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Sufficiency and Duality for E-differentiable Multiobjective Programming Problems Involving Generalized V-E-invex Functions

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Abstract

In this paper, a class of E-differentiable multiobjective programming problems with both inequality and equality constraints is considered. For E-differentiable functions, the concepts of V-E-pseudo-invexity, strictly V-E-pseudo-invexity and V-E-quasi-invexity are introduced. Based upon these notions of generalized V-E-invex functions, the sufficiency of the so-called E-Karush-Kuhn-Tucker optimality conditions are established for the considered E-differentiable vector optimization problems with both inequality and equality constraints. Furthermore, the so-called vector Mond-Weir E-dual problem is defined for the considered E-differentiable multiobjective programming problem and several E-duality theorems in the sense of Mond-Weir are derived also under appropriate generalized V-E-invexity assumptions.

Key words: V-E-invex function, Generalized V-E-invexity, E-differentiable function, E-optimality conditions, E-Mond-Weir duality.

AMS Subject Classification: 90C26, 90C30, 90C46, 26B25

1 Introduction

Convexity notion plays an important role to derive the optimality conditions and duality results for various scalar and vector optimization problems. However, many operational research problems that are modeled by various optimization problems are not convex. During the past decades, therefore, generalized convex functions received much attention. Various classes of differentiable and nondifferentiable generalized convex functions have appeared in literature, not only for scalar optimization problems, but also for multiobjective programming problems. Optimality conditions and duality theorems for differentiable and nondifferentiable optimization problems have been studied extensively in the literature (see, for example, [1], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [18], [19], [21], [22], [23], [24], [25], [26], and others).

One of such important generalizations of the convexity notion is the concept of invexity introduced by Hanson [15] for scalar optimization problems. Jeyakumar and Mond [17] introduced a new class of nonconvex differentiable vector-valued functions, namely V-invex functions, in order to resolve the difficulty of demanding the same function η for objective and constraint functions in extremum problems dealing with the concept of invexity introduced by Hanson [15] for scalar optimization problems. They established sufficient optimality criteria and duality results in the multiobjective static case for weak Pareto solutions under V-invexity hypotheses. Megahed et al. [20] presented the concept of an E-differentiable convex function which transforms a (not necessarily) differentiable convex function to a differentiable function based on the effect of an operator $E : R^n \to R^n$. Antczak and Abdulaleem [5] proved the so-called E-optimality conditions and Wolfe E-duality for E-differentiable vector optimization problems with both inequality and equality constraints. Abdulaleem [1] introduced a new concept of generalized convexity as a generalization of the notion of E-differentiable E-convexity. Namely, he defined the concept of E-differentiable E-invexity in the case of (not necessarily) differentiable vector optimization problems with E-differentiable functions. Recently, Abdulaleem [2] introduced a new concept of generalized convexity as a generalization of the E-differentiable E-invexity notion and the concept of V-invexity. Namely, he defined the concept of E-differentiable V-E-invexity in the case of (not necessarily) differentiable vector optimization problems with E-differentiable functions.

In this paper, a new class of nonconvex E-differentiable vector optimization problems with both inequality and equality constraints is considered in which the involved functions are generalized V-E-invex. Therefore, the concepts of V-E-pseudo-invex, strictly V-E-pseudo-invex and V-E-quasi-invex functions for E-differentiable vector optimization problems are introduced. Further, the sufficiency of the so-called E-Karush-Kuhn-Tucker optimality conditions are derived for the considered E-differentiable vector optimization problem under appropriate generalized V-E-invexity hypotheses. Furthermore, the so-called vector E-dual problems in the sense of Mond-Weir is defined for E-differentiable vector dual problems. Then, several Mond-Weir E-duality results are established between the considered E-differentiable multicriteria optimization problem and its Mond-Weir vector dual problem also under appropriate generalized V-E-invexity hypotheses.

2 Preliminaries

Let R^n be the n-dimensional Euclidean space and R^n_+ be its nonnegative orthant. The following convention for equalities and inequalities will be used in the paper. For any vectors $x = (x_1, x_2, ..., x_n)^T$ and $y = (y_1, y_2, ..., y_n)^T$ in R^n, we define:

(i) $x = y$ if and only if $x_i = y_i$ for all $i = 1, 2, \cdots, n$;

(ii) $x > y$ if and only if $x_i > y_i$ for all $i = 1, 2, \cdots, n$;

(iii) $x \geqq y$ if and only if $x_i \geqq y_i$ for all $i = 1, 2, \cdots, n$;

(iv) $x \geq y$ if and only if $x_i \geqq y_i$ for all $i = 1, 2, \cdots, n$ but $x \neq y$;

(v) $x \not> y$ is the negation of $x > y$.

Definition 2.1 *[1] Let $E : R^n \to R^n$. A set $M \subseteq R^n$ is said to be an E-invex set if and only if there exists a vector-valued function $\eta : M \times M \to R^n$ such that the relation*

$$E(u) + \lambda\eta(E(x), E(u)) \in M$$

holds for all $x, u \in M$ and any $\lambda \in [0, 1]$.

Let M be a nonempty E-invex subset of R^n.

Definition 2.2 *[20] Let $E : R^n \to R^n$ and $f : M \to R$ be a (not necessarily) differentiable function at a given point $u \in M$. It is said that f is an E-differentiable function at u if and only if $f \circ E$ is a differentiable function at u (in the usual sense), that is,*

$$(f \circ E)(x) = (f \circ E)(u) + \nabla(f \circ E)(u)(x - u) + \theta(u, x - u)\|x - u\|, \tag{2.1}$$

where $\theta(u, x - u) \to 0$ as $x \to u$.

Definition 2.3 *[2] Let $E : R^n \to R^n$ and $f : M \to R^k$ be an E-differentiable function on M. It is said that f is a V-E-invex function (a strictly V-E-invex function) with respect to η at $u \in M$ on M if, there exist functions $\eta : M \times M \to R^n$ and $\alpha_i : M \times M \to R_+ \setminus \{0\}$, $i = 1, 2, ..., k$, such that, for each $x \in M$ $(E(x) \neq E(u))$, the inequalities*

$$f_i(E(x)) - f_i(E(u)) \geqq \alpha_i(E(x), E(u))\nabla f_i(E(u))\eta(E(x), E(u)) \quad (>) \tag{2.2}$$

hold. If inequalities (2.2) are fulfilled for any $u \in M$ $(E(x) \neq E(u))$, then f is V-E-invex (strictly V-E-invex) with respect to η on M. Each function f_i, $i = 1, ..., k$, for which (2.2) is fulfilled is said to be α_i-E-invex (strictly α_i-E-invex) with respect to η at u on M.

Remark 2.4 *Note that the Definition 2.3 generalizes and extends several generalized convexity notions, previously introduced in the literature. Indeed, there are the following special cases:*