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Sensors characterizations for regional boundary detectability in distributed parameter systems

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Abstract

The purpose of this paper is to explore the concept of asymptotic (respectively exponential) regional boundary detectability in connection with the characterizations of sensors. We consider a class of parabolic distributed systems and we give various results related with different types of measurements, of domains and boundary conditions. We also present some original results concerning diffusion systems which allow the possibility to construct an asymptotic (respectively exponential) regional boundary observer.

Keywords: Sensors; Measurements; Boundary detectability; Observer; Diffusion system

1. Introduction

The important problem of detecting distributed parameter systems has received much attention in the literatures ([1] and references therein) in order to determinate the asymptotic (respectively exponential) state of the considered systems. These systems are the general representation of several physical systems described by partial differential equations or differential equations with delays [2]. Many works are devoted to the study of control problems of systems [3]. Later, the notion of asymptotic (respectively exponential) regional ω-detectability was developed by Al-Saphory and El Jai [4,5] and was concentrated the detection in a part ω of the domain Ω by using characterizations of sensors. In this paper, we study the concept of asymptotic (respectively exponential) regional boundary detectability, in the case of a given region Γ located on the boundary $\partial\Omega$. Moreover, we show that there is a link between this notion and the number of sensors, their locations and the geometrical domains. Section 2 concerns the formulation problem, considered systems and strategic sensors. In Section 3, we introduce regional boundary detectability problem. Then we study the relationship between this notion and regional boundary observability and sensors structures. Thus, we show that the asymptotic (respectively exponential) regional boundary detectability on Γ can be extended from asymptotic (respectively exponential) regional detectability in ω. In Section 4, we give an application to diffusion system and in Section 5, we characterize an asymptotic (respectively exponential) regional boundary observer on Γ by the use of asymptotic (respectively exponential) regional boundary detectability on Γ and we present an application of these results.

2. Problem formulation

We consider a parabolic distributed parameter system and we suppose that the following assumptions are given:

- An open regular and bounded set Ω of $\mathbb{R}^n$ with smooth boundary $\partial\Omega$.
- A non-empty subset $\Gamma \subset \partial\Omega$, with positive measurement.
- For a given $T > 0$ let us set $\mathcal{Q} = \Omega\times]0,\infty[$, $\Theta = \partial\Omega\times]0,\infty[$.
- Separable Hilbert spaces $X, U, \mathcal{O}$, where X is the state space, U the control space and $\mathcal{O}$ is the observation space. We consider $X = H^1(\Omega)$, $U = L^2(0,\infty,\mathbb{R}^p)$ and $\mathcal{O} = L^2(0,\infty,\mathbb{R}^q)$, where p and q hold for the number of actuators and sensors.
- A second order linear differential operator A which generates a strongly continuous semi-group $(S_A(t))_{t\geq 0}$ on the Hilbert space $H^1(\Omega)$ and is self-adjoint with compact

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resolvent. The considered system is described by the following parabolic equation:

$$\begin{cases} \dfrac{\partial x}{\partial t}(\xi,t) = Ax(\xi,t) + Bu(t), & \mathcal{Q} \\ x(\xi,0) = x_0(\xi), & \overline{\Omega} \\ \dfrac{\partial x}{\partial v}(\eta,t) = 0, & \Theta \end{cases} \tag{2.1}$$

where $\overline{\Omega}$ holds for the closure of Ω and $x(\xi,0)$ is supposed to be in $H^1(\overline{\Omega})$ and unknown. The system (2.1) is defined with a Neumann boundary condition, $\partial x/\partial v$ holds for the outward normal derivative. The measurements may be obtained by the use of zone, pointwise or lines sensors which may be located inside of Ω (or on the boundary $\partial\Omega$) (see [1,6]). Then the output function can be written in the form

$$z(\cdot,t) = Cx(\cdot,t) \tag{2.2}$$

where the operators $B \in \mathscr{L}(U, H^1(\Omega))$ and $C \in \mathscr{L}(H^1(\overline{\Omega}), \mathcal{O})$ depend on the characterization of actuators and sensors employed with $u \in L^2(0,\infty,U)$ and $z \in L^2(0,\infty,\mathcal{O})$. The system (2.1) has a unique solution given by

$$x(\xi,t) = S_A(t)x_0(\xi) + \int_0^t S_A(t-\tau)Bu(\tau)\,\mathrm{d}\tau \tag{2.3}$$

The problem is how to detect asymptotically (respectively exponentially) the current state on Γ, by using structures sensors, i.e. to construct an asymptotic (respectively exponential) estimation to the restriction of the state $x(\xi,t)$ to Γ. We recall the following:

- A sensor is defined by any couple (D,f), where D, a non-empty closed subset of $\overline{\Omega}$, is the spatial support of the sensor and f defines the spatial distribution of the sensing measurements on D. Let us consider the following points:
  - The operator K defined by

    $$\begin{aligned} K : H^1(\Omega) &\to L^2(0,\infty,\mathbb{R}^q) \\ x &\to CS_A(t)x \end{aligned} \tag{2.4}$$

    and in the case of internal zone sensors is linear and bounded with an adjoint

    $$\begin{aligned} K^* : L^2(0,\infty,\mathbb{R}^q) &\to H^1(\Omega) \\ z^* &\to \textstyle\int_0^t S_A^*(\tau)C^*z^*(\cdot,\tau)\,\mathrm{d}\tau \end{aligned} \tag{2.5}$$

  - The trace operator of order zero

    $$\gamma_0 : H^1(\Omega) \to H^{1/2}(\partial\Omega)$$

    is linear, subjective and continuous with adjoint denoted by γ_0^*.
  - A subregion Γ of $\partial\Omega$ and let χ_Γ be the function defined by

    $$\begin{aligned} \chi_\Gamma : H^{1/2}(\partial\Omega) &\to H^{1/2}(\Gamma) \\ x &\to \chi_\Gamma = x|_\Gamma \end{aligned} \tag{2.6}$$

    where $x|_\Gamma$ is the restriction of the state x to Γ. We denote by χ_Γ^* the adjoint of χ_Γ.
  - The operator $H_\Gamma : L^2(0,\infty,\mathbb{R}^q) \to H^{1/2}(\Gamma)$ is given by

    $$H_\Gamma = \chi_\Gamma \gamma_0 K^*$$

- The autonomous system associated to Eqs. (2.1) and (2.2) is said to be exactly (respectively weakly) regionally boundary observable on Γ if

  $$\mathrm{Im} H_\Gamma = H^{1/2}(\Gamma) \quad (\text{respectively } \overline{\mathrm{Im} H_\Gamma} = H^{1/2}(\Gamma))$$

- The suite $(D_i, f_i)_{1\le i\le q}$ of sensors is said to be Γ-strategic if the system (2.1) together with the output function (2.2) is weakly regionally boundary observable on Γ [7]. For the dual results concerning the actuators structures, see [8–12].

3. Asymptotic (respectively exponential) regional boundary detectability

The main reason for introducing asymptotic (respectively exponential) regional boundary detectability in a given region Γ is (see Fig. 1), the possibility to observe asymptotically (respectively exponentially) the current state of the original system. This work is an extension of [5].

3.1. Definitions and characterizations

- The semi-group $(S_A(t))_{t\ge 0}$ is said to be asymptotically (respectively exponentially) stable on the space $H^{1/2}(\partial\Omega)$, if for every initial state $x_0 \in H^1(\Omega)$ the solution x corresponding to the autonomous system associated to Eq. (2.1), converges asymptotically (respectively exponentially) to zero as t tends to ∞. It is easy to see that the system (2.1) is exponentially stable on $\partial\Omega$ if and only if, for some positive constants M and α, we have

  $$\| \gamma_0 S_A(t) \|_{H^{1/2}(\partial\Omega)} \le M\,\mathrm{e}^{-\alpha t}, \quad \forall t \ge 0$$

  If $(S_A(t))_{t\ge 0}$ is an asymptotically (respectively exponentially) stable semi-group on $H^{1/2}(\partial\Omega)$, then for all $x_0 \in H^1(\Omega)$, the solution of the associated autonomous system satisfies

  $$\lim_{t\to\infty} \| x \|_{H^{1/2}(\partial\Omega)} = \lim_{t\to\infty} \| \gamma_0 S_A(t)x_0 \|_{H^{1/2}(\partial\Omega)} = 0 \tag{3.1}$$

- The system (2.1) is said to be asymptotically (respectively exponentially) $\partial\Omega$-stable, if the operator A generates a semi-group which is asymptotically (respectively exponentially) stable on $H^{1/2}(\partial\Omega)$. In the finite dimensional

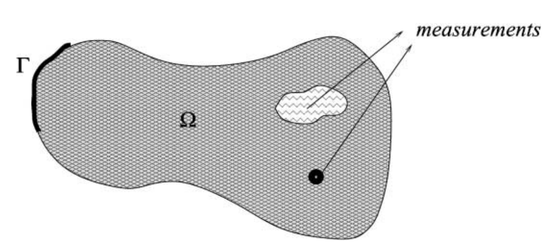

Fig. 1. How to detect regional boundary state on Γ.

linear systems, the concept of exponential boundary stability is equivalent to asymptotic boundary stability. It is not the case if the state space X is infinite dimensional.

- The system (2.1) together with the output (2.2), is said to be asymptotically (respectively exponentially) $\partial\Omega$-detectable, if there exists an operator $H_{\partial\Omega} = \gamma_0 K^* : \mathcal{O} \to H^{1/2}(\partial\Omega)$ such that $A - H_{\partial\Omega}C$ generates a strongly continuous semi-group $(S_{H_{\partial\Omega}}(t))_{t\geq 0}$ which is asymptotically (respectively exponentially) stable on $H^{1/2}(\partial\Omega)$.
- If a system is asymptotically (respectively exponentially) boundary detectable, then it is possible to construct an asymptotic (respectively exponential) boundary observer for the original system. If we consider the system

$$\begin{cases} \dfrac{\partial y}{\partial t}(\xi,t) = Ay(\xi,t) + Bu(t) \\ \qquad\qquad + H_{\partial\Omega}(z(\cdot,t) - Cy(\xi,t)), & \mathcal{Q} \\ y(\xi,0) = y_0(\xi), & \overline{\Omega} \\ \dfrac{\partial y}{\partial v}(\eta,t) = 0, & \Theta \end{cases} \tag{3.2}$$

then $y(\xi,t)$ estimates asymptotically (respectively exponentially) the state $x(\xi,t)$ because the error $e(\xi,t) = x(\xi,t) - y(\xi,t)$ satisfies $\partial e/\partial t(\xi,t) = (A - H_{\partial\Omega}C)e(\xi,t)$ with $e(\xi,0) = x_0(\xi,t) - y_0(\xi,t)$. Then if the system is asymptotically (respectively exponentially) boundary detectable, it is possible to choose $H_{\partial\Omega}$ which realizes $\lim_{t\to\infty} \| e \|_{H^{1/2}(\partial\Omega)} = 0$.
- The system (2.1) together with the output function (2.2) is said to be asymptotically (respectively exponentially) boundary observable if there exists a dynamical system which is an asymptotic (respectively exponential) boundary observer for the systems (2.1)–(2.2).

Remark 1. In this paper, we only need the relation (3.1) to be true on a given subregion Γ of the boundary $\partial\Omega$:

$$\lim_{t\to\infty} \| x \|_{H^{1/2}(\Gamma)} = \lim_{t\to\infty} \| \chi_\Gamma \gamma_0 S_A(t) x_0 \|_{H^{1/2}(\Gamma)} = 0 \tag{3.3}$$

We may refer to this as asymptotic (respectively exponential) regional boundary stability on Γ (or Γ-stability).

Definition 1. The system (2.1) is said to be asymptotically (respectively exponentially) regionally boundary stable on Γ, if the operator A generates a semi-group which is asymptotically (respectively exponentially) regionally Γ-stable.

Definition 2. The system (2.1) together with the output function (2.2) is said to be asymptotically (respectively exponentially) regionally boundary detectable on Γ (or Γ-detectable) if there exists an operator

$$H_\Gamma : \mathcal{O} \to H^{1/2}(\Gamma)$$

such that $A - H_\Gamma C$ generates a strongly continuous semi-group $(S_{H_\Gamma}(t))_{t\geq 0}$ which is asymptotically (respectively exponentially) stable on $H^{1/2}(\Gamma)$. Hence, the following are clear:

1. A system which is exponentially regionally Γ-detectable, is asymptotically regionally Γ-detectable.
2. A system which is asymptotically (respectively exponentially) regionally Γ-detectable, is asymptotically (respectively exponentially) regionally Γ_1-detectable, for every subset Γ_1 of Γ.

However, one can deduce the following important results.

Corollary 1. *If a system is exactly regionally observable in $\overline{\omega}$, then it is asymptotically (respectively exponentially) regionally observable in $\overline{\omega}$.*

Corollary 2. *If the system* (2.1) *together with output function* (2.2) *is exactly regionally Γ-observable, then it is asymptotically (respectively exponentially) regionally Γ-detectable.*

From this result, we can easily deduce that there exists $\gamma > 0$ such that

$$\| CS_A(t)x \|_{L^2(0,T,\mathcal{O})} \geq \gamma \| \chi_\Gamma \gamma_0 S_A(t) x \|_{H^{1/2}(\Gamma)}, \quad \forall x \in H^{1/2}(\Gamma)$$

Thus, the notion of asymptotically (respectively exponentially) regionally Γ-detectability is a weaker property than the exact regional observability on Γ. For more details, see [4,5,17].

3.2. Sensors and regional boundary detectability

In this section, we study the relation between internal and boundary regional detectability by using various sensors and we give a sufficient condition of Γ-detectability.

3.2.1. Preliminaries

In this subsection, we present a method which allows the detection of the current state $x(\xi,t)$ on Γ, based on the internal regional detectability. This method is an extension to [1,5]. Suppose that the measurements are given by different sensors, may be pointwise and zone (internal or on the boundary $\partial\Omega$). For the considered systems, we can show that it is possible to link internal regional detectability and regional boundary detectability on Γ. In this case, we consider the following:

- The extension operator (see [13]).

  $$\mathcal{R} : H^{1/2}(\partial\Omega) \to H^1(\Omega)$$

  is continuous, linear and is defined by

  $$\gamma_0 \mathcal{R} h(\xi,t) = h(\xi,t), \quad \forall h \in H^{1/2}(\partial\Omega) \tag{3.4}$$

- Let $r > 0$ be an arbitrary and sufficiently small real number and let

  $$E = \bigcup_{x\in\Gamma} B(x,r) \quad \text{and} \quad \omega_r = E \cap \Omega$$

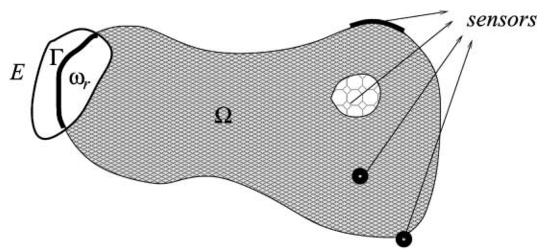

Fig. 2. Domain Ω, subdomain ω_r, considered region Γ and sensors.

where $B(x,r)$ is the ball of radius r centered in $x(\xi,t)$, and Γ is a part of $\overline{\omega}$ (Fig. 2).

Proposition 1. *If the system* (2.1) *together with output function* (2.2) *is asymptotically (respectively exponentially) regionally $\overline{\omega}_r$-detectable, then it is asymptotically (respectively exponentially) regionally Γ-detectable.*

Proof. Let $x(\xi,t)\in H^{1/2}(\Gamma)$ and $\overline{x}(\xi,t)$ be an extension to $H^{1/2}(\partial\Omega)$. So, using Eq. (3.4) and the trace theorem, there exists $\mathscr{R}\overline{x}(\xi,t)\in H^1(\Omega)$ with a bounded support such that

$$\gamma_0(\mathscr{R}\overline{x}(\xi,t))=\overline{x}(\xi,t) \tag{3.5}$$

Since the systems (2.1)–(2.2) is asymptotically (respectively exponentially) regionally $\overline{\omega}_r$-detectable, then it is asymptotically (respectively exponentially) regionally ω_r-detectable [5]. Thus, there exists an operator $\chi_{\omega_r}K^*:\mathscr{O}\to H^1(\omega_r)$ defined by

$$H_{\omega_r}z(\cdot,t)=\chi_{\omega_r}K^*z(\cdot,t)$$

such that $A-H_{\omega_r}C$ generates a strongly continuous semi-group $(S_{H_{\omega_r}}(t))_{t\geq 0}$ which is asymptotically (respectively exponentially) stable on $H^1(\omega_r)$. For every $z\in\mathscr{O}$, we then have

$$\chi_{\omega_r}K^*z(\cdot,t)=\chi_{\omega_r}\mathscr{R}\overline{x}(\xi,t)$$

and hence

$$\chi_\Gamma(\gamma_0\chi_{\omega_r}K^*z)(\cdot,t)=x(\xi,t) \tag{3.6}$$

Consequently there exists an operator

$$H_\Gamma=\chi_\Gamma(\gamma_0\chi_{\omega_r}K^*z):\mathscr{O}\to H^{1/2}(\Gamma)$$

such that $A-H_\Gamma C$ generates un semi-group $(S_{H_\Gamma}(t))_{t\geq 0}$ which is asymptotically (respectively exponentially) stable on $H^{1/2}(\Gamma)$. Finally, the systems (2.1)–(2.2) is asymptotically (respectively exponentially) regionally Γ-detectable.

3.2.2. Sufficient condition for Γ-detectability

In this section, we characterize the concept of asymptotically (respectively exponentially) Γ-detectability in connection with sensors structures as in [5]. For that purpose, we consider the system (2.1) with measurements taken by q sensors. The output (2.2) is given by $z(t)=z_1(t),\ldots,z_q(t)$. In the case of pointwise sensors, $z_i(t)=x(b_i,t)$ with $b_i\in\overline{\Omega}$ for $1\leq i\leq q$, and in the case of zone sensors, $z_i(t)=\int_{D_i}x(\xi,t)f_i(\xi)\,\mathrm{d}\xi$ with $D_i\subset\overline{\Omega}$. For boundary zone sensors, we have $z_i(t)=\int_{\Gamma_i}x(\eta,t)f_i(\eta)\,\mathrm{d}\eta$ with $\Gamma_i\subset\partial\Omega$, for $1\leq i\leq q$. Assume that there exists a complete set of eigenfunctions (φ_i) of A, associated to the eigenvalues λ_i with a multiplicity m_i such that $m=\sup_i m_i$ is finite. For $\xi=(\xi_1,\ldots,\xi_n)\in\Omega$ and $i=(i_1,\ldots,i_n)\in IN^n$, let $\overline{\xi}=(\xi_1,\ldots,\xi_{n-1})$ and $\overline{i}=(i_1,\ldots,i_{n-1})$. Suppose that the functions $(\psi_{\overline{i}})$ defined by $\psi_{\overline{i}}(\overline{\xi})=\chi_\Gamma\gamma_0\varphi_i(\xi)$, is a complete set in $H^{1/2}(\Gamma)$. If the system (2.1) has J unstable modes, then we have the following result.

Theorem 1. *Suppose that there are q sensors $(D_i,f_i)_{1\leq i\leq q}$ and that the spectrum of A contains J eigenvalues with non-negative real parts. The system* (2.1) *together with the output function* (2.2) *is Γ-detectable if and only if*

1. $q\geq m$
2. *rank* $G_i=m_i,\quad \forall i,\ i=1,\ldots,J$ *with*

$$G=G_{ij}=\begin{cases}\langle\varphi_j,f_i\rangle_{D_i} & (\textit{zone sensor})\\ \varphi_j(b_i) & (\textit{pointwise sensor})\\ \langle\varphi_j,f_i\rangle_{\Gamma_i} & (\textit{boundary zone sensor})\end{cases}$$

where $j=1,\ldots,m_i$.

Proof. For brevity, the proof is limited to the case of zone sensors. In this case, the output function (2.2) is given by

$$z_i(t)=\int_{D_i}x(\xi,t)f_i(\xi)\,\mathrm{d}\xi \tag{3.7}$$

Under the assumptions in Section 2, the system (2.1) may be decomposed into the following forms:

$$\begin{cases}\dfrac{\partial x_1}{\partial t}(\xi,t)=A_1x_1(\xi,t)+PB\,u(t), & \mathscr{Q}\\ x_1(\xi,0)=x_{1_0}(\xi), & \overline{\Omega}\\ \dfrac{\partial x_1}{\partial v}(\eta,t)=0, & \Theta\end{cases} \tag{3.8}$$

where $x_1(\xi,t)$ is the component state of the unstable part of the system (2.1) and

$$\begin{cases}\dfrac{\partial x_2}{\partial t}(\xi,t)=A_2x_2(\xi,t)+(I-P)Bu(t), & \mathscr{Q}\\ x_2(\xi,0)=x_{2_0}(\xi), & \overline{\Omega}\\ \dfrac{\partial x_2}{\partial v}(\eta,t)=0, & \Theta\end{cases} \tag{3.9}$$

where $x_2(\xi,t)$ is the component state of the stable part of the system (2.1), where P and $I-P$ are the projections of the unstable (respectively stable) subsystems of Eq. (2.1). Thus, the state vector may be given by $x(\xi,t)=[x_1(\xi,t)\,x_2(\xi,t)]^{\mathrm{T}}$ and the operator A_1 is represented by a matrix of order $(\sum_{i=1}^J m_i,\sum_{i=1}^J m_i)$ defined by $A_1=\mathrm{diag}[\lambda_1,\ldots,\lambda_1,\lambda_2,\ldots,\lambda_2,\ldots\lambda_J,\ldots,\lambda_J]$ and $PB=[G_1^{\mathrm{T}},G_2^{\mathrm{T}},\ldots,G_J^{\mathrm{T}}]$. By using the condition (2) of this theorem, we deduce that the suite $(D_i,f_i)_{1\leq i\leq q}$ of sensors is Γ-strategic for the unstable part of the system (2.1), the subsystem (3.8) is weakly regionally boundary observable on Γ, and since it is finite dimensional, then it is exactly regionally boundary observable on Γ. Therefore, it is asymptotically (respectively exponentially) regionally Γ-detectable, and hence there exists an operator H_Γ^1 such that $A_1 - H_\Gamma^1 C$ which is satisfied the following:

$$\exists M_\Gamma^1, \alpha_\Gamma^1 > 0 \text{ such that } \| \mathrm{e}^{(A_1 - H_\Gamma^1 C)t} \|_{H^{1/2}(\Gamma)} \leq M_\Gamma^1 \, \mathrm{e}^{-\alpha_\Gamma^1 (t)}, \quad \forall t$$

and then we have

$$\| x_1 \|_{H^{1/2}(\Gamma)} \leq M_\Gamma^1 \, \mathrm{e}^{-\alpha_\Gamma^1 (t)} \, \| \chi_\Gamma \gamma_0 P x_0 \|_{H^{1/2}(\Gamma)}, \quad \forall t$$

Since the semi-group generated by the operator A_2 is asymptotically (respectively exponentially) regionally boundary stable, then there exist M_Γ^2, $\alpha_\Gamma^2 > 0$ such that

$$\| x_2 \|_{H^{1/2}(\Gamma)} \leq M_\Gamma^2 \, \mathrm{e}^{-\alpha_\Gamma^2 (t)} \, \| (I - P) x_0 \|_{H^{1/2}(\Gamma)} + \int_0^t M_\Gamma^2 \, \mathrm{e}^{-\alpha_\Gamma^2 (t-\tau)} \, \| (I - P) x_0 \|_{H^{1/2}(\Gamma)} \| \, u(\tau) \, \| \, \mathrm{d}\tau$$

and therefore $x(\xi, t)$ converges to zero as $t \to \infty$. Thus, the system (2.1) together with the output function (3.7) is asymptotically (respectively exponentially) regionally Γ-detectable.

Reciprocally, if the system (2.1) together with the output function (3.7) is asymptotically (respectively exponentially) regionally Γ-detectable, there exists an operator $H_\Gamma \in \mathscr{L}(L^2(0, \infty, \mathbb{R}^q), H^{1/2}(\Gamma))$, such that $A - H_\Gamma C$ generates a strongly continuous semi-group $(S_{H_\Gamma}(t))_{t \geq 0}$, asymptotically (respectively exponentially) regionally Γ-stable on the space $H^{1/2}(\Gamma)$, then there exists M_Γ and $\alpha_\Gamma > 0$ such that

$$\| \chi_\Gamma \gamma_0 S_{H_\Gamma}(t) \|_{H^{1/2}(\Gamma)} \leq M_\Gamma \, \mathrm{e}^{-\alpha_\Gamma (t)}$$

Thus, the unstable subsystem (3.8) is asymptotically (respectively exponentially) regionally Γ-detectable. Since this subsystem is of finite dimensional, then it is exactly regionally boundary observable. Therefore, Eq. (3.8) is weakly regionally observable and hence the suite $(D_i, f_i)_{1 \leq i \leq q}$ of sensors is Γ-strategic, i.e. $[K \gamma_0^* \chi_\Gamma^* x^* = 0 \Rightarrow x^* = 0]$ [14]. For $x^* \in H^{1/2}(\Gamma)$, we have

$$K \gamma_0^* \chi_\Gamma^* x^* = \left(\sum_{j=1}^{J} \mathrm{e}^{\lambda_j t} \langle \varphi_j, \gamma_0^* \chi_\Gamma^* x^* \rangle_\Gamma \langle \varphi_j, f_i \rangle_{D_i} \right)_{1 \leq i \leq q}$$

If the suite $(D_i, f_i)_{1 \leq i \leq q}$ of sensors is not Γ-strategic for the unstable system (3.8), there exists $(x^* \neq 0) \in H^{1/2}(\Gamma)$, such that $K \gamma_0^* \chi_\Gamma^* x^* = 0$, this leads

$$\sum_{j=1}^{J} \langle \chi_\Gamma \gamma_0 \varphi_1, x^* \rangle_\Gamma \langle \varphi_j, f_i \rangle_{D_i} = 0$$

The state vectors x_i may be given by

$$x_i = [\langle \psi_1, x^* \rangle_\Gamma \langle \psi_J, x^* \rangle_{D_i}]^{\mathrm{T}} \neq 0$$

we then obtain $G_i x_i = 0$ for all $i \in \{1, \ldots, J\}$ and therefore rank $G_i \neq m_i$, $\forall i$.

Remark 2. From this theorem, we can deduce the following:

1. The number of sensors is greater than or equal to the largest multiplicity of the eigenvalues.
2. The multiplicity of the eigenvalues may be reduced to one [15]. Consequently, an asymptotically (respectively exponentially) regionally Γ-detectability can be guaranteed by using one sensor.

4. Applications to diffusion systems

In this section, we consider the distributed diffusion systems defined on Ω. We explore various results related to different types of measurements, domains and boundary conditions. However, if we suppose that

$$\frac{\alpha^2}{\beta^2} \notin IQ \tag{4.1}$$

then $m = 1$ and one sensor may be sufficient for Γ-detectability. The considered systems may be described in the following forms.

Case of a rectangular domain $\Omega =]0, \alpha[\times]0, \beta[$:

$$\begin{cases} \dfrac{\partial x}{\partial t}(\xi_1, \xi_2, t) = \dfrac{\partial^2 x}{\partial \xi_1^2}(\xi_1, \xi_2, t) + \dfrac{\partial^2 x}{\partial \xi_2^2}(\xi_1, \xi_2, t) + x(\xi_1, \xi_2, t), & \mathscr{Q} \\ \dfrac{\partial x}{\partial v}(\eta_1, \eta_2, t) = 0, & t > 0 \\ x(\xi_1, \xi_2, 0) = x_0(\xi_1, \xi_2), & \overline{\Omega} \end{cases} \tag{4.2}$$

with measurements obtained by the output function given as in Eq. (2.2). Let $\Gamma =]0, \alpha[\times \{\beta\}$ be a region of $]0, \alpha[\times]0, \beta[$ with $\alpha = \xi_1 + l_1$ and $\beta = \xi_2 + l_2$. In this case, the eigenfunctions of the dynamic of the system (4.2) for Dirichlet boundary conditions, are given by

$$\psi_{ij}(\xi_1, \xi_2) = \frac{2}{\sqrt{\alpha\beta}} \cos\left(i\pi \frac{\xi_1}{\alpha} \right) \cos\left(j\pi \frac{\xi_2}{\beta} \right) \tag{4.3}$$

the associated eigenvalues are

$$\lambda_{ij} = -\left(\frac{i^2}{\alpha^2} + \frac{j^2}{\beta^2} \right) \pi^2 \tag{4.4}$$

Case of a disk $\Omega = D(0, 1)$:

$$\begin{cases} \dfrac{\partial x}{\partial t}(r, \theta, t) = \dfrac{\partial^2 x}{\partial r^2}(r, \theta, t) + \dfrac{\partial^2 x}{\partial \theta^2}(r, \theta, t) + x(r, \theta, t), & \mathscr{Q} \\ x(1, \theta, t) = 0, & \theta \in [0, 2\pi], \ t > 0 \\ x(r, \theta, 0) = x_0(r, \theta), & \overline{\Omega} \end{cases} \tag{4.5}$$

In this case, $\Gamma = D(1, \theta_i)_{2 \leq i \leq q}$ is a region of $\partial\Omega$ with $\theta_i \in [0, 2\pi]$. Thus, the eigenfunctions and eigenvalues concerning the dynamic of the system (4.5) are given by

$$\lambda_{ij} = -\beta_{ij}^2, \quad i \geq 0, \ j \geq 1 \tag{4.6}$$

where β_{ij} are the zeros of the Bessel functions J_i and

$$\begin{cases} \psi_{0j}(r,\theta) = J_0(\beta_{0j} r), & j \geq 1 \\ \psi_{ij_1}(r,\theta) = J_i(\beta_{ij_1} r)\cos(i\theta), & i, j_1 \geq 1 \\ \psi_{ij_2}(r,\theta) = J_i(\beta_{ij_2} r)\sin(i\theta), & i, j_2 \geq 1 \end{cases} \tag{4.7}$$

with the multiplicity $m_i = 2$ for all $i, j \neq 0$ and $m_i = 1$ for $i, j = 0$. It is necessary therefore to consider at least two sensors $(D_i, f_i)_{2 \leq i \leq q}$, where $D_i = (r_i, \theta_i)_{2 \leq i \leq q}$ in order to detect the system on Γ. If we consider the case of Dirichlet or mixed boundary conditions, we obtain different functions. We give some practical examples, the results follow from symmetry considerations.

4.1. Case of a boundary sensor

This section concerns the locations of the boundary pointwise (respectively zone) sensor for ensuring Γ-detectability.

4.1.1. Pointwise sensor

Consider the system (4.2) with Neumann boundary conditions. We then study the following cases.

Case of Fig. 3:

Suppose that the sensor (b, δ_b) is located on $b = (b_1, 0)$. The output function is given by

$$z(t) = \int_{\partial\Omega} x(\eta_1, \eta_2, t)\delta(\eta_1 - b_1, \eta_2)\, d\eta_1\, d\eta_2 \tag{4.8}$$

Thus, we obtain the result.

Corollary 3. *If the sensor support* $b \in \partial\Omega$, *then the systems* (4.2)–(4.8) *is* Γ*-detectable if there exists an integer* $i, 1 \leq i \leq J$ *such that* $2ib_1/\alpha$ *is even.*

Case of Fig. 4:

We consider the system (4.5) together with the output function

$$z_i(t) = \int_{\partial\Omega} x(1, \theta_i, t) f(1, \theta_i)\, d\theta_i, \quad 0 \leq \theta_i \leq 2\pi, \ t > 0 \tag{4.9}$$

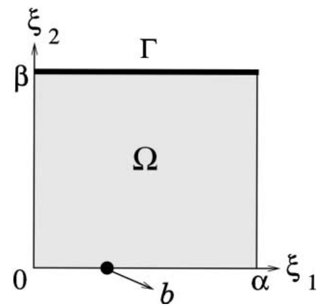

Fig. 3. Rectangular domain, region Γ and location b of boundary pointwise sensor.

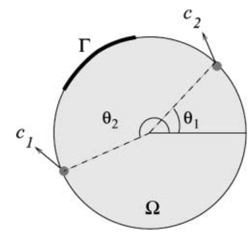

Fig. 4. Circular domain, region Γ and locations c_1, c_2 of boundary pointwise sensors.

where $i = 2, \ldots, q$. The eigenfunctions and eigenvalues are given as in the Eqs. (4.6) and (4.7) with multiplicity $m_i = 2$ for $i, j \neq 0$ and $m_i = 1$ for all $i, j = 0$. In this case the Γ-detectability is required at least two pointwise sensors may be located at the polar coordinates $c_i = (1, \theta_i)$, where $\theta_i \in [0, 2\pi]$ and $2 \leq i \leq q$. We have the following result.

Corollary 4. *The systems* (4.5)–(4.9) *is* Γ*-detectable if for every* $i, 1 \leq i \leq J$, $i(\theta_1 - \theta_2)/\pi$ *is not an integer.*

4.1.2. Zone sensor

Let us consider the system (4.2) with the Neumann boundary conditions and output function (2.2). We study this case with different geometrical domains.

Case of Fig. 5:

Now the output function (2.2) is given by

$$z(t) = \int_{\Gamma_0} x(\eta_1, \eta_2, t) f(\eta_1, \eta_2)\, d\eta_1\, d\eta_2 \tag{4.10}$$

where $\Gamma_0 \subset \partial\Omega$ is the boundary support of the sensor and $f \in L^2(\Gamma_0)$. In the case where the support of the sensor (D, f) is on one side (see Fig. 5), i.e. supp $(f) = \{0\} \times [\eta_{1_0} - l_1, \eta_{1_0} - l_2] = \Gamma_0$, then we have the following corollary.

Corollary 5. *If the function* f *is symmetric with respect to* $\eta_1 = \eta_{1_0}$, *then the system* (4.2) *together with the output function* (4.10) *is* Γ*-detectable if there exists an integer* $i, 1 \leq i \leq J$ *such that* $i\eta_{1_0}/a$ *is even.*

When the support of the sensor is on two sides, i.e. $\overline{\Gamma} = [0, \overline{\eta}_{1_0} + l_1] \times \{0\} \bigcup \{0\} \times [0, \overline{\eta}_{2_0} + l_2] = \Gamma_1 \bigcup \Gamma_2$, where $\overline{\Gamma} \subset \partial\Omega$ (see Fig. 5). We obtain the following result.

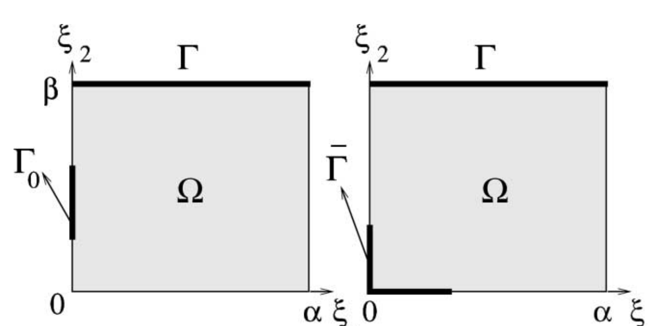

Fig. 5. Domain Ω, region Γ and locations $\Gamma_0, \overline{\Gamma}$ of boundary zone sensors.

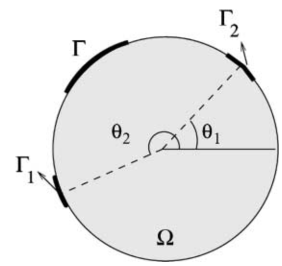

Fig. 6. Domain Ω, region Γ and locations Γ_1, Γ_2 of boundary zone sensors.

Corollary 6. *Suppose that the function $f|_{\Gamma_k}$ is symmetric with respect to $\eta_k = \overline{\eta}_{k_0}$, $k = 1, 2$. Then the system* (4.2) *together with the output function* (4.10) *is Γ-detectable if there exist two integer i, j, $1 \leq i, j \leq J$ such that $2i\overline{\eta}_{1_0}/\alpha$ and $2i\overline{\eta}_{2_0}/\beta$ are even.*

Case of Fig. 6:

Here, we consider the system (4.5) with the output function

$$z_i(t) = \int_{\Gamma_i} x(1, \theta_i, t) f(1, \theta_i)\, \mathrm{d}\theta_i,$$
$$0 \leq \theta_i \leq 2\pi, \quad t > 0 \tag{4.11}$$

In this case, it is necessary to have at least two boundary zone sensors $(\Gamma_i, f_i)_{2 \leq i \leq q}$ with $\Gamma_i = (1, \theta_i)$ and if the function $f|_{\Gamma_i}$ is symmetric with respect to $\theta = \theta_i$ as in (Fig. 6). So, we have the following corollary.

Corollary 7. *The system* (4.5) *together with the output function* (4.11) *is Γ-detectable if for every i, $1 \leq i \leq J$, $i(\theta_1 - \theta_2)/\pi$ is not an integer.*

4.2. Case of an internal sensor

4.2.1. Internal pointwise sensor

This section concerns the locations of the internal pointwise (respectively zone) sensor may be discussed in the following cases.

Case of Fig. 7:

The system (4.2) is augmented with the following output

$$z(t) = \int_{\Omega} x(\xi_1, \xi_2, t)\delta(\xi_1 - b_1, \xi_2 - b_2)\, \mathrm{d}\xi_1\, \mathrm{d}\xi_2 \tag{4.12}$$

where $b = (b_1, b_2)$ is the location of the pointwise sensor in Ω, defined as in Fig. 7. Then we obtain the following result.

Corollary 8. *If the sensor is located in $b = (b_1, b_2)$, then the systems* (4.2)–(4.12) *is not Γ-detectable if there exists $i, j \in \{1, \ldots, J\}$ such that ib_1/α and jb_2/α are integers.*

Case of Fig. 8:

Consider the system (4.5) with the output function defined by

$$z_i(t) = \int_{\Omega} x(r_i, \theta_i, t) f_i(r_i, \theta_i)\, \mathrm{d}r_i\, \mathrm{d}\theta_i,$$
$$0 \leq \theta_i \leq 2\pi, \quad 0 < r_i < \frac{1}{2} \tag{4.13}$$

where $2 \leq i \leq q$ and $t > 0$. The sensors may be located in $c_1 = (r_1, \theta_1)$ and $c_2 = (r_2, \theta_2) \in \Omega$ (see Fig. 8).

Corollary 9.

1. *The systems* (4.5)–(4.13) *is Γ-detectable if there exists $i, j \in \{1, \ldots, J\}$ such that $i(\theta_1 - \theta_2)/\pi$ is an integer.*
2. *If $r_1 = r_2$, the systems* (4.5)–(4.13) *is not Γ-detectable if there exists $i, j \in \{1, \ldots, J\}$ such that $i(\theta_1 - \theta_2)/\pi$ is an integer.*

4.2.2. Filament sensors

We consider the case where $\Omega =]0, \alpha[\times]0, \beta[$ and $\Gamma =]0, \alpha[\times\{\beta\} \subset \partial\Omega$. If the observation recovered by the filament sensor (σ, δ_σ), where $\sigma = \mathrm{Im}(\Gamma_0)$ with $\Gamma_0 \in C^1(0, 1)$ (Fig. 9), then we have the following corollary.

Corollary 10. *If the curve σ is symmetric with respect to the line $\xi = \xi_0$, the systems* (4.2)–(4.12) *is not Γ-detectable if there exists $i, j \in \{1, \ldots, J\}$ such that $i\xi_{1_0}/\alpha$ and $j\xi_{2_0}/\beta$ are integers.*

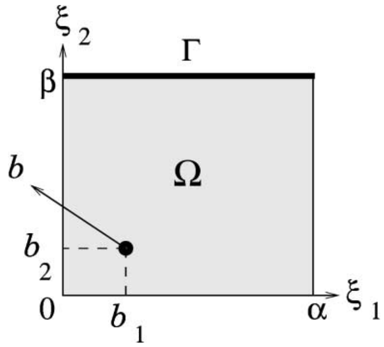

Fig. 7. Domain Ω, region Γ and locations b of internal pointwise sensors.

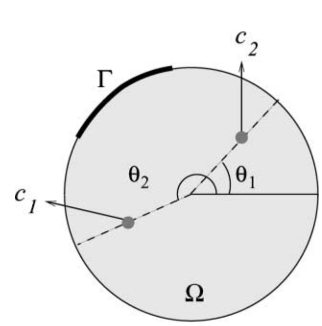

Fig. 8. Domain Ω, region Γ and locations c_1, c_2 of internal pointwise sensors.

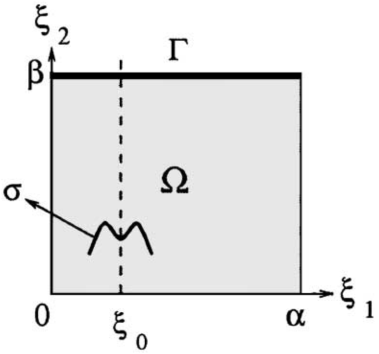

Fig. 9. Domain Ω, region Γ and location σ of internal filament sensor.

4.2.3. Internal zone sensor

In this case different domains are considered.

Case of Fig. 10:

The output function (2.2) can be written in the form

$$z(t) = \int_D x(\xi_1, \xi_2, t) f(\xi_1, \xi_2)\, \mathrm{d}\xi_1\, \mathrm{d}\xi_2 \tag{4.14}$$

where $D \subset \Omega$ is the location of the zone sensor and $f \in L^2(D)$. In this case (Fig. 10), the eigenfunctions and the eigenvalues are given, respectively, in the Eqs. (4.3) and (4.4). If the measurement support is a rectangle $D = [\xi_{1_0} - l_1, \xi_{1_0} + l_1] \times [\xi_{2_0} - l_2, \xi_{2_0} + l_2]$, we then have the following result.

Corollary 11. *Suppose that f_k is symmetric with respect to $\xi_k = \xi_{k_0}$, $k = 1, 2$. The system* (4.2) *together with the output function* (4.14) *is not Γ-detectable if there exists $i, j \in \{1, \ldots, J\}$ such that $i\xi_{1_0}/\alpha$ and $j\xi_{2_0}/\beta$ are integers.*

Case of Fig. 11:

Consider the system (4.2) augmented by the following output function:

$$z_i(t) = \int_{D_i} x(r_i, \theta_i, t) f(r_i, \theta_i)\, \mathrm{d}r_i\, \mathrm{d}\theta_i,$$

$$0 \le \theta_i \le 2\pi, \;\; 0 < r_i < \frac{1}{2}, \;\; 2 \le i \le q \tag{4.15}$$

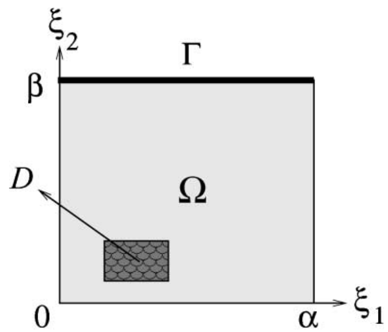

Fig. 10. Domain Ω, region Γ and location D of internal zone sensor.

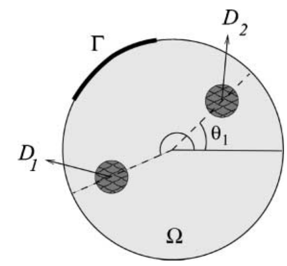

Fig. 11. Domain Ω, region Γ and locations D_1, D_2 of internal zone sensors.

where $D_i = (r_i, \theta_i)_{2 \le i \le q} \subset \Omega$ is defined as in (Fig. 11). In this case, for the regional detectability in Γ, at least two zone sensors are required. Thus, we have the following result.

Corollary 12. *Suppose that f_i and D_i, are symmetric with respect to $\theta = \theta_i$, for all $i, 2 \le i \le q$. Then the systems* (4.5)–(4.15) *is not Γ-detectable if there exists $i, j \in \{1, \ldots, J\}$ such that $i_0(\theta_1 - \theta_2)/\pi$ is an integer.*

Remark 3. The results given in this paper, can be extended to the case of Dirichlet boundary conditions as in [5]. In the next section, we illustrate an application regional boundary observer to this case.

5. Γ-observer and Γ-detectability

In this section, we give an approach which allows to determinate a regional asymptotic (respectively exponential) estimator of $Tx(\xi, t)$ on Γ, based on the Γ-detectability. This approach derives from Luenberger observer type as introduced in [16]. For that purpose, we recall some definitions concerning the regional boundary observer on Γ (see [17]).

5.1. Definitions and characterizations

Definition 3. Suppose that there exists a dynamical system with state $y(\xi, t) \in Y$ (a Hilbert space) given by

$$\begin{cases} \dfrac{\partial y}{\partial t}(\xi, t) = F_\Gamma y(\xi, t) + G_\Gamma u(t) + H_\Gamma z(\cdot, t), & \mathcal{Q} \\ y(\xi, 0) = y_0(\xi), & \overline{\Omega} \\ y(\eta, t) = 0, & \Theta \end{cases} \tag{5.1}$$

where F_Γ generates a strongly continuous semi-group which is asymptotically (respectively exponentially) regionally Γ-stable on the space Y, $G_\Gamma \in \mathscr{L}(U, Y)$ and $H_\Gamma \in \mathscr{L}(\mathcal{O}, Y)$. The system (5.1) defines an asymptotic (respectively exponential) regional Γ-estimator for $T_\Gamma = \chi_\Gamma \gamma_0 Tx$ if the following conditions hold:

1. $\lim_{t\to\infty}[T_\Gamma x(\xi, t) - y(\xi, t)] = 0, \quad \xi \in \Gamma.$
2. T_Γ maps $D(A)$ into $D(F_\Gamma)$, where $x(\xi, t)$ and $y(\xi, t)$ are the solutions of Eqs. (2.1), (2.2) and (5.1).

Definition 4. The system (5.1) specifies an asymptotical (respectively exponential) observer in Γ (or Γ-observer) for the system (2.1) together with the output function (2.2) if the following conditions hold:

1. There exists $R_\Gamma \in \mathscr{L}(\mathcal{O}, H^{1/2}(\Gamma))$ and $S_\Gamma \in \mathscr{L}(H^{1/2}(\Gamma))$ such that $R_\Gamma C + S_\Gamma T_\Gamma = I_\Gamma$.
2. $T_\Gamma A - F_\Gamma T_\Gamma = G_\Gamma C$ and $H_\Gamma = T_\Gamma B$.
3. The system (5.1) defines an asymptotic (respectively exponential) regional Γ-estimator for $T_\Gamma x(\xi, t)$.

Definition 5. The system (5.1) is said to be an identity Γ-observer for the system (2.1) together with the output function (2.2) if $T_\Gamma = I_\Gamma$ and $X = Y$.

Definition 6. The system (5.1) is said to be a reduced-order a Γ-observer for the system (2.1) together with the output function (2.2) if $X = \mathcal{O} \oplus Y$.

Definition 7. The system (2.1) together with the output function (2.2) is an asymptotically (respectively exponentially) regionally Γ-observable if there exists a dynamical system which is Γ-observer for the original system.

Proposition 2. *Suppose that the system* (2.1) *together with output function* (2.2) *is an asymptotically (respectively exponentially) regionally Γ-detectable then, the dynamical system*

$$\begin{cases} \dfrac{\partial y}{\partial t}(\xi, t) = Ay(\xi, t) + Bu(t) - H_\Gamma(Cy(\xi, t) - z(\cdot, t)), & \mathcal{Q} \\ y(\xi, 0) = 0, & \overline{\Omega} \\ y(\eta, t) = 0, & \Theta \end{cases} \tag{5.2}$$

is a Γ-observer of the systems (2.1) and (2.2), *if*

$$\lim_{t\to\infty}[x(\xi, t) - y(\xi, t)] = 0, \quad \xi \in \Gamma$$

Proof. Let

$$\phi(\xi, t) = x(\xi, t) - y(\xi, t)$$

where $y(\xi, t)$ is the solution of the system (5.2). Deriving the above equation and using the Eqs. (2.1) and (5.2), we obtain

$$\frac{\partial \phi}{\partial t}(\xi, t) = \frac{\partial x}{\partial t}(\xi, t) - \frac{\partial y}{\partial t}(\xi, t) = (A - H_\Gamma C)\phi(\xi, t)$$

The system (2.1) is Γ-detectable, there exists an operator $H_\Gamma \in \mathscr{L}(\mathcal{O}, H^{1/2}(\Gamma))$, such that $A - H_\Gamma C$ generates a strongly continuous semi-group $(S_{H_\Gamma}(t))_{t\geq 0}$ which is Γ-stable on $H^{1/2}(\Gamma)$, that is

$$\exists M_\Gamma, \alpha_\Gamma > 0 \text{ such that } \| \chi_\Gamma \gamma_0 S_{H_\Gamma}(t) \|_{H^{1/2}(\Gamma)} \leq M_\Gamma \,\mathrm{e}^{-\alpha_\Gamma(t)}, \quad \forall t$$

Finally, we have

$$\| \phi \|_{H^{\frac{1}{2}}(\Gamma)} \leq \| \chi_\Gamma \gamma_0 S_{H_\Gamma}(t) \|_{H^{1/2}(\Gamma)} \| \phi_0 \| \leq M_\Gamma \,\mathrm{e}^{-\alpha_\Gamma(t)} \| \phi_0 \| \tag{5.3}$$

with $\phi_0(\xi) = x_0(\xi) - y_0(\xi)$, and hence Eq. (5.3) allows the following result:

$$\lim_{t\to\infty}[x(\xi, t) - y(\xi, t)] = 0, \quad \xi \in \Gamma$$

The dynamical system (5.2) may be considered as an (identity) Γ-observer for the system (2.1)–(2.2) without needing the stability on Γ of the system (2.1).

Thus, the following statements are clear:

1. A system which is exactly regionally boundary observable on Γ, is asymptotically (respectively exponentially) regionally boundary Γ-observable.
2. A system which is asymptotically (respectively exponentially) observable in $\overline{\omega}$, is asymptotically (respectively exponentially) regionally boundary Γ-observable.
3. A system which is asymptotically (respectively exponentially) regionally boundary Γ-observable, is asymptotically (respectively exponentially) regionally boundary Γ_1-observable for every subset Γ_1 of Γ. For more details, see [5,17].

5.2. Application to Γ-observer for diffusion systems

Consider the case of two-dimensional distributed parameter diffusion system defined in $\Omega =]0,1[\times]0,1[$ and described by

$$\begin{cases} \dfrac{\partial x}{\partial t}(\xi_1, \xi_2, t) = \gamma\Delta(\xi_1, \xi_2, t) + \upsilon x(\xi_1, \xi_2, t), & \mathcal{Q} \\ x(\xi_1, \xi_2, 0) = x_0(\xi_1, \xi_2), & \overline{\Omega} \\ x(\eta_1, \eta_2, t) = 0, & t > 0 \end{cases} \tag{5.4}$$

where υ is a real number and the above system represents the heat-conduction problem (see [18]). and $\beta(\xi_1, \xi_2)$ is a real-valued function. Let $\Gamma =]0,1[\times\{0\}$ be a subregion of $\partial\Omega$ and suppose that there exists a sensor (b, δ_b) with $b = (b_1, b_2) \in]0,1[\times]0,1[$ (see Fig. 12).

Thus, the augmented output function may be written in the form

$$z(t) = x(b, t) \tag{5.5}$$

The eigenfunctions of the operator $(\gamma\Delta + \upsilon I)$ for the Dirichlet boundary conditions are defined by

$$\psi_{ij}(\xi_1, \xi_2) = \sqrt{2}\sin i\pi(b_1)$$

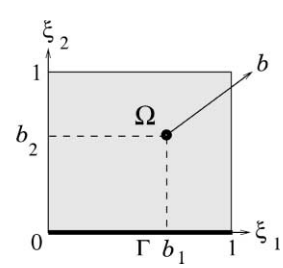

Fig. 12. Domain Ω, region Γ and sensor location $b = (b_1, b_2)$.

associated with the eigenvalues

$$\lambda_{ij} = \upsilon - \gamma i^2 \pi^2$$

Thus, using Remark 3, the system

$$\begin{cases} \dfrac{\partial y}{\partial t}(\xi_1, \xi_2, t) = \gamma \Delta(\xi_1, \xi_2, t) + \upsilon y(\xi_1, \xi_2, t) \\ \qquad\qquad - H_\Gamma(Cy(\xi_1, \xi_2, t) - z(t)), & \mathcal{Q} \\ y(\xi_1, \xi_2, 0) = y_0(\xi_1, \xi_2), & \overline{\Omega} \\ y(\eta_1, \eta_2, t) = 0, & \Theta \end{cases} \tag{5.6}$$

is a Γ-observer for the systems (5.4)–(5.5) if this system is Γ-detectable, that means that ib_1 is not an integer and then we have

$$\lim_{t\to\infty}[x(\xi, t) - y(\xi, t)] = 0, \quad \xi \in \Gamma$$

6. Conclusion

The concept developed in this paper is related to the asymptotic (respectively exponential) regional boundary Γ-detectability, in connection with the sensors characterizations, based to the internal asymptotic (respectively exponential) regional ω-detectability. A sufficient condition for this concept has been presented and applied to diffusion distributed systems in various situations of structures sensors. Moreover, these results are extended to the boundary Dirichlet conditions and applied to the case of an asymptotical (respectively exponential) Γ-observer. The dual results concerning the choice of actuators structures for the regional boundary stabilizability is under consideration.

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