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On the $\omega b-$compactspace

Raad Aziz Hussain, Zain AL-abdeen AbbasNasser

Al-Qadisiyah University, College of Computer Science and Mathematics ,Department of Mathematics

Zain.alsafi@qu.edu.iq

ABSTRACT:

The main aim of our paper is introduced new conceptofcompactspaceis called ωb-compact Space,for this aim,the conceptof b-compact space and ω-compactspace introduced. and find that everyis relationshipsamong compact, b-compact, ω-compact spaces. andthe converse is not not true in generals.and wedefine nearlyωb-compact space and we prove some resultsabout subject.

Keywords: $b-$open,; $\omega-$open; $\omega b-$open, $\omega b-$compact ;nearly ωb-compact

1.Introduction and Preliminaries

The concept of ω-open sets in topological spaces was introduced in1982 byHdeib[1], In1996 Andrjivic [2] gave a new typeof generalized open setin topological spacecalled b-open sets,In 2008,Noiri,Al-Omari and Noorani [3] introduced the conceptof $\omega b-$open and the complement of an $\omega b-$open set is said to be $\omega b-$closed [11] the intersection of all $\omega b-$closed sets of X, containing A is calledthe ωb-closure of A and is denoted by $\overline{A}^{\omega b}$.The union of all $\omega b-$open sets. of X contained in A is calledtheωb-interior of A and is denoted by $A^{\circ\omega b}$, In [14] Burbaki studied

the concept of compact space.In this workwe introduced definition theconcept of ωb-compact

space, ωb-cluster point,and we introduced definitionωb-converge point,ωb-accumulatepoint and we prove some theoremabout subject .

Definition (1.1): [1]

A subset$A$ is said to be $\omega-$open set if for each $x \in A$,there existsan open setU_x such that $x \in U_x$ and $U_x - A$ is countable,thecomplement of ω-open set is called ω-closed the family of ω-open sets denoted by $\omega O(X)$.

Definition (1.2): [2]

LetX be topological space$A$ is called b-open set in X, iff $\overline{\mathring{A}} \cup \mathring{\overline{A}}$ the complementof b-open set is

Called b-closedand it is easy to see that A is b-closedsetiff$\overline{\mathring{A}} \cap \mathring{\overline{A}} \subseteq A$,thefamily of b-opensets

denoted by $BO(X)$.

Definition(1.3): [3]

A subset A of a space Xis said to be ωb-open,if for every $x \in A$,there exists a b-open subset$U_x \subseteq X$containing x,such that$U_x - A$ is countable,thecomplement of an ωb-opensubsetis said

to be ωb-closed,thefamily of ωb-open sets denoted by $\omega bO(X)$.

Definition (1.4): [4]

Let $f: X \rightarrow Y$ be a function of a space X into a space Y,thenf is called an open function,iff(A) is an open set inY,for every open.set$A$ in X.

Definition (1.5): [4]

Let $f: X \rightarrow Y$be a function ofa space$X$ into a space Y,then f is called an closed function,if $f(A)$ is an closed set in Y,for every closed set A in X.

Definition (1.6):[5]

Let X be a topological space and $A \subseteq X$,A is called regular open set in X,if $A = \mathring{\overline{A}}$,the

Complement ofregular open set is called regular closed,and it is easy to see that A is regular closed if $A = \overline{\mathring{A}}$.

Definition (1.7): [4]

Let $f: X \rightarrow Y$ be a function ofa space X into a space Y,then f is called a continuousfunction,iff$^{-1}(A)$ is an openset in X, for every openset A in Y.

Definition (1.8): [8]

A space X is called T_2-space (Hausdorff space) if for each $x \neq y$ in X, there exists disjoint an open sets U and V such that $x \in U, y \in V$.

Definition (1.9): [6]

A function $f:(D,\geq) \longrightarrow X$, from a directset $(D,\geq)$ to anon-empty set X is called a net on X, and it denoted by $\{X_\alpha\}_{\alpha\in D}$ $\forall\, \alpha \in D\ \exists\, X_\alpha \in X \ni f(\alpha) = X_\alpha$

Definition (1.10): [7]

ATopological space X issaid to be compact,if everyopen cover of X,has a finite sub cover.

Theorem (1.11): [7]

1- Every closed subset of a compact space is compact.

2- In any topological space the intersection of a compact subset with closedsubset is compact.

3- Every compact subset of a Hausdorff spaceis closed.

Definition (1.12): [8]

Atopological space X is said to be b-compact,ifevery b-open cover of X,hasafinite sub cover.

Remark (1.13): [8]

It is clear that every b-compact space is compact,however, butthe converse is not true in general as the following example shows.

Example (1.14): [8]

Let B be an infinite set such that $a \notin B, X = B \cup \{a\}$,let $\tau = \{X, \emptyset, \{a\}\}$be atopologyon X,such that $(X,\tau)$is compact space,where it is not ab-compact since$\{\{a,b\}: b \in B\}$ is a b-open cover of X, which has no finite sub cover.

Definition (1.15): [9]

Atopological space X is said to beω-compact,if every ω-open cover of X,has a finite sub cover.

Definition (1.16): [10]

A topological spaceX is called nearly compact if for every regular open cover ofX,hasfinite sub cover .

$2 - \omega b -$ compact space

Definition (2.1):

Afunction $f: X \longrightarrow Y$ is said to be ωb-open,for every open subsetAof X,if $f(A)$ is an ωb-open set inY.

Definition (2.2):

A function $f: X \longrightarrow Y$ is said to beωb-closed,for every closedsubset A of X, if $f(A)$ is an $\omega b -$ closed set inY,

Definition (2.3):

Let Xbe a space and $A \subseteq X$,theintersection of all ωb-closed sets of X containingAis called ωb-closure of Adefinedby $\overline{A}^{\omega b} = \cap \{B: B\ \omega b\text{– closed in} X \text{ and } A \subseteq B\}$

Definition (2.4):

Let X be a space and $A \subseteq X$,the union of all ωb-open sets of X containing A is called ωb-Interior of A denoted by $A^{\circ\omega b}$ or $\omega b - In(A) A^{\circ\omega b} = \cup \{B: B\ \omega b - \text{open in} X \text{and } B \subseteq A\}$.

Definition (2.5):

Let Xbe topological space and $A \subseteq X$,A is called regular-ωb-open set in Xif $A = {\overline{A}^{\omega b}}^{\circ\omega b}$ the complement of regular-ωb-open setis called regular-ωb-closed and it is easy to see that A is regular-ωb-closed set if $A = \overline{A^{\circ\omega b}}^{\omega b}$.

Definition (2.6):

Let$f$: $X \to Y$ be a function of a space X into a space Y then f is called an ωb-continuous function if $f^{-1}(A)$ is an ωb-open set in X, for every open set$A$ in Y.

Definition (2.7):

Let $f: X \to Y$ be a function of a topological space(X, τ) into a topological space (Y, τ'),then f iscalled an ωb-irresolutefunction if $f^{-1}(A)$ is an ωb-open set in X,for every ωb-open set$A$ in Y.

Definition (2.8):

A space X is called ωbT_2-space (ωb-Hausdorff space) if for each$x \neq y$in X,there exists disjoint an ωb-open sets U, V such that $x \in U, y \in V$

Definition (2.9):

Atopological space X is said to be ωb-compact,if every ωb-open cover ofX,hasafinite subcover

Remark (2.10):

1- It is clear that every ωb-compact space is compact.

2- It is clear that every ω-compact space is compact.

but the converse is not true in general as following example shows:

Example (2.11):

Let X= R withthe topology, $\tau = \{X, \emptyset, Q, Q^c\}$ then $(X, \tau)$is compact space,but it is not aωb-compact,since the family $\{Q \cup X - x \notin Q$ is ωb-open cover of $\mathcal{R}$,thus X= ∪Q∪X,but it is has no finite sub cover.

Definition (2.12):

A topological spaceX is said to be nearlyωb-compact if every ωb-regular open cover of X,hasfinite sub cover .

Remark (2.13):

1- Every b-compact is not true in general ω-compact

2- Every b-compact is not true in general ωb-compact

as the following

Example (2.14):

Let $X = Z$,be the integer number with topological ,$\tau = \{X, \emptyset, Z^+, Z^-\}$,then

$BO(X)=\{A \subseteq X: 0 \notin A\} \cup \{X\}$,thus X is b-compact,since $\omega o(X) = \omega BO(X) =$

$\{A: A \subseteq X\}$,therefore X is notω-compact and ωb-compact

Remark (2.15):

1- Every ω-compact is not true in general b-compact.

2- Every ω-compact is not true in general ωb-compact.

as the following

Example (2.16)

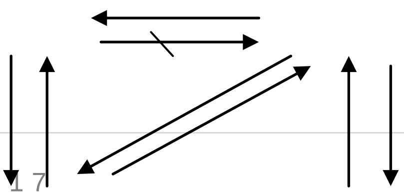
Let B is an un countable, X = B∪{a}, $a \notin B$ and, $\tau = \{\emptyset, X, \{a\}\}$, then

$\omega o(X) = \{\emptyset, X, \{a\}\} \cup \{G \subseteq X : G^c \text{ is finite }\}$, thus X is $\omega -$ compact, since

$BO(X) = \omega BO(X) = \{\{a, b\}: b \in B\}$, then X is not b-compact and ωb-compact

The following diagram shows the relations among the difference types of compact space.

Compact b-compact



$\omega - \text{compact} \qquad \omega b - \text{compact}$

Theorem (2.17):

Let $f: X \to Y$be an onto,ωb-continuous function,ifX is ωb-compact then Y is compact.

Proof

Let $\{G_\lambda: \lambda \in I\}$be an open cover of Y,then $\{f^{-1}(G_\lambda): \lambda \in I\}$ is an ωb-open cover of X,since X is ωb-compact,thusX has finite sub cover say$\{f^{-1}(G_{\lambda i}): i = 1,2,..,n\}$, and $G_{\lambda i} \in \{G_\lambda: \lambda \in I\}$

,hence $\{G_{\lambda i}: i = 1,2,\ldots,n\}$is afinite sub cover of Y, therefore Y is compact.

Proposition (2.18):

For any topological spaceX,the following statement are equivalent:

1-X is ωb-compact.

2- Every family of ωb-closed sets $\{V_\alpha: \alpha \in \Lambda\}$ of X,such that $\bigcap_{\alpha \in \Lambda} V_\alpha = \emptyset$,then there exists

a finite subset$\Lambda_0 \subseteq \Lambda$ such that $\bigcap_{\alpha \in \Lambda_0} V_\alpha = \emptyset$.

Proof

(1)→(2)

Assume that X is ωb-compact,let$\{V_\alpha: \alpha \in \Lambda\}$be a family of ωb-closed subset of X, such that $\bigcap_{\alpha \in \Lambda} V_\alpha = \emptyset$, then thefamily $\{X - V_\alpha: \alpha \in \Lambda\}$is ωb-open coverof theωb-compact(X, τ)there

exists a finite subsetΛ_0 of Λ,thus $X = \bigcup\{X - V_\alpha: \alpha \in \Lambda_0\}$ therefore $\emptyset = X - \bigcup\{X - V_\alpha: \alpha \in \Lambda_0\}$

$= \bigcap\{X - (X - V_\alpha): \alpha \in \Lambda_0\} = \bigcap\{V_\alpha: \alpha \in \Lambda_0\}$

(2)→(1)

Let $U = \{U_\alpha: \alpha \in \Lambda\}$ be an ωb-open coverof the space (X, τ),then $X - \{U_\alpha: \alpha \in \Lambda\}$ is a family of ωb-closed subset of(X, τ) with $\bigcap \{X - U_\alpha: \alpha \in \Lambda\} = \emptyset$by assumpmption,there exists a

a finite subset $\Lambda_0$of Λ,hence $\bigcap\{X - U_\alpha: \alpha \in \Lambda_0\} = \emptyset$,so $X = X - \bigcap\{X - U_\alpha: \alpha \in \Lambda_0\} = \bigcup\{U_\alpha: \alpha \in \Lambda_0\}$,therefore X is ωb-compact.

Proposition (2.19):

If $f: X \to Y$ is ωb-irresolute function, and X is ωb-compact space,then $f(X)$ is $\omega b - \text{compact}$.

Proof

Let $\{B_\lambda: \lambda \in I\}$ be an ωb-open cover of $f(X)$,then $f(X) \subseteq \bigcup_{\lambda \in I} B_\lambda$such that $f^{-1}(f(X)) \subseteq$

$f^{-1}(\bigcup_{\lambda \in I} B_\lambda)$=$\bigcup_{\lambda \in I} f^{-1}(B_\lambda) \subseteq X$,thus $X = \bigcup_{\lambda \in I} f^{-1}(B_\lambda)$since $B_\lambda$is ωb-open set inY,∀

$\lambda \in I$ and, since $f$is ωb-irresolute hence $f^{-1}(B_\lambda)$ is ωb-openset in X,∀ $\lambda \in I$, $\{f^{-1}(B_\lambda): \lambda \in I\}$

is $\omega b -$ open cover of X, sinceX is $\omega b -$ compact space $\exists \lambda_1\ \lambda_2, \ldots\ \lambda_n \in I$ such that

$X = \bigcup_{i=1}^{n} f^{-1}(B_{\lambda i}), f(X) = \bigcup_{i=1}^{n} f(f^{-1}(B_{\lambda i})) \subseteq \bigcup_{i=1}^{n} B_{\lambda i}$,therefore $f(X)$ is ωb-compact .

Definition (2.20):

A subset Bof a topological spaceX,is said to be ωb-compact relative to X,if every cover of B by $\omega b -$ open sets of X,has finite sub cover of B,the subset B is ωb-compact,if it is ωb-compact as a subspace.

Theorem (2.21):

The following statements are equivalent,for any topological space

1- X isωb-compact .

2- Every anyfamily Fofωb-open sets, ifnofinite subfamilyofF coversX,thenF doesnot cover X .

3- Every anyfamily F ofωb-closed sets,if F satisfiesthefinite intersectioncon-dition then$\cap\{A : A \in F\} \neq \emptyset$

4- Every anyfamilyF of subsetsofX,if F satisfies the finite intersectioncon-dition then$\cap\{ \overline{A}^{\omega b} : A \in F\} \neq \emptyset$

Proof

(1)⇔(2) and(2)⇔(3) are obvious (3) ⇒(4) if F⊂P(X) satisfies the finiteintersection condition,then$\cap\{ \overline{A}^{\omega b} : A \in F\}$isa family ofωb-closedsets which

,obviously satisfies the finite intersectioncondition.

(4)⇒(3)

Follows fromthefact that$A=\overline{A}^{\omega b}$ for every ωb-closed set A .

Proposition (2.22):

Let Ybe $\omega b-$open subspace of a space X,and $B \subseteq Y$,then B is ωb-compact set inY,iff B is

ωb-compact in X.

Proof

Let B an ωb-compact set inY,and let $\{V_\alpha : \alpha \in \Lambda\}$ be $\omega b-$open cover of$B$ in X,then$B \subseteq \cup_{\alpha\in\Lambda} V_\alpha$,since $B \subseteq Y, B \subseteq \cup \{Y \cap V_\alpha : \alpha \in \Lambda\}$since $Y \cap V_\alpha$ is $\omega b-$open relative to Y,thus

$\{Y \cap V_\alpha : \alpha \in \Lambda\}$ is $\omega b-$opencover of B relative to Y, we have B$\subseteq (Y \cap V_\alpha) \cup \ldots \cup (Y \cap V_{\alpha_n})$,therefore B is ωb-compact in X .

Conversely:

Let B be ωb-compact set in X,and let$\{U_\alpha : \alpha \in \Lambda\}$ be an $\omega b-$open cover of B in Y,then $B \subseteq \cup_{\alpha\in\Lambda} U_\alpha$,thus there exists $V_\alpha$is$\omega b-$open relative toX,such that $U_\alpha = Y \cap V_\alpha, \forall \alpha \in \Lambda$, hence

$B \subseteq \cup_{\alpha\in\Lambda} V_\alpha$where$\{V_\alpha : \alpha \in \Lambda\}\omega b-$open cover of B relative to X,since B is $\omega b-$compact

set inX ,$\exists \alpha_1, \alpha_2, \ldots \alpha_n \in \Lambda$ such that $\exists B \subseteq \cup_{i=1}^{n} V_{\alpha_i}$ since $B \subseteq Y, B \subseteq Y \cap \{V_{\alpha_1} \cup V_{\alpha_2}, \ldots \cup V_{\alpha_n}\} = (Y \cap V_{\alpha_1}) \cup \ldots \cup (Y \cap V_{\alpha_n})$since $Y \cap V_{\alpha_i} = U_i$,therefore Bis ωb-compact inY.

Theorem(2.23):

For any topological spaceX, the following statement are equivalent :

1-X is nearly ωb-compact.

2- Every ωb-open cover $\mu = \{V_\alpha : \alpha \in \Lambda\}$ of X,there exists a finite subset$\Lambda_0 \subseteq \Lambda$ suchthat . $X = \overline{U_{\alpha\in\Lambda o}}^{\omega b\,\circ\omega b}$.

Proof

(1)→(2)

Let $\mu = \{V_\alpha : \alpha \in \Lambda\}$ be ωb-open cover of X, then $\left\{\overline{V_\alpha}^{\omega b\,\circ\omega b} : \alpha \in \Lambda\right\}$ is ωb-regularopen cover of the nearlyωb-compact space X, thus there exists a finite subset$\Lambda_0 \subseteq \Lambda$ such that $X = \cup_{\alpha\in\Lambda_0} \overline{V_{\alpha\in\Lambda o}}^{\omega b\,\circ\omega b}$.

(2)→(1)

It is clear since ωb-regular open set is ωb-open.

Definition (2.24):

A point x∈ X is said to be ωb-cluster point of a net $\{X_\alpha\}_{\alpha\in\Delta}$ if $\{X_\alpha\}_{\alpha\in\Delta}$ isFrequently in every

ωb-open set containing x .we denote by ωb-cp$\{X_\alpha\}_{\alpha\in\Delta}$ the set of all ωb-cluster pointsof a net

$\{X_\alpha\}_{\alpha\in\Delta}$.

Theorem (2.25):t

Atopological spaceX is ωb-compact,iffeach net$\{X_\alpha\}_{\alpha\in\Delta}$in X, hasat least one ωb-cluster

point.

Proof

Let X be a ωb-compactspace,assume that thereexists somenet$\{x_\alpha\}_{\alpha\in\Delta}$ in X ,such that

ωb-cp$\{x_\alpha\}_{\alpha\in\Delta}$ is empty,let$x\in X$,then there exist $G(x)\in\omega BO(X,x)$is not frequently,thus

there exist $\alpha(x)\in\Delta$,such that$x_\lambda\notin G(x)$, whenever, $\lambda\geq\alpha(x), \lambda\in\Delta$,thefamily$\{G(x): x\in X\}$is acover ofX byωb-open sets and has afinite subcoversay,$\{G_k: k=1,2,\dots n\}$where, $G_k=G(x_k)$for $k=1,2,\dots n$,$\{x_k: k=1,2,\dots n\}$,let us take $\alpha\in\Delta$,hence$\alpha\geq\alpha x_k$,for every $k\in 1,2,\dots n$;forevery$\lambda\in\Delta$such that $\lambda\geq\alpha$ we have ,$x_\lambda\notin G_x$,k= 1,2,....n, hence$x_\lambda\notin X$ which isa contradiction.

Conversely:

IfX is not ωb-compactthen there exists$\{G_i: i\in I\}$acover of X , by ωb-open sets

whichhas no finite subcover;let$P(I)$be the family of every finite subsetsof I

clear$(P(I),\subseteq)$ is adirected set, foreach $J\in\mathcal{J}$wemay choose $x_j\in X-\cup\in\{G_i: i\in J\}$

let usconsider thenet $\{x_j\}_{j\in p(I)}$ byhypothesisthesetωb-cp$\{x_j\}_{j\in p(I)}$ isnonempty,let $x\in\omega b-cp\{x_j\}_{j\in p(I)}$andlet $i_0\in I$,hence $x\in G_{i0}$,bythe definition ofωb-clusterpoint

,for each$J\in P(I)$thusthereexists $J^*\in P(I)$suchthat$J\subset J^*$and $x_{j^*}\in G_{i0}$for $J=\{i_0\}$,

There exists$J^*\in P(I)$ such that$i_0\in J^*$ and $x_{j^*}\in G_{i0}$but$x_{j^*}\in X-\cup\{G_i: i\in j^*\}$

$\subset X-G_{i0}$ iscontradiction, therefore Xis ωb – compact .

In the following wewillgive acharacterizationof ωb –compact,by means offilterbases,letus we recallthat anonempty family $\mathcal{F}$ a of subsets ofX,issaid to be afilterbaseon X,if$\emptyset\notin\mathcal{F}$ andeach intersection of twomembersof $\mathcal{F}$ contains thirdmember of $\mathcal{F}$,notice that each chain inthe familyof every filterbaseonX.has an

upper bound,the union ofevery membersof thechainthenby Zorn'slemma,thefamily

of everyfilterbaseson X,hasat leastone maximal element Similarly,the family ofevery filterbaseson X, containinga given filterbase$\mathcal{F}$ has at least one maximal

element

Definition (2.26):

A filterbase$\mathcal{F}$ on a topological spaceX,is said to be:

1- ωb-convergetoa point$x\in X$,ifforeach .ωb-opensetU containingx, there exists $B\in\mathcal{F}$

such that $B\subset U$.

2- ωb-accumulate at $x\in X$, if $U\cap B\neq\emptyset$ for every ωb-open set U containing x and every $B\in\mathcal{F}$

Lemma (2.27):

If a maximalfilterbase $\mathcal{F}$ ωb-accumulate at, $x\in X$, then $\mathcal{F}$ ωb-convergeto x .

Proof

Let $\mathcal{F}$beamaximal filterbase with ωb-accumulate at$x\in X$,if $\mathcal{F}$ is not ωb-convergeto x,thenthereexistsa ωb-open setU_0 containing x,suchthat$U_0\cap B\neq\emptyset$nd $(X-U_0)\cap B\neq\emptyset$ for every$B\in\mathcal{F}$,thus $\mathcal{F}\cup\{U_0\cap B : B\in\mathcal{F}\}$is a filterbasewhich contains $\mathcal{F}$, which is contradiction

Theorem (2.28):

Let X be topological space, then following statements are equivalent:

1- X is ωb-compact.

2- Every maximal filterbase ωb-convergesto some points of X .

3- Every filter base ωb-accumulates atsome points ofX .

Proof

(1) ⇒(2)

Let $\mathcal{F}_0$ be a maximal filterbase on X,suppose that$\mathcal{F}_0$is not ωb-convergestoanypoint ofX,thenby lemma (2.27),$\mathcal{F}_0$is not ωb-accumulatesat anypointof X,foreach $x\in X$,thenthere exists aωb-open set U_x containing x and $B_x\in\mathcal{F}_0$ hence $U_x\cap B_x=\emptyset$ the family

$\{U_x: x\in X\}$is a cover of X,byωb-open sets ,by (1) thus there existsa finite subset$\{x_1, x_2, \dots, x_n\}$of X,hence$X=\cup\{U_{xk}: k1,2,\dots\}$, since$\mathcal{F}_0$is afilterbase ,there exists$B_0\in\mathcal{F}_0$

such that $B_0 \subset \cap \{B_{x_k}: k = 1,2,\dots n\} = X - \cup \{U_{x_k}: k = 1,2,..\}$, hence $B_0 = \emptyset$

which iscontradiction.

(2) ⇒ (3)

Let $\mathcal{F}$ be a filterbaseon X,thenthere existsa maximal filterbase$\mathcal{F}_0$, hence$\mathcal{F} \subset \mathcal{F}_0$ by (2),$\mathcal{F}_0$ is ωb-converges to some point$x_0 \in X$,let $B \in \mathcal{F}$ for everyU $\in BO(X, x_0)$ thus there exists $B_U \in \mathcal{F}_0$such that$B_u \subset U$,hence$U \cap B \neq \emptyset$, Since itis containsthe member$B_U \cap B$ of $\mathcal{F}_0$,this that $\mathcal{F}$ ωb-accumulates atx_0.

(3) ⇒ (1)

Let$\{V_i: i \in I\} = \emptyset$ be any family of ωb-closed setssuch that $\cap \{V_i: i \in I\} = \emptyset$ weprovethat thereexistsa finitesubset I_0ofI,hence $\cap \{V_i: i \in I\}$by theorem (2.21)(1),letP(I) bethe

family offinite subsets ofI, assume that$\cap \{V_i: i \in J\} = \emptyset$ for everyJ$\in$P(I)....∗∗ thus

the family$\mathcal{F} = \{\cap \{V_i: i \in J\}: J \in P(I)\}$ is afilter baseon X by (3),$\mathcal{F}$ is ωb-accumulates to somepoint$x_0 \in X$, Since$\{X - V_i: I \in I\}$isacover ofX, thereexists$i_0 \in I$ hence$x_0 \in X - V_{i0}$, $X - V_{i0}$is ωb – open set contains x_0, $V_{i0} \in \mathcal{F}$and $(X - V_{i0}) \cap V_{i0} = \emptyset$ which is contradiction withthe fact that $\mathcal{F}$ωb-accumulatesat x_0 shows that(∗∗)is false.

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