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On Generalization of Hollow Acts

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Abstract In this paper, we introduced a generalization of the hollow act which is known as hollow-lifting acts and obtained new properties and characterizations for this notion. It's known that a unitary right S-act A over S which denoted by A_S could be a non-empty set with a function $f: A \times S \longrightarrow A$ specified $f(a,s) \mapsto$ as and also the following properties hold: (1) a•1= a . (2) a(st) = (as)t for all $a \in A$ and $s, t \in S$. An S-act M_S is stated as hollow-lifting if every subact N of M_S such that M_S/N is hollow includes a coessential subact that's a retract subact of M_S. Conditions under which subacts are inheriting the property of the Hollow-lifting act are examined. Further because the relationship between hollow acts and hollow-lifting is taken into account. Ultimately, the notion of the indecomposable act is employed to coincide these classes. The conclusion of our work is clarified within the last section.

Keywords: Hollow-lifting acts, coessential subacts , lifting acts, supplement act, strong supplement, Fully invariant.

AMS Subject Classification: 20M30, 20M99, 08B30, 06F05.

INTRODUCTION

The notion of hollow acts was first studied by R. Khosravi and M. Roueentan in August 2019 [1] which was a generalization of [2,3,4,5]. Also as in [6], we completed early results on hollow acts by R. Khosravi and M. Roueentan and obtained new properties and characterizations for this notion. Besides, a reformulation for the definition of hollow acts was clarified. In everywhere of this paper, every S-acts is unitary right S-acts with zero element Θ represented by M_S where S is commutative monoid. We refer the reader to the references for more details per S act which are used here (for basic definitions and terminology) [7-17].

Let M_S be an S-act and N_S be any subact of M_S, then N_S defines Rees congruence ρ_N on M, by setting $a\rho_N a^{/}$ if $a, a^{/} \in N_S$ or $a = a^{/}$. The resulting factor act is mentioned as Rees factor of M_S by subact N_S and it represented by M_S/N_S ([18],p.52). A subact B_S of an S-act M_S is said as coessential subact of A_S in M_S if A_S/B_S is small in M_S/B_S [16].

A right S-act B_S may be a retract of a right S-act A_S if and on condition that there exists a subact W of A_S and epimorphism $f: A_S \longrightarrow W$ such $B_S \cong W$ and f(x) = x for each $x \in W$ [18,P.84]. A subact N of a right S-act M_S is termed fully invariant if $f(N) \subseteq N$ for each endomorphism f of M_S and M_S is named duo if every subact of M_S is fully invariant [19].

An S-act M_S is spoken as θ-simple act if it contains no subacts aside from M_S and one element subact. Besides, M_S is said as simple if it contains no subacts apart from M_S itself ([18],p50). Let S be a semigroup. A nonempty subset K of S is named left ideal of S if SK⊆K ; a right ideal of S if KS⊆K; an ideal of S if SK⊆K and KS⊆K([18],p.20). An element s of a semigroup S is stated as (right) nilpotent if there exists n∈N specified $S^n = z \in S$ where z could be a (right) zero of S. A semigroup S is referred to as (right) nil if all elements of S are (right) nilpotent([18], p.29).

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A proper subact N_S of an S-act M_S is stated as maximal if for every subact K_S of M_S with $N_S \subseteq K_S \subseteq M_S$ implies either $K_S=N_S$ or $K_S=M_S$[20]. A right S-act is spoken as local if it contains exactly one maximal subact, also, a monoid S is additionally called right (left) local if it contains exactly one maximal right (left) ideal[1]. An S-act A_S is obseverd as cyclic (or principal) act if it's generated by one element and it denotes by $A_S = < u >$ where $u \in A_S$, then $A_S=uS$ ([18],P.63). An S-act M_S is named decomposable if there exist two subacts A_S,B_S of M_S such that $M_S = A_S \cup B_S$ and $A_S \cap B_S = \Theta$. During this case, $A \cup B$ is stated as decomposition of M_S. Otherwise M_S is said as indecomposable ([18],p.65). Every cyclic act is indecomposable. An S-act M_S is termed semisimple if and provided that every subact of M_S could be a retract or it's union of simple subacts[21].

A subact N of a right S-act M_S is stated as small (or superfluous) in M_S just in case of for each subact H of M_S, $N \cup H=M_S$ implies $H=M_S$[22]. A subact B_S of an S-act M_S is named coessential subact of A_S in M_S if A_S/B_S is small in M_S/B_S [16]. An S-act M_S is named to as lifting, if for each subact N_S of M_S contains a retract H_S of M_S specified N_S/H_S is small in M_S/H_S [16]. In other words, an S-act M_S is mentioned as lifting or satisfies (D1), if for each subact N_S of M_S there exists a retract H_S of M_S specified H_S could be a coessential subact of N_S in M_S. In [6], we present hollow acts where an S-act M_S is spoken as hollow act if whenever N_1, N_2 are subacts of M_S and $N_1 \cup N_2 = M_S$ implies that either $N_1= M_S$ or $N_2= M_S$. This paper is consists of four sections. In section 2, we start by showing some general properties and characterizations of hollow-lifting acts. In Section 3, we will be concerned with the connection of hollow lifting acts and a few concepts like hollow act and others more. Section 4 is dedicated to the conclusions of this paper.

HOLLOW-LIFTING ACTS

Motivated by [6], shaymaa who defined the hollow act, we would like to generalize this notion to hollow-lifting S- act as shown below, but before we'd like the subsequent definitions:
Recall that a subact B_S of an S-act M_S is said as a **coessential** subact of A_S in M_S if A_S/B_S is small in M_S/B_S [16].
Definition (2.1): An S-act M_S is called hollow-lifting if each subact N of M_S such that M_S/N is hollow has a coessential subact that is a retract subact of M_S.
Recall that the radical of the S-act M_S is the union of all small subacts of M_S .Accordingly, we mention that by Rad(M). A monoid for which Rad(M)=Θ for every right S-act M_S is called a right V-monoid [6]. Besides, M is called a radical act if Rad(M) = M_S.
In the next proposition, we denote N is that the union of all radical subacts of M_S. It's easy to determine that N may be a fully invariant subact of M_S and it's always radical.

Definition (2.2): Let M_S be S-act. Let A and B be two subacts of M_S. We called that B is **a strong supplement** of A in M_S if B is a supplement of A in M_S and $B \cap A$ is a retract subact of A.

Proposition (2.3): Let M_S be S-act. Assume that M_S is hollow-lifting act. If N is a retract subact of M_S, then N and $M_S/_N$ both are hollow-lifting act.
Proof: It is enough to show that N is hollow-lifting act. Let A be a subact of M_S with $M_S = N \dot{\cup} A$. Let B be a subact of N such $N/_B$ is hollow act. Thus $\frac{M_S}{(B \dot{\cup} A)}$ is hollow act. Since M_S is hollow-lifting act, so there exists a subact H of M_S such that H is a strong supplement of $B \dot{\cup} A$ in M_S. Hence $(B \dot{\cup} A) \dot{\cup} H = M_S$ and $\frac{N}{B} \cong \frac{M_S}{(B \dot{\cup} A)} \cong \frac{H}{(B \dot{\cup} A) \cap H}$. Hence, H is hollow radical and H is subact N. Thus, $N = B \dot{\cup} H$. Since, $H \cap B = (B \dot{\cup} A) \cap H$ and $H \cap B$ is a retract of B, so H is strong supplement of B in N. Therefore, N is hollow-lifting act.

Before the next proposition, we need the subsequent definition:
Proposition (2.4): Let N be a subact of S-act M_S. The following are equivalent:
(i)N has a strong supplement in M_S;
(ii)N has a coessential subact that is a retract of M_S.
Proof: (i ⟹ ii) Let A be a strong supplement of N in MS and let K be subact of MS such that $(N \cap A) \ddot{\cup} K = N$. Then, $M_S = K \ddot{\cup} A$. Moreover, if the Rees factor $\frac{A}{K} \dot{\cup} \frac{X}{K} = \frac{M_S}{K}$, then $A \ddot{\cup} X = M_S$ and $(A \cap K) \ddot{\cup} K \ddot{\cup} X = M_S$. Since, $A \cap K$ is small subact of K, we have $K \ddot{\cup} X = M_S$. Hence $X = M_S$. Therefore N/K is small in M_S/K and then we get the required.

(ii $\Longrightarrow$ i) Let A be a coessential subact of N that is a retact of M_S. Let B be a subact of M_S with $M_S = A \dot{\cup} B$. Thus, $N = A \dot{\cup}(B \cap N)$ and $N \dot{\cup} B = M_S$. If $(N \cap B) \dot{\cup} X = B$ then $A \dot{\cup}(N \cap B) \dot{\cup} X = M_S$. Thereby, $N \dot{\cup} X = M_S$ and $N \dot{\cup} X \dot{\cup} A = M_S$. Since N/A is small in M_S/A, so we have $X \dot{\cup} A = M_S$. But X is subact of B, then $X = B$. Therefore, $N \cap B$ is small in B. Consequently, B is a strong supplement of N in M_S.

Corollary (2.5): Let M_S be any S-act. The following are equivalent:

(i)M is hollow-lifting;

(ii)Every subact N of M_S such that M_S is hollow act has a strong supplement in M_S.

Proposition (2.6): Let M_S be S-act. The following are equivalent:

(i) M_S is hollow-lifting;

(ii) Every subact A of M_S such that M_S/A is hollow act can be written as $A = B \dot{\cup} H$ with B is a retract subact of M_S and H is a small subact of M_S.

Proof: (i $\Longrightarrow$ii) Let A be a subact of M_S such that M_S/A is hollow act. Since M_S is hollow-lifting act, there exists a retract subact H of M_S such that H is subact of A and
A/H is small subact of M_S/H. Let L be a subact of M_S with $M_S = H \dot{\cup} L$. So $A = H \dot{\cup} (L \cap A)$. Further, if X is subact of L with $(L \cap A) \dot{\cup} X = L$, then $A \dot{\cup} X = M_S$. Since A/H is small subact in M_S/H, so we have $X \dot{\cup} H = M_S$. Thus, $X = M_S$ and $L \cap A$ is small subact of L. It suffices to take $K = L \cap A$.

(ii $\Longrightarrow$i)Let A be a subact of M_S such that M_S/A is hollow act. Then, A can be written as $A = B \dot{\cup} H$ with B is a retract subact of M_S and H is small subact in M_S. Let X be a subact of M_S such that B is subact of X and assume that $A/B \dot{\cup} X/B = M_S/B$ (Since B is retract subact of M_S, so there exists subact K of M_S such that $M_S = B \dot{\cup} K$. Since $A = B \dot{\cup} H$, so we obtain $A \dot{\cup} K = (B \dot{\cup} K) \dot{\cup} H$. Thereafter $A \dot{\cup} K = M_S$ and A is retract of M_S. Also, since B is subact of X and $B \dot{\cup} K = M_S$. Then, $M_S \cup X = (B \cup X) \dot{\cup} K$. Hence, $M_S = X \dot{\cup} K$ and X is retract of M_S)). Thus, $A \dot{\cup} X = M_S$. Thereby, $B \dot{\cup} H \dot{\cup} X = M_S$ and $B \dot{\cup} X = M_S$. But B is subact of X. Then, $X = M_S$ and A/B is small subact of M_S/B. Hence, M_S is hollow-lifting act.

Proposition (2.7): Let $M_1, \ldots, M_n$ be S-acts having no hollow factor acts. Then, $M_S = M_1 \dot{\cup} M_2 \dot{\cup} \ldots \dot{\cup} M_n$ is hollow-lifting act.

Proof: Assume that M_S has a subact A such that M_S/A is hollow act. Since $\frac{M_1 \dot{\cup} A}{A} \dot{\cup} \frac{M_2 \dot{\cup} A}{A} \dot{\cup} \ldots \dot{\cup} \frac{M_n \dot{\cup} A}{A} = \frac{M_S}{A}$, so there exists $i \in \{1, \ldots, n\}$ such that $\frac{M_i \dot{\cup} A}{A} = \frac{M_S}{A}$ is hollow act. So M_i has a hollow factor act, and this is implies to a contradiction. Thus, M_S is hollow-lifting act.

Proposition (2.8): Let M_S be a hollow-lifting act such that M_S has a non-small hollow subact. Then M_S has a hollow retract subact.

Proof: Let A be a non-small hollow subact of M_S. Then, there is a proper subact B of M_S such that $M_S = A \dot{\cup} B$ .Since M_S is hollow-lifting act, there is a retract subact H of M_S such that B/H is small subact of M_S/H. It is easy to see that M_S/H is hollow act. Now $M_S = H \dot{\cup} L$ for some subact L of M_S. Thus, L is a hollow retract subact of M_S.

Lemma (2.9): Let M_S be a hollow-lifting act having a maximal subact A. Then M_S has a local retract subact.

Proof: Since M_S is hollow-lifting act and M_S/A is simple, so there is a subact B of M_S. that is a strong supplement subact of A in M_S. Thus B is a retract subact of M_S, $M_S = A \dot{\cup} B$ and $\frac{B}{A \cap B} \cong \frac{M_S}{N}$ is simple. Thus, B is local because $A \cap B$ is small subact in B.

Proposition (2.10): Let $M_S = M_1 \dot{\cup} M_2$ be act. Suppose that for every proper subact A of M_S if $M_S = A \dot{\cup} M_2$,then $M_S \neq A \dot{\cup} M_1$. Then there is no epimorphism from M_1 to M_2.

Proof: Let that there is an epimorphism $f: M_1 \longrightarrow M_2$. Define $A = \{(m_1, f(m_1)) \mid m_1 \in M_1\}$. Then, $M_S = A \dot{\cup} M_2$. Since f is epic, so $M_S = A \dot{\cup} M_1$ and this is a contradiction. Hence, there is no epimorphism from M_1 to M_2.

RELATIONSHIP BETWEEN HOLLOW-LIFTING S-ACTS WITH OTHER CLASSES

Next proposition explains important property that if S-act M_S is direct summand of hollow acts, then it'll be hollow lifting if and providing it's lifting

Proposition (3.1): Let N_1 and N_2 be hollow acts. The following are equivalent for the act $M_S = N_1 \dot{\cup} N_2$

(i) M_S is hollow-lifting;

(ii)M_S is lifting.

Proof: (i $\Longrightarrow$ ii) Let A be subact of M_S. Let $\pi_1: M_S \longrightarrow A_1$ and $\pi_1: M_S \longrightarrow A_2$ be two projection maps. If $\pi_1(A) \neq N_1$and $\pi_1(A) \neq N_2$, then A is small subact of M_S. Now, assume that $\pi_1(A) \neq N_1$. Then $M_S = A \dot{\cup} N_2$. Therefore, the Rees factor M_S/A is hollow act. Thereby, there exists a retract K of M_S such that K is subact of A and the Rees factor A/K is small subact of M_S/K. Thus M_S is lifting.

(ii $\Longrightarrow$ i) It is obvious.

Proposition (3.2): Let M_S be an indecomposable act. The following are equivalent:
(i) M_S is hollow-lifting act;
(ii) M_S is hollow act, or M has no hollow factor acts.
Proof: (i $\Longrightarrow$ii) Assume that M_S has a hollow factor act. Then there exists a subact A of M_S and it is not equal to M_S such that M_S/A is hollow act. Since M_S is hollow-lifting act, so there is a retract subact B of M_S such that A/B is small subact in M_S/B. But M_S is indecomposable act, so $B = \Theta$ and A is small subact in M_S. Thereby, M_S is itself a hollow act.
(ii $\Longrightarrow$i) It is obvious.

Proposition (3.3): Let M_S be S-act. If $M_S = M_1 \dot{\cup} M_2$, then $\frac{M_S}{N} = \frac{M_1 \dot{\cup} N}{N} \dot{\cup} \frac{M_2 \dot{\cup} N}{N}$ for every fully invariant subact N of M_S.
Proof: Let N be a fully invariant subact of M. Then $N = (N \cap M_1) \dot{\cup} (N \cap M_2)$ (since N is a fully invariant subact of M).Thus, $(N \dot{\cup} M_1) \cap (N \dot{\cup} M_2)$ is subact of $(M_1 \dot{\cup} M_2 \dot{\cup} N) \cap N \dot{\cup} (M_1 \dot{\cup} N \dot{\cup} N) \cap M_2 =$
$= N \dot{\cup} [M_1 \dot{\cup} (N \cap M_1) \dot{\cup} (N \cap M_2)] \cap M_2 = N$. Therefore, $\frac{M_S}{N} = \frac{M_1 \dot{\cup} N}{N} \dot{\cup} \frac{M_2 \dot{\cup} N}{N}$.
Before the following lemma, we'd like the subsequent concept:
Definition (3.4): An S-act M_S is completely hollow-lifting act if every retract subact of M_S is hollow-lifting act.
Lemma (3.5): Let M_S be a duo hollow-lifting act. Then M_S is completely hollow-lifting act.■
Direct sum of two hollow-lifting acts need not be a hollow-lifting act as we see in the following example:
Example *(3.6):* Let M_S be the Z–act such that $M_S = 2Z \dot{\cup} 8Z$. Since 2Z and 8Z are hollow acts, so they are hollow-lifting acts. But M_S is not hollow-lifting act. It is easy to check that 2Z is not 8Z-projective.
Proposition (3.7): Let $M_S = M_1 \dot{\cup} M_2$ be a duo act. Then M_S is hollow-lifting act if and only if M_1 and M_2 are hollow-lifting.
Proof: ($\Longrightarrow$) It is clear by lemma(3.5).
($\Longleftarrow$) Let N be subact of M_S with M_S/N hollow act. By proposition (3.3), we have
$\frac{M_S}{N} = \frac{M_1 \dot{\cup} N}{N} \dot{\cup} \frac{M_2 \dot{\cup} N}{N}$. Since M_S/N is hollow, we can assume that $\frac{M_S}{N} = \frac{M_1 \dot{\cup} N}{N}$. Then M_2 is subact of N.
Since $\frac{M_1 \dot{\cup} N}{N} \cong \frac{M_1}{N \cap M_1}$ and M1 is hollow-lifting act, there exists a retract H1 of M1 such that H1 is subact of $N \cap M_1$ and $\frac{(N \cap M_1)}{H_1}$ is small subact of $\frac{M_1}{H_1}$. Since $N = (N \cap M_1) \dot{\cup} (N \cap M_2)$, so we get $\frac{N}{(H_1 \dot{\cup} M_2)}$ is small subact of $\frac{M_S}{(H_1 \dot{\cup} M_2)}$ .Moreover, it is easily seen that $H_1 \dot{\cup} M_2$ is a retract subact of M_S. Thus M_S is hollow-lifting act.
Lemma (3.8): Let $M_S = M_1 \oplus ... \oplus M_n$ be a duo act. Then M_S is hollow-lifting act if and only if M_i is hollow-lifting act for all i = 1, 2,…, n.

Proof: It is easily to prove this lemma by induction on n and it is based on the fact that any retract subact of a duo act is duo.
The following example shows that in proposition (3.7), duo is essential:
Example (3.9): Consider the Z–act M_S in is equal to $2Z \dot{\cup} 8Z$ as in example (3.6). Then M_S is not duo. To prove this, let $f: 2Z \dot{\cup} 8Z \longrightarrow 2Z \dot{\cup} 8Z$ be the homomorphism defined by $f(\overline{x}, \overline{y}) = (\overline{x+y}, \overline{2y})$.Then, $f(\overline{\Theta}, \overline{1}) = (\overline{1}, \overline{2})$. Thereby, $f(\Theta \dot{\cup} 8Z) \nsubseteq \Theta \dot{\cup} 8Z$.

CONCLUSION

In this article, we presented a brand new notion which was hollow-lifting acts and obtained several interesting results. From these results, we deduced that retract subacts and also the Rees factor of hollow lifting act are going to be hollow lifting. Also, we concluded that direct summand of hollow lifting act are hollow lifting if S-act could be a duo. Additionally, direct summand of hollow act are hollow-lifting if and providing S-act is lifting. A disjoint union of acts which they need no hollow factor acts are going to be hollow lifting. An S-act will contain a hollow retract subact whenever S-act is hollow lifting and has a non-small hollow subact. Also, S-act will contain a local retract subact if it's a hollow lifting act and has maximal subact. Besides, we deduced that an S-act must be indecomposable as a condition to coincide classes of hollow lifting acts with hollow acts.

Conflict of Interest: The author declares that there is no conflict of interest.

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