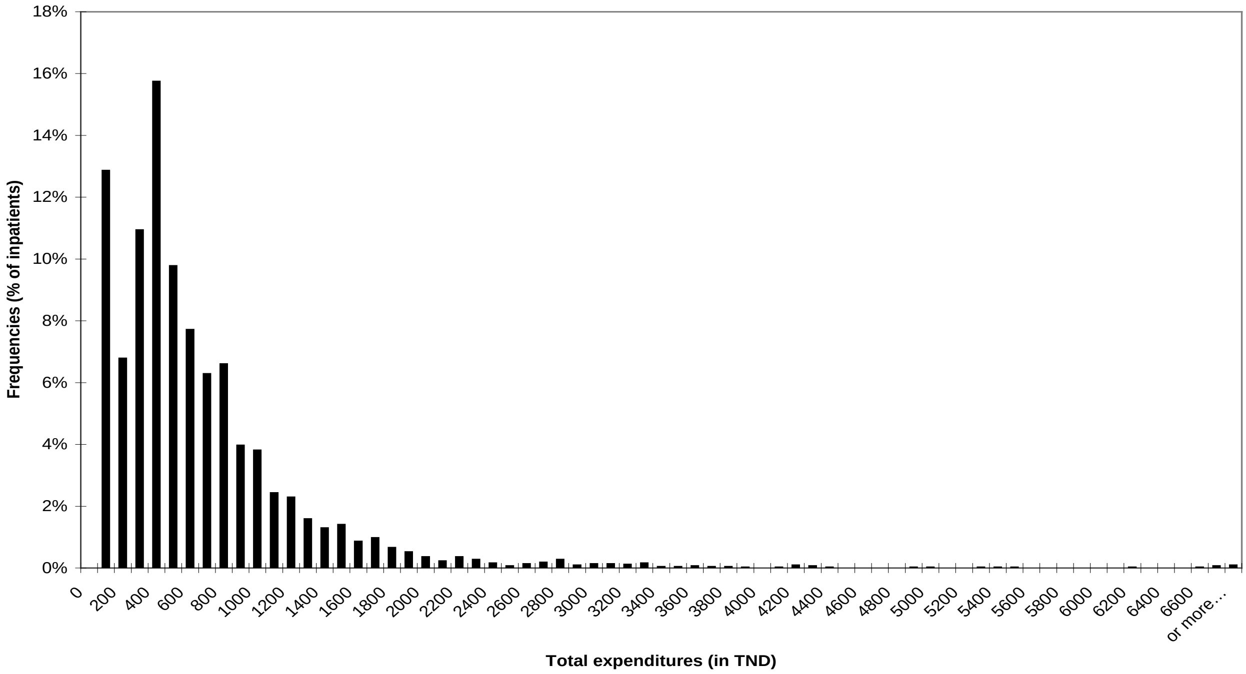

Figure 1. Distribution of total expenditures among inpatients

Patients admitted for one day of medical monitoring without, receiving any formal therapeutic care could overestimate the technical inefficiency score and should be considered as outliers. Indeed, we found that such patients are always efficient in view of the inputs used.

The total expenditure diagram (Figure 1) allows the detection of these outliers. The first bar of the diagram reports the frequency of one-day inpatient expenditure that can be reassumed as outliers. The estimation of the FDH technology production set is very sensitive to outliers, and for efficiency scores. The outlier problem has been avoided by transforming the data into ranks before the analysis as proposed by [29]. The first quartile of the patient sample (patients set of the first bar) is excluded.

4. Results

4.1. Descriptive Results

Table 2. Average technical inefficiencies for each ward by input types

Ward	number of inpatients	Inefficiency Biological assessment, B	Inefficiency Specialized medical care, KE	Inefficiency Inpatient days, ID
W1	568	55.26%	57.51%	52.09%
W2	882	68.85%	74.20%	55.84%
W3	565	56.02%	55.85%	56.94%
W4	1025	80.38%	40.65%	56.60%
W5	1838	30.93%	42.66%	50.54%
Total	4878	58.29%	54.17%	54.40%

This section sets out the inefficiency scores obtained from using the linear FDH model (programme 8). We report on inefficiency for each input used as well as overall inefficiency for each ward. Results for the average inefficient scores at ward level are summarized in Table 2. For the three inputs (B, KE and ID), the average inefficiency for all wards together was at least 54% with a minimum score of inefficiency of 31% (W5; input B) and a maximum score around 80% (W4; input B) and 74% (W2, input KE).

4.2. Concentration Analysis

Concentration analysis can explain the source of input inefficiencies regarding treated patients. This analysis is performed for three inputs and for all inpatients. For this analysis, the cumulative share of inpatients is reported on the horizontal axis, and the cumulated frequencies of the input inefficiency on the vertical axis of Figure 2 to Figure 4.

Figure 2 presents the concentration of the distribution of input B inefficiencies among inpatients. For input B, inefficiencies at ward level are related to approximately 50% of inpatients. It shows a high concentration for wards W4 and W2 and average concentration for the remaining wards. The technical inefficiencies recorded for W4 and W2 are concentrated respectively on 35% and 40% of total inpatients. For W1 and W3, 100% of technical inefficiencies is concentrated on roughly 40% of inpatients; 50 % of patients admitted at W5 generate 100% of technical inefficiencies.

The second concentration curve (Figure 3) is linked to KE input. Here, in W4, only 30% of inpatients caused 100% of inefficiencies and 90% of inefficiencies are concentrated among 10% inpatients. The situation is not the same for W1 and W3, where 60% of inpatients account for 100% inefficiencies. W5 provides another view. Its inefficiency is caused by only 30% of inpatients and 80% of this ward inefficiency is related to only 10% of inpatients. W5 shows that the 100% of the technical inefficiencies is concentrated among 50% of inpatients.

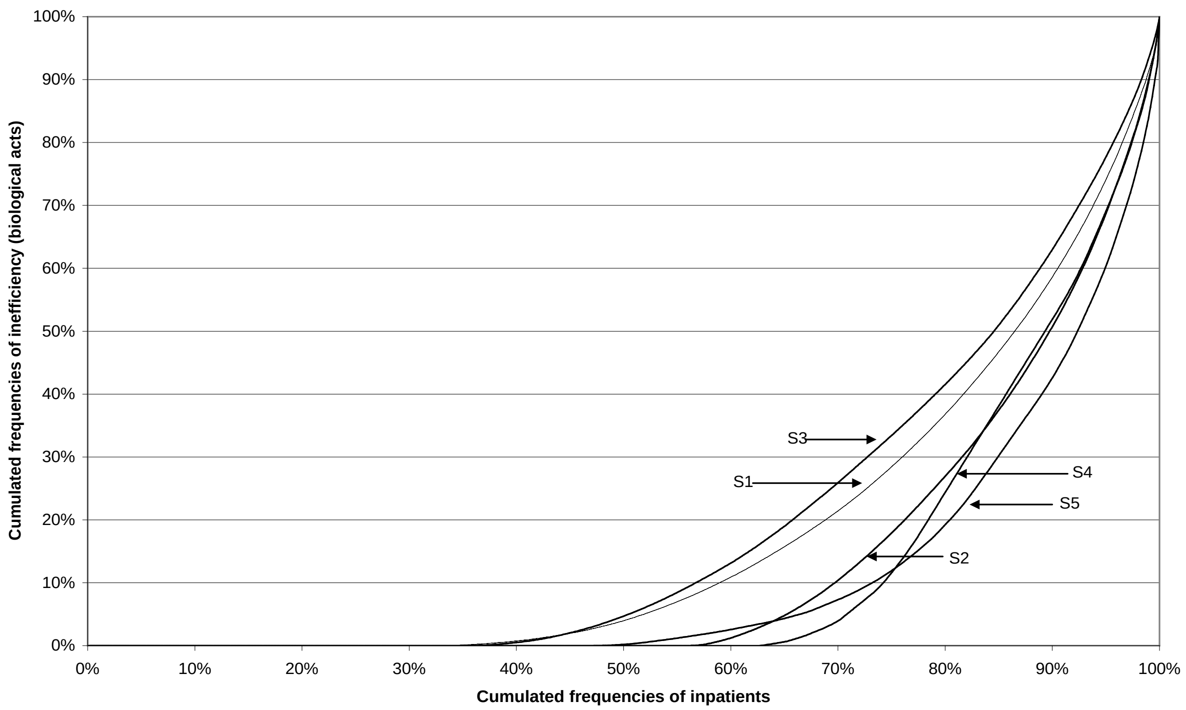

Figure 2. Concentration of inefficiency on biological acts (B)

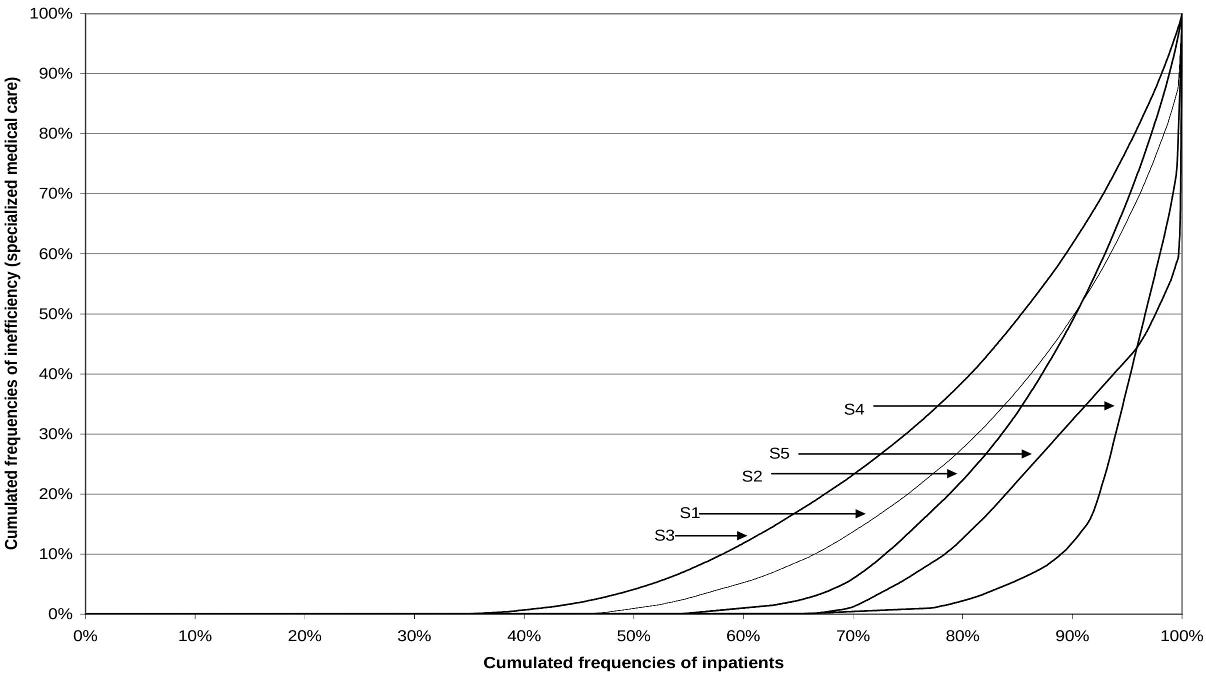

Figure 3. Concentration of inefficiency on specialized medical care (KE)

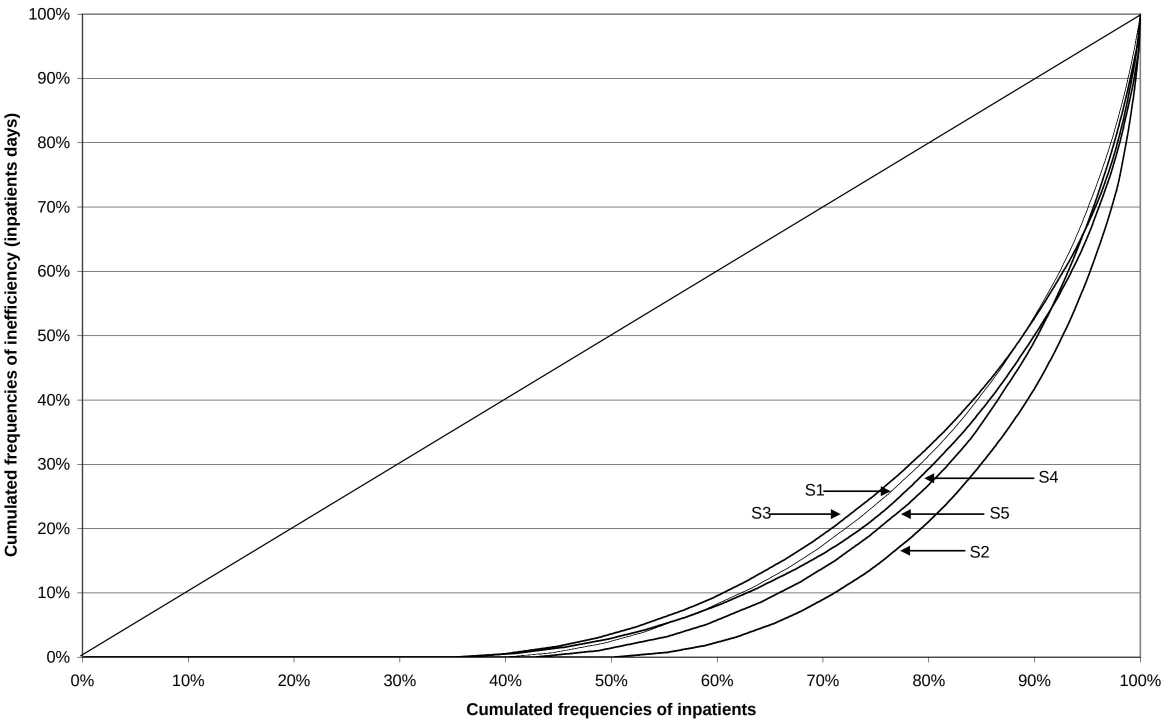

Figure 4. Concentration of inefficiency on inpatients days (ID)

The last concentration curve (Figure 4) allows analysis of inefficiencies of ID input among inpatients. The concentration analysis shows the 50 % of inpatients explain 100% of W2 inefficiencies in W2. Therefore, only 60 % of inpatients explain to total inefficiencies for the remaining wards.

4.3. Ward Inefficiencies by Major Diagnoses

Given the high inefficient scores for all wards and the strong concentration among specific groups of inpatients, we assume that inefficiency could be related to specific disease patterns according to ICD-10. For the five cardiovascular wards, we find 217 major diagnoses from which 50 are clearly implicated as sources of inefficiency for all the wards. These diagnoses explain 95-98% of inefficiencies. In W1, 39 diagnoses are implicated for more than 95% of inefficiencies. For W2, at least 22 diagnoses are sources of 97% of its inefficiencies and 32 diagnoses are at least sources of 95% of W3 inefficiencies. W4 accounts for only 14 diagnoses as sources of 98 % of its inefficiencies. Table 3 summarizes the disparities on the implication of diagnoses on ward inefficiencies.

Table 3. Number of diagnoses as sources of inefficiencies for each ward by three inputs

Inputs	Diagnoses and inefficiency (in brackets)				
	W1	W2	W3	W4	W5
Input B	38 (95.4%)	25 (98.3%)	36 (97.3%)	14 (98.3%)	28 (95.5%)
Input KE	39 (98.2%)	24 (97.9%)	34 (95.3%)	15 (99.7%)	26 (96.9%)
Input ID	38 (96.8%)	22 (97.5%)	32 (94.9%)	15 (98.6%)	29 (95.8%)

Table 1. Main diagnoses sources of wards inefficiency

Diagnoses	Inputs	W1 Inefficiency	W2 Inefficiency	W3 Inefficiency	W4 Inefficiency	W5 Inefficiency
UA	B	21.9%	26.2%	6.5%	29.1%	0.3%
	KE	22.8%	28.9%	7.9%	30.4%	0.4%
	ID	18.2%	24.0%	3.2%	34.8%	-
Total		**62.9%**	**79.1%**	**17.6%**	**94.3%**	**0.7%**
AMIU	B	10.5%	8.1%	11.4%	33.8%	5.4%
	KE	10.6%	7.6%	10.7%	24.8%	2.6%
	ID	8.5%	7.9%	7.4%	28.8%	2.0%
Total		**29.6%**	**23.6%**	**29.5%**	**87.4%**	**10.0%**

Table 4 shows that the distribution of inefficiencies in input B shows that the diagnoses Unstable Angina (UA) and Acute Myocardial Infraction Unspecified (AMIU) are the main sources of inefficiency for wards W1, W2 and W4. Ward inefficiency related to KE input confirms the implication of two types of diagnoses (UA and AMIU) in the higher inefficiency scores for wards W1, W2, W3 and W4 but not for W5.

For the third input (ID), the diagnoses AU and the AMIU remain the main sources of the technical inefficiency for W1, W2 and W4 but not for W5. The essential hypertension diagnosis was the main source of inefficiencies for W5 (26.6%) and contributes highly to the technical inefficiency for W3 (14.5%) and W2 (11.5%).

The diagnoses UA and AMIU are involved in the inefficiency observed in biological acts, specialized acts as well as in inpatients days. These higher scores seem to be linked to the three prestigious cardiology wards: W1, W2 and W4. The patients treated with those diagnoses contribute a lower value of inefficiency for the ward W3 and an even smaller value for W5 (Table 4).

5. Discussion

In this paper, a non-parametric methodology is developed to measure efficiency at patient level for five cardiology wards, based on the estimation of an input directional distance function, DDF.

The major advantage of the methodology is its application to measure efficiency at patient level and then to aggregate at ward level. It may be criticized by the heterogeneity in patients' health status as well as the ability and the length of time needed to recover that on a patient level, errors and misspecification could be expected to be substantial. Since efficiency measures are, whatever methodology is chosen, based on a residual concept, it is obvious that they capture output heterogeneity. The results of this paper indicate similar views of heterogeneity as in Table 4. High proportions of inefficiency are due to few numbers of diagnoses.

Results of ward inefficiencies by input types show that two wards, located in the largest university hospitals and providing all medical acts are the most inefficient users of B and KE input. For instance, staff number in one of these two wards is twice that number compared to the other wards. The second less inefficient in KE inputs (40.65 %), may be due to proximity of a highly specialized ward (within the same building), exclusively devoted to cardiology medical acts (functional exploration and hemodynamic).

The more technically efficient ward is the one recently created (W5), its medical staff and equipment are still reduced and it is not a referral ward. Whilst the two wards registering the highest rate of inefficiency are particularly affected by, patients referred for surgical acts. On the other hand, these wards are particularly involved in health personnel training. Medical acts could be repeatedly ordered for the same patients, by different staff members (residents, assistants, professors, etc.) indicating a lack of coordination. Furthermore, we noted that medical staff on cardiology wards uses the input KE differently, given the specialized medical practices multiplies the process for the same diseases.

For inpatient days, all wards have recorded high inefficiency scores, varying from 50% to 57%, leading to potential saving by reducing the length of stay by half. The disparity of wards' technical inefficiencies scores suggests a key role for factors such as staff practices, inpatients' health status, patients' profiles and illness severity. The concentration analysis shows more concentration prevailing in the wards W4 and W2. This disparity may be due to a higher amount of heterogeneity in the kind of biological assessments realized by these wards' laboratories, which are better at assessing a wide variety of biological exams. In addition, the observed concentration in these two wards could be explained by their ability to use more sophisticated specialized medical acts than the other wards.

In comparison with wards W1, W2, W4, the recently created W5 shows a lower score of inefficiency for these complicated diagnoses (AMIU and AU), partly due to the lack of sufficient qualified staff. On the other hand, it becomes clear that W5 has some difficulty addressing common non-treated diagnoses (not shown in Table 4) such as elevated blood pressure (HTA) (rated 26.6% vs. only 0.5% at W4 in terms of inpatient days inefficiency) or some other non-rheumatic mitral valve disorders (rated 35% vs. 0.3% at W1) in terms of specialized medical care acts inefficiency.

They are a high life-threatening diagnosis and are supposed to be treated and/or transferred to W2 or W4.Therefore, the high inefficiency score observed at W4 could be more due to a high rate of inpatient recruitment and case mix severity, than to the lack of performance. This result confirms the observed relationship between the high level of technical inefficiency and some diagnoses. At patient level, inefficiency could be reduced, by improving the management care of patients with those pathologies [30]. This kind of observation and analysis may be of major interest among comparable wards, on basic criteria such as wards' medical specificity, staff qualification and effective operational capabilities. It may also depend on the priority of the medical staff, in their choice of inpatients recruitment.

The results reported in this study demonstrate the existence of technical inefficiency in Tunisian Public Hospitals. The government must not only pursue cost containment policies but also focus on enhancing productive efficiency at hospitals and ward level in order to get a high return of investment in the treatment of NCDs.

Cardiovascular wards differ in their performance, and it is not easy to explain this disparity, without diagnosis related groups and a clear process of medical production. Clinical practice is quite different among physicians, even when they are treating the same case-mix patients, or when physician performs widely different clinical practices. We assume that the inefficiencies were related to the type of management care within each ward rather than to clear and standardized procedures. This study shows inefficiencies regarding inpatient days, caused by a specific clinical practice. Regarding functioning of the wards, two specific issues arise. First, one issue pertains to the optimum size related to input's used for a particular health care unit and how services should be organized to ensure a higher level of efficiency. Second, whether there are aspects of organization that have an impact on health care delivery that need to be elucidated.

Given the Tunisian public hospital context, adjusted staff numbers can lead to greater ward efficiencies. For a wide range of wards' interventions, there is a clear relationship between volume and outcome i.e., wards treating more patients provide better treatment; be it through practice or availability of standardized routines, better equipment, or some other factor. Wards could also obtain better results because they treat less seriously diseases, without UA and AMIU pathologies. The first arguments explain the higher inefficiencies for wards W2 and W4 and the latter one justifies the less inefficiency for W5.

Factors such as ward specificity, illness severity, junior medical staff, ward size and organization characteristics, actually can lead to an (in) efficient ward and bad/good quality of healthcare. There is a need to give modern comprehensive recommendations concerning the structure, organization, and function of the cardiology wards. These include the need of specially trained cardiologists and cardiac nurses who can manage patients with acute cardiac conditions. The optimum number of physicians, nurses, and other personnel working in the ward should be included. Specific recommendations are also to be included for the minimal number of beds, monitoring system, admissions, length of stay, and relocation policy.

6. Conclusion

This study has shown how Free Disposal Hull methods can be applied at hospital ward level of to gain insight into variations in efficiency across hospitals using a description of clinical practices as a non-convex production technology. A linear FDH program applied to weighted Färe-Lovell input Directional Distance Function provides a measure of efficiency at patient level and then aggregated at wards levels. Our study contributes to the growing interest on efficiency analysis and offer a rigorous methodology to measure efficiency of hospital 'wards.

In Tunisia, public hospitals are changing and facing challenges of high costs, particularly for non-communicable diseases such as cardiovascular. The results show that, on average, cardiology ward could save more than 50% of the used inputs. By directing attention to hospital's wards, it is possible to gain insight into efficiencies of clinical practices for the used inputs and related costs. The approach presented in this study provides managers with relevant information for wide-ranging evaluations of the system and to investigate why the clinical practices are differently in these wards and why inefficiencies are high.

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