Large basis shell model calculations of some nuclei around doubly-magic ^{56}Ni

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Abstract: The level schemes and binding energy of $^{59\text{-}67}$Cu isotopes were studied by performing large-scale shell model calculations in the f5/2pg9/2 space by employing *jun45* and *jj44b* effective residual interactions. The recent shell model code NushellX@MSU has been employed in the present work in which the core is taken ^{56}Ni. The comparison between our theoretical results for the level schemes agreed reasonably well with their corresponding experimental data. The experimental binding energies are very well reproduced by the current shell model calculations.

Keywords: shell model; binding energies; NushellX@MSU.

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1 Introduction

The nuclear shell model assumes that each of the nucleons in an atomic nucleus move independently in an average field created by the other nucleons (Smirnova et al., 2004). The nuclear shell model succeeded because the average potential is approximated by the harmonic oscillator potential centrifugal term (*l*–*l*) and spin orbit interaction (*l*–*s*), to reproduce the so-called magic numbers, i.e., the numbers of protons and neutrons that gives an additional stability to a nuclear system, which are 2, 8, 20, 28, 50, 82 and 126. The inclusion of the correlations beyond the mean-field approximation by means of the effective residual two-body interactions gives us an excellent description of the properties of atomic nuclei in the vicinity of β stability (Caurier et al., 1999).

There is a growing interest in nuclei in the vicinity of the double closed shell nucleus ^{56}Ni which shows that particle excitations from *f*7*l*2 shell have important contribution to the dynamical behaviour of these nuclei. In particular, they could account for the electromagnetic transition rates between low-lying states. These transition rates can hardly be described by models which do not involve such excitations (Cohen et al., 1967). Rudolph et al. (1999) studied the high spin states of nuclei near to doubly-magic ^{56}Ni by means of the reaction ^{28}Si(*n*, *p*)^{28}Al. They compared their experimental findings with the full *fp*-shell model calculations, and concluded that new experiments are necessary to pin down the unnatural parity states and, hence, the $1g_{9/2}$ single particle state in ^{57}Ni. Rai et al. (2015) performed shell model calculations for even–even ^{60,62,64,66}Zn isotopes in the $f_{5/2}pg_{9/2}$ model space with different effective interactions. They compare their calculated excitation energies, reduced transition probabilities B(E2), mean life time and quadruple moment with the available experimental data. Their results are in good global agreement with some discrepancies in comparison with the experimental data. Kaneko et al. (2015) introduced an effective shell model interaction, where the pairing-plus multipole Hamiltonian with the monopole interaction has been achieved by using an empirical fits starting from the monopole-based universal force (PMMU) and applied systemically to nuclei lies in the $f_{5/2}pg_{9/2}$ shell region. The calculations prove that they successfully describe a wide range of experimental data reasonably well for low lying and high-excitation spectra, (E2) transitions, quadruple moments, and the binding energies for Ni, Cu, Zn, Ga, Ge, As and Se isotopes with A = 64–80 (Kaneko et al., 2015). Shell model calculations were performed by Majeed et al. (2012) to study the energy levels and reduced transition probabilities $B\left(E2;0_1^+ \rightarrow 2_1^+\right)$ for even–even $^{66\text{-}76}$Ni isotopes. Their results show reasonable agreement with the experimental data. Smirnova et al. (2004) investigated the variations in the nuclear mean field, in neutron-rich nuclei; the change is distinguished to originate mainly from the monopole part of the effective two-body proton–neutron interaction. Application to the low-lying states in odd-A Cu nuclei was demonstrated and compared using schematic and realistic forces. The

monopole shifts were also compared with the results obtained from large-scale shell model calculations, using the same realistic interaction in order to study two-body correlations beyond the proton mean-field variations. Very recently, Majeed and Obaid (2017) had performed unrestricted large scale shell model calculations to study excitation spectra, binding energies and reduced transition probabilities $B\left(E2;0_1^+ \to 2_1^+\right)$ for isotopes ^{136,138}Xe and ^{138}Ba. A large scale shell model calculation has been performed by Almayyali (2016) to study the energy levels and reduced transition probabilities for neutron rich even–even $^{62-76}$Zn. They reported that their calculations agree well with the experimental data and they confirm that using fixed values for proton and neutron effective charges was unable to reproduce the experimental reduced transition probabilities, which proves the limitation of the model space and effective interactions that were used in their study. Unrestricted large scale shell model calculations have been performed by Majeed (2017) in which he studied the level excitation spectra, $B\left(E2;0_1^+ \to 2_1^+\right)$, and binding energies for even–even $^{210-212}$Pb, $^{210-212}$Po isotopes in the neutron deficient region $\pi\left(hfpi\right)\nu\left(igdsj\right)$ valence space above the ^{208}Pb core using four effective interactions *khpba*, *khpbu*, *khp* and *khpe*.

The purpose of this paper is to study the energy levels for the positive and negative parity and binding energies of $^{59-67}$Cu isotopes using a new variation of NushellX@MSU for Windows (Brown and Rae, 2007) without any restriction imposed the model space with effective interactions *jun45* (Honma, 2009) and *jj44b* (Brown and Lisetskiy, 2009), to test the ability of the present effective interactions to reproduce the experiment in this mass region.

2 Shell model calculations

2.1 Energy levels

The core is taken at ^{56}Ni for all isotopes under the study with valence nucleon distributed *p*3/2 *f*5/2 *p*1/2 and *g*9/2 valance space by employing *jun45* and *jj44b* effective interactions using the shell model code NushellX@MSU. The comparison of the calculated energy level for the positive and negative parity for $^{59-67}$Cu isotopes is presented in Figures 1–7 by employing *jun45* and *jj44b* effective interactions with the recent available experimental data.

Figure 1 presents the comparison between our calculations and the corresponding experimental data using the mentioned effective interactions for the positive and negative parity states, for ^{59}Cu isotope. We noticed that both effective interactions correctly reproduce the ground-state spin of $3/2^-$. There are many unconfirmed values of the experimental data after the $9/2^+$ that have been confirmed in our theoretical calculations, and both effective interactions agree reasonably well in comparison with their corresponding experimental data.

The calculations of the excitation energies for positive and negative parity states are shown in Figure 2 for ^{60}Cu isotope. The ground-state spin of 2^+ could be reproduced with both effective interactions. There are many unconfirmed values of the experimental data after the level 1^+ have been confirmed in our theoretical. The calculations using both effective interactions are in reasonable agreement. The effective interaction of *jj44b*

is better agreement than *jun45* effective interaction with the experimental data. The low-lying energy level has performed using the same effective interactions by considering the core at ^{56}Ni compared with the experimental data is presented in Figures 3, 4 and 5, respectively. The ground state for this isotope is correctly reproduced by using the effective interactions. The ordering of the low-lying spin states for ^{61,63,65}Cu isotopes is correctly reproduced by *jj44b* effective interaction, while *jun45* predicts $7/2^-$ lower than $5/2^-$ which is in disagreement with the experimental data. The effective interaction *jj44b* reproduces the correct ordering of $7/2^-$ and $5/2^-$ states for ^{61,63,65}Cu isotopes in comparison with the experimental data. There are many unconfirmed values of the experimental data that have been confirmed in our theoretical. The calculations using both effective interactions are in reasonable agreement, but the effective interaction of *jj44b* is in better agreement than *jun45* effective interaction with the experimental data.

Figure 1 Comparison of calculated and experimental low-lying spectra for ^{59}Cu isotope with *jun45* and *jj44b* effective interactions. Experimental data take from NNDC (2018)

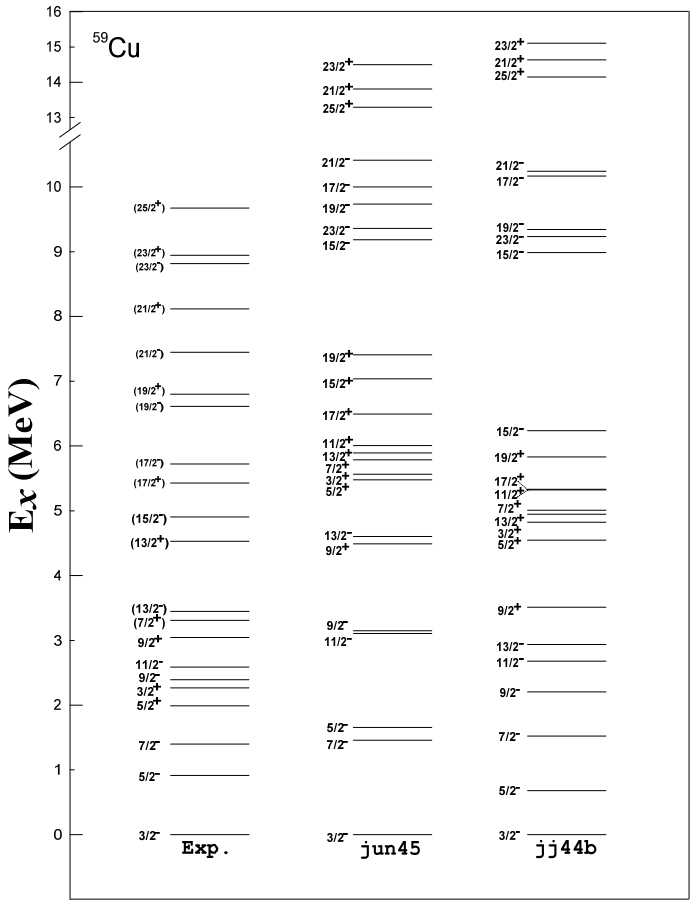


Figure 2 Comparison of calculated and experimental low-lying spectra for ^{60}Cu isotope with *jun45* and *jj44b* effective interactions. Experimental data take from NNDC (2018)

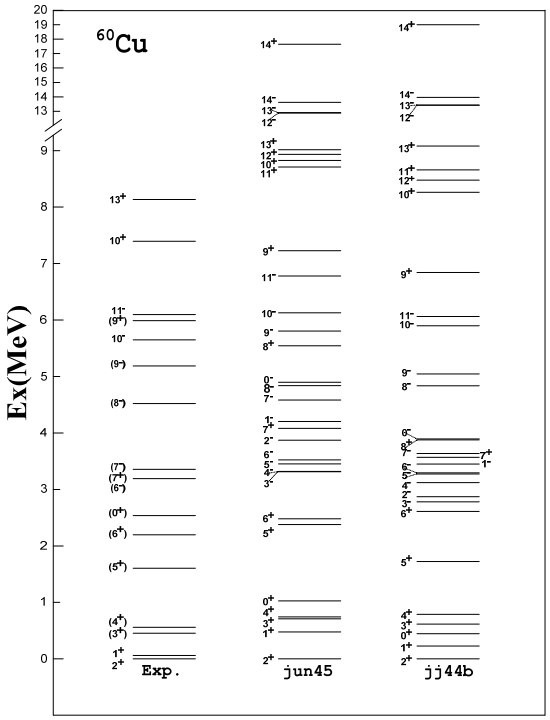


Figure 3 Comparison of calculated and experimental low-lying spectra for ^{61}Cu isotope with *jun45* and *jj44b* effective interactions. Experimental data take from NNDC (2018)

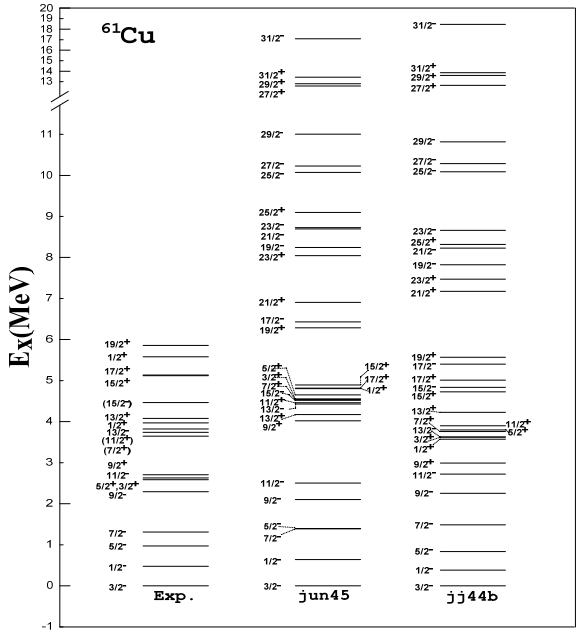


Figure 4 Comparison of calculated and experimental low-lying spectra for ^{63}Cu isotope with *jun45* and *jj44b* effective interactions. Experimental data take from NNDC (2018)

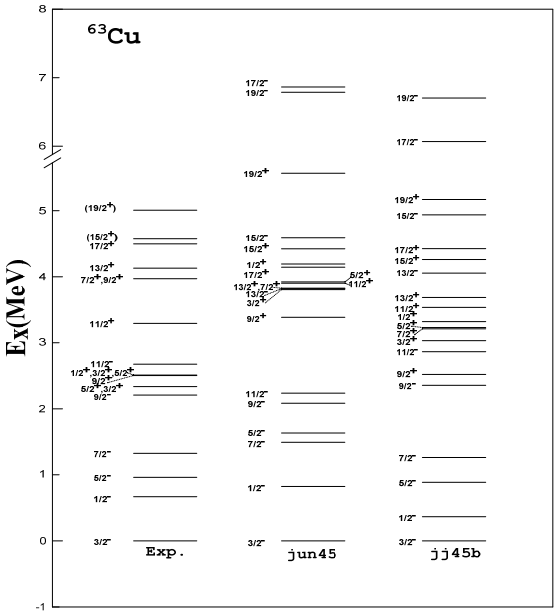
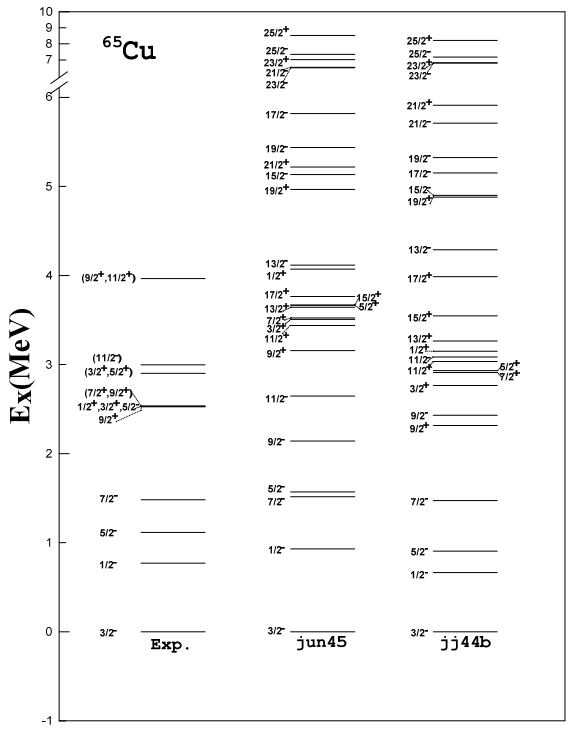


Figure 5 Comparison of calculated and experimental low-lying spectra for ^{65}Cu isotope with *jun45* and *jj44b* effective interactions. Experimental data take from NNDC (2018)



In Figures 6, 7 and 8, the calculated energy levels for positive and negative parity for ^{62,64,66}Cu isotopes obtained using *jun45* and *jj44b* together with the experimental data are shown. The two interactions used in the present calculations aren't able to predict correct ground state as observed in experimental, and many unconfirmed values of the experimental data have been confirmed in our theoretical. The calculations with *jj44b* are closer to the experimental values than *jun45* for these states.

Figure 6 Comparison of calculated and experimental low-lying spectra for ^{62}Cu isotope with *jun45* and *jj44b* effective interactions. Experimental data take from NNDC (2018)

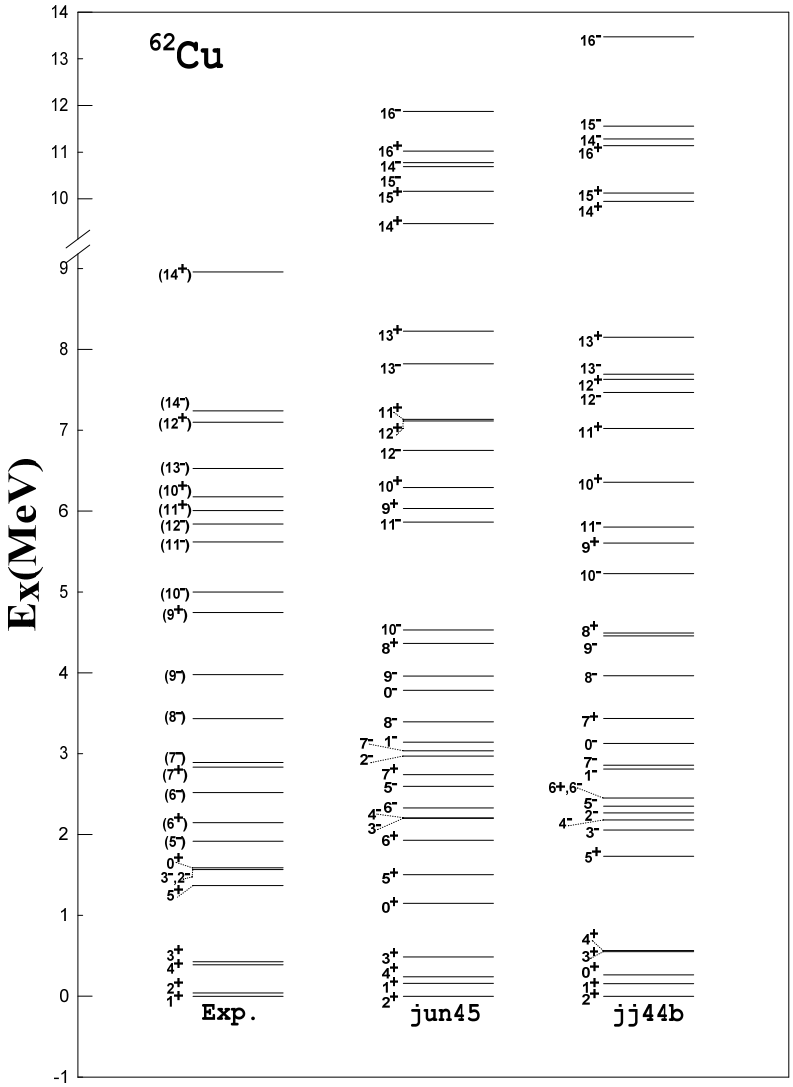


Figure 7 Comparison of calculated and experimental low-lying spectra for ^{64}Cu isotope with *jun45* and *jj44b* effective interactions. Experimental data take from NNDC (2018)

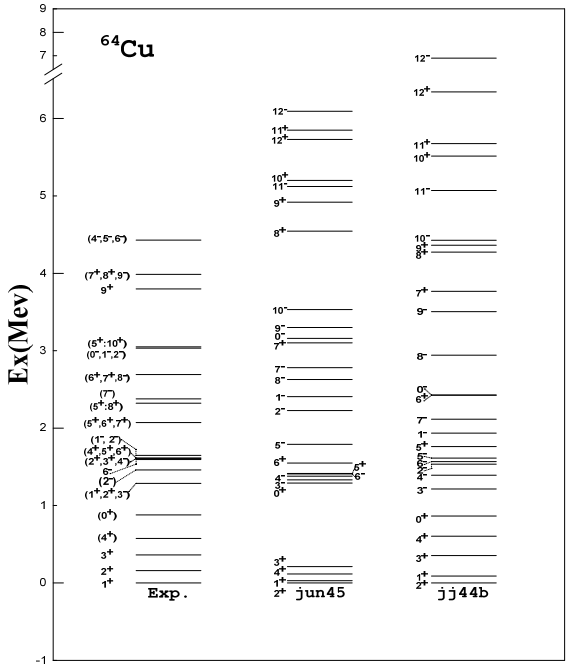


Figure 8 Comparison of calculated and experimental low-lying spectra for ^{66}Cu isotope with *jun45* and *jj44b* effective interactions. Experimental data take from NNDC (2018)

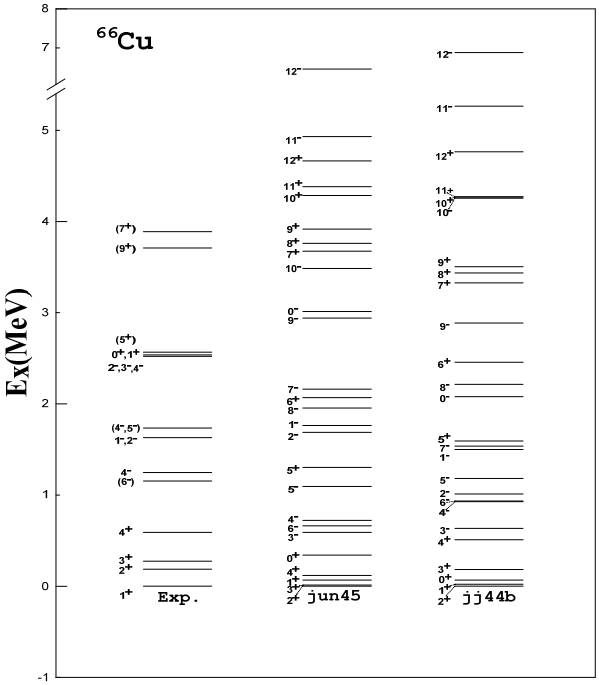
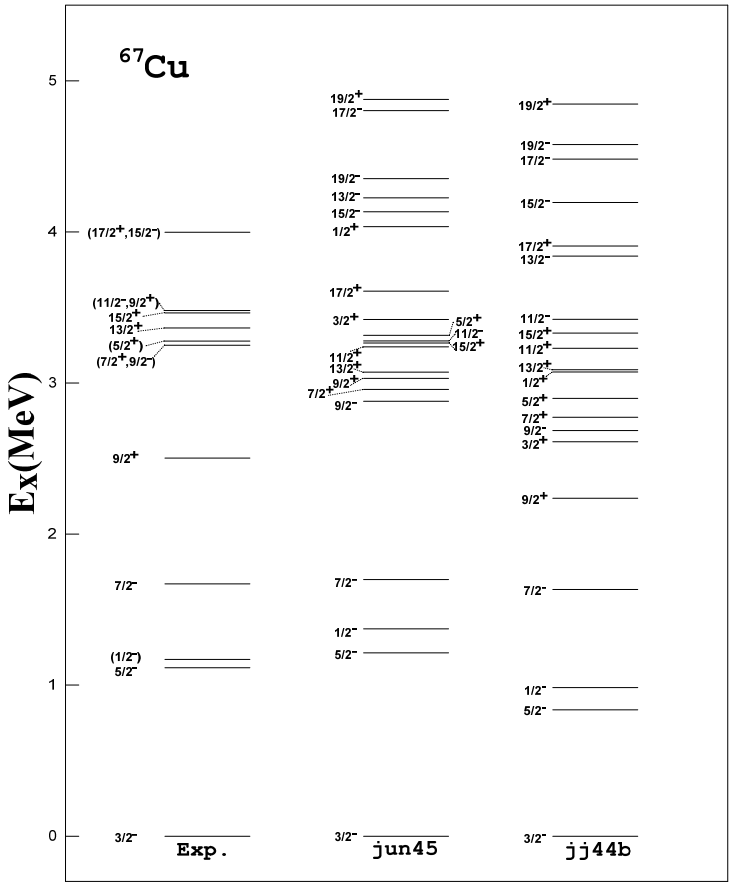


Figure 9 displays the comparison between our calculations with the experimental data for ^{67}Cu isotope. The two interactions used the present work are able two reproduce the ground state spin $3/2^-$. Both effective interactions are able to reproduce the correct ordering of the low-lying spins of $5/2^-$ and $7/2^-$. There are many unconfirmed values of the experimental data that have been confirmed in our theoretical. We had noticed that *jj44b* is better than *jun45* in comparison with the experimental data.

Figure 9 Comparison of calculated and experimental low-lying spectra for ^{67}Cu isotope with *jun45* and *jj44b* effective interactions. Experimental data take from NNDC (2018)



2.2 Binding energy

The binding energies with respect to the ^{56}Ni core have been calculated for the ground state of the Cu isotopes. The comparison of the calculated binding energy for all isotopes under the study using *jun45* and *jj44b* with the corresponding experimental binding energies (Huang et al., 2017) is tabulated in Table 1. The *jun45* and *jj44b* effective interactions are capable of reproducing the binding energies accurately for all the isotopes examined in the present study.

Table 1 Comparison between the calculated binding energy for each effective interaction in unit (MeV) and compared it with the experimental data taken from Huang et al. (2017)

Isotope	*Exp.*	*jun45*	*jj44b*
^{59}Cu	509.877	518.963	518.665
^{60}Cu	519.935	528.98	528.976
^{61}Cu	531.646	540.152	540.513
^{62}Cu	540.531	549.006	549.765
^{63}Cu	551.384	559.734	560.561
^{64}Cu	559.300	567.565	568.739
^{65}Cu	569.211	577.675	578.657
^{66}Cu	576.277	584.558	585.793
^{67}Cu	585.409	594.101	595.175

3 Conclusion

In the present work, unrestricted large-scale shell model calculations have been performed to study the energy levels and binding energies for the $^{59-67}$Cu isotopes using the shell model code NushellX@MSU by adopting the model space *jj44* with *jun45* and *jj44b* effective interactions. In our work there is no restriction imposed on the valance nucleons and all basis were included in the calculations. A conclusion can be drawn that the effective interactions are an adequate choice for nuclei lying in this mass region. The effective interaction *jj44b* is more consistent in reproducing the experimental data for nuclei investigated in the present study. The experimental binding energies are very well reproduced by the current shell model calculations.

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