Generalization of Newton's Forward Interpolation Formula

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Abstract- In this paper we generate new Newton's Forward Interpolation Formula`s using 12 , 13 and 14 points , that help us to calculate any numerical integration with very much less amount of error`s , the idea is increase the coefficients instead of making many intervals .
We used fractions not decimals in our results, since this proved to be useful.

Index Terms- Newton's Forward Interpolation , Numerical integration, *Maple* , *MATLAB* , Numerator and Denominator .

I. Introduction

The most commonly encountered mathematical models in engineering and science are in the form of differential equations. The dynamic of physical systems that have one independent variable can be modeled by ordinary differential equations, whereas systems with two, or more, independent variables require the use of partial differential equations. Several types of ordinary differential equations, and a few partial differential equations, render themselves to analytical (closed-form) solutions. These methods have been developed thoroughly in differential calculus. However, the great majority of differential equations, especially the nonlinear ones and those that involve large sets of simultaneous differential equations, do not have analytical solutions but require the application of numerical techniques for their solution.
There are several numerical methods for differentiation, integration, and the solution of ordinary and partial differential equations .These methods are based on the concept of finite differences. Therefore, the purpose of the Finite Difference Methods and Interpolation is to develop the systematic terminology used in the calculus of finite differences and to derive the relationships between finite differences and differential operators, which are needed in the numerical solution of ordinary and partial differential equations.
The calculus of finite differences may be characterized as a "two-way street" , 'that enables the user to take a differential equation and integrate it numerically by calculating the values of the function at a discrete (finite) number of points. Or, conversely, if a set of finite values is available, such as experimental data, these may he differentiated, or integrated, using the calculus of finite differences. It should be pointed out, however, that numerical differentiation is inherently less accurate than numerical integration.
Another very useful application of the calculus of finite differences is in the derivation of interpolation/extrapolation formulas, the so-called ***interpolating polynomials*** which can he used to represent experimental data when the actual functionality of these data is not known.
The word interpolation refers to interpolating some unknown information from a given set of known information. The technique of interpolation is widely used as a valuable tool in science and engineering. The problem is a classical one and dates back to the time of Newton and Kepler, who needed to solve such a problem in analyzing data on the position of stars and planets.
Mathematical applications of interpolation include derivation of computational techniques for • Numerical differentiation• Numerical integration • Numerical solutions of differential equations .

This paper constructs of :

1) Abstract
2) Introduction
3) PRELIMINARIES
4) MAIN NEW RESULT`S
5) Conclusions

II. PRELIMINARIES

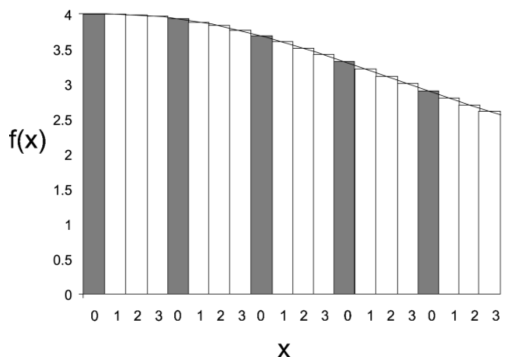
Numerical Integration

In numerical analysis, numerical integration constitutes a broad family of algorithms for calculating the numerical value of a definite integral, and by extension, the term is also sometimes used to describe the numerical solution of differential equations. The basic problem considered by numerical integration is to compute an approximate solution to a definite integral. It is different from analytical integration in two ways: first it is an approximation and will not yield an exact answer; Error analysis is a very important aspect in numerical integration. Second it does not produce an elementary function with which to determine the area given any arbitrary bounds; it only produces a numerical value representing an approximation of area.

Elements of Numerical Integration

If f(x) is a smooth well-behaved function, integrated over a small number of dimensions and the limits of integration are bounded, there are many methods of approximating the integral with arbitrary precision. We consider an indefinite integral $\int f(x)dx$.

Numerical integration methods can generally be described as combining evaluations of the integral to get an approximation to the integral. The integral is evaluated at a finite set of points called integration points and a weighted sum of these values is used to approximate the integral. For instance if we use rectangles as our shape:



In this example the definite integral is thus approximated using areas of rectangles. The integration points and weights depend on the specific method used and the accuracy required from the approximation. An important part of the analysis of any numerical integration method is to study the behavior of the approximation error as a function of the number of integral evaluations. A method which yields a small error for a small number of evaluations is usually considered superior. Reducing the number of evaluations of the integral reduces the number of arithmetic operations involved, and therefore reduces the total round-off error. Also, each evaluation takes time, and the integral may be arbitrarily complicated. Note that if one were to take an infinite number of divisions this would approach the analytical function (derived in calculus) representing the area under the curve. We do not do this in practice as an infinite number of divisions would require a prohibitively expensive amount of computing power is rarely ever needed to be exact.

Newton's Forward Interpolation Formula

Statement: If $x_0, x_1, x_{2,} \dots, x_n$ are given set of observations with common difference h and let $y_0, y_1, y_{2,} \dots, y_n$ are their corresponding values, where $y = f(x)$ be the given function then

$$f(x) = y_0 + p\,\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots + \frac{p(p-1)(p-2)\dots(p-(n-1))}{n!}\Delta^n y_0$$

Where $p = \frac{x-x_0}{h}$

Previous study

There are so many studies that show the importance of numerical methods to adopt to solve bounded integrals using Newton's Forward Interpolation Formula law, there is no room for listed all of what we care about is what to do with finding the equations and laws of ranks high in dealing with computer processors, and briefly we will show that where each of G.Dahlquist and A.Bjorck work schedule which shows the number of points used against the resulting equation coefficients, with the error value $c_{n,d}$.

n	d	A	c_0	c_1	c_2	c_3	c_4	c_5	c_6	$c_{n,d}$
1	1	1/2	1	1						−1/12
2	3	1/3	1	4	1					−1/90
3	3	3/8	1	3	3	1				−3/80
4	5	2/45	7	32	12	32	7			−8/945
5	5	5/288	19	75	50	50	75	19		−275/12 096
6	7	1/140	41	236	27	272	27	236	41	−9/1400

We can see from the table above , that if $n = 1$ gives as the Trapezoidal rule , and so on .
And mentioned the general equation for Newton Cotes open which is: -

$$\int_0^{nh} f(x)dx = h\sum_{i=1}^{n-1} w_i f(ih) + R_{n-1,n}(h)$$

Alkis Constantinides & Navid Mostoufi also touched to a group of important algorithms to find Newton Forward Interpolation equation using 9 points (dividing the 8 periods) using MATLAB software, with a comparison with previous methods.

Remark :- For all the equations

$$x_1 \le \xi \le x_{n+1} , f_1 = f(x_1) , f_2 = f(x_2) , \dots , f_n = f(x_n) ,\ h = \tfrac{x_{n+1} - x_1}{n} .$$

So far what was to be found are: -

1) Ueberhuber Using 5 point`s

$$\int_{x_1}^{x_5} f(x)dx = \tfrac{2}{45}h(7f_1 + 32\, f_2 + 12f_3 + 32f_4 + 7f_5) - \tfrac{8}{945}h^7 f^{(6)}(\xi)$$

2) Abramowitz and Stegun Using 6 point`s

$$\int_{x_1}^{x_6} f(x)dx = \tfrac{5}{288}h(19f_1 + 75\, f_2 + 50f_3 + 50f_4 + 75f_5 + 19f_6) - \tfrac{275}{12096}h^7 f^{(6)}(\xi)$$

3) Abramowitz and Stegun Using 7 point`s

$$\int_{x_1}^{x_7} f(x)dx = \tfrac{1}{140}h(41f_1 + 216\, f_2 + 27f_3 + 272f_4 + 27f_5 + 216f_6 + 41f_7) - \tfrac{9}{1400}h^9 f^{(8)}(\xi)$$

4) Abramowitz and Stegun Using 8 point`s

$$\int_{x_1}^{x_8} f(x)dx = \tfrac{7}{17280}h(751f_1 + 3577\, f_2 + 1323f_3 + 2989f_4 + 2989f_5 + 1323f_6 + 3577f_7 + 751f_8) - \tfrac{8183}{518400}h^9 f^{(8)}(\xi)$$

5) Abramowitz and Stegun Using 9 point`s

$$\int_{x_1}^{x_9} f(x)dx = \tfrac{4}{14175}h(989f_1 + 5888\, f_2 - 928f_3 + 10496f_4 - 4540f_5 + 10496f_6 - 928f_7 + 5888f_8 + 989f_9) - \tfrac{2368}{467775}h^{11} f^{(10)}(\xi)$$

6) Ueberhuber Using 10 point`s

$$\int_{x_1}^{x_{10}} f(x)dx = \tfrac{9}{89600}h(\,2857(f_1 + f_{10}) + 15741(\,f_2 + f_9) + 1080(f_3 + f_8\,) + 19344(f_4 + f_7) + 5778(f_5 + f_6)) - \tfrac{173}{14620}h^{11} f^{(10)}(\xi)$$

7) Ueberhuber Using 11 point`s

$$\int_{x_1}^{x_{11}} f(x)dx = \tfrac{5}{299376}h(\,16067(f_1 + f_{11}) + 106300(\,f_2 + f_{10}) - 48525(f_3 + f_9\,) + 272400(f_4 + f_8) - 260550(f_5 + f_7) + 427368\, f_6))$$

$$- \tfrac{11346350}{326918592}h^{13} f^{(12)}(\xi)$$

III. Main new result`s

1) . Using 12 point`s

$$\int_{x_1}^{x_{12}} f(x)dx = \frac{11}{87091200} h(2171465 f_1 + 13486539 f_2 - 3237113 f_3 + 25226685 f_4 - 9595542 f_5 + 15493566\, f_6 + 15493566 f_7 - 9595542 f_8 + 25226685 f_9 - 3237113 f_{10} + 13486539 f_{11} + 2171465 f_{12}) - \frac{2224234463}{237758976000} h^{13} f^{(12)}(\xi)$$

Hence the error is = $\frac{2224234463}{237758976000} h^{13} f^{(12)}(\xi)$.

2) . Using 13 point`s

$$\int_{x_1}^{x_{13}} f(x)dx = \frac{1}{5255250} h\big(1364651 f_1 + 9903168\, f_2 - 7587864\, f_3 + 35725120\, f_4 - 51491295\, f_5 + 87516288\, f_6 - 87797136\, f_7 + 87516288\, f_8 - 51491295\, f_9 + 35725120\, f_{10} - 7587864\, f_{11} + 9903168\, f_{12} - 1364651\, f_{13}\big) - \frac{3012}{875875} h^{15} f^{(14)}(\xi)$$

Hence the error is = $\frac{3012}{875875} h^{15} f^{(14)}(\xi)$.

3) . Using 14 point`s

$$\int_{x_1}^{x_{13}} f(x)dx = \frac{13}{402361344000} h(8181904909 f_1 + 56280729661 f_2 - 31268252574 f_3 + 156074417954 f_4 - 151659573325 f_5 + 206683437987 f_6 - 43111992612 f_7 - 43111992612 f_8 + 206683437987 f_9 - 151659573325\, f_{10} + 156074417954\, f_{11} - 31268252574 f_{12} + 56280729661\, f_{13} + 8181904909 f_{14}) - \frac{2639651053}{344881152000} h^{15} f^{(14)}(\xi)$$

Hence the error is = $\frac{2639651053}{344881152000} h^{15} f^{(14)}(\xi)$.

IV CONCLUSION

In this study we attained the following results :

1) We Evaluated the numerical integration $\int_a^b f(x)\, dx$, with more accuracy and much more less amount of error`s .

2) The high order Newton Forward Interpolation Formula`s makes a great tool to find the trigonometrically fitted formulae for long-time integration of orbital problems. This is true for trigonometrically-fitted formulae for the numerical solution of the Schodinger equation, for also trigonometrically-fitted formulae of high order for long-time integration of orbital problems . This is addition to multilayer symplectic integrators .

3) We made a great deal of combination between Mathematics and the computers science represented by computer programs languages such as $pascal$, c & c^{++} and computer programs *Maple* & *MATLAB* to compute the big amount of numbers and coefficients .

4) We gave a useful tool for a large number of researcher`s and student`s in many areas of science, engineering, and business. Mathematicians who need to calculate any bounded integration (the area under a curve) That`s not all , but they can choose any number of Decimal places since any equation is written in the form of Numerator and Denominator .

5) The study provided a great potential to the Engineers , scientists and researchers to find the solution of their Differential equations and the Partial Differential equations with more significance results , knowing that a small error in calculation leads to big functional errors .

6) This study will open a big door to the researcher`s to find the next height order equation of Newton's Forward Interpolation Formula .

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