



# Online Analytical Processing Operations via Neutrosophic Systems

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## Abstract

In this paper, a neutrosophic fuzzy data warehouse modelling approach is presented to support the neutrosophic analysis of the publishing house for books which allows integration of neutrosophic concept in dimensions and facts without affecting the core of a crisp data warehouse. Also we describe a method is presented which includes guidelines that can be used to convert a crisp data warehouse into a neutrosophic fuzzy domain. Finally we have presented an OLAP system that implements a neutrosophic multidimensional model to represent imprecision using neutrosophic concept in hierarchies and facts and achieving knowledge discovery from imperfect data. The use of neutrosophic structures and the definition of the OLAP operations (roll-up, drill-down, slice, and dice) enable the model to manage the imprecision of data and hide the complexity of the model and provide the user with a more understandable result.

**Keywords:** Fuzzy Sets, Neutrosophic Fuzzy Data Warehouse, Neutrosophic Fuzzy Cube, Neutrosophic Fuzzy OLAP Operation.

## 1. Introduction

In business scenarios, where some of the data or the business attributes are neutrosophic, it may be useful to construct a warehouse that can support the analysis of neutrosophic data. Accurate information about an organization's state is necessary in order to make strategic decisions. Information contains historical data derived from transaction data, but it usually includes data from other sources such as relational databases, spreadsheets, mainframes, mail systems or even paper files. Each of these data stores tends to serve a subset of the enterprise for decision making. An increasing number of heterogeneous information systems makes retrieving meaningful information more difficult. In order to gather, store and process this information, various information systems are used. The enterprise information system map shows often numerous, heterogeneous and complex information system constellations. Often, for operational use relational database systems are used and for analytical purposes a data warehouse is used. Bill Inmon [1] is cited very often and seems to be the father of the term Data Warehouse. In fact, Inmon's definition goes back to the first edition of his book "Building The Data Warehouse" from 1993, Wolfgang Lehner [2], a researcher in data warehousing, has recently published a profound and comprehensive book on data warehouse systems in German. He references Bill Inmon, but his book also contains a more elaborate definition of data warehouse systems. In addition to a relational database, a data warehouse environment includes an Extraction, Transformation, and Loading (ETL) solution, an Online Analytical Processing (OLAP) engine, client analysis tools, and other applications that manage the process of gathering data and delivering it to business users. This analytical view on data finally enables the enterprise to have a more global sight on its business environment than operational systems can provide. Therefore, data warehouses are often used as systems for decision making [3]. Besides positive aspects of centralized processing of business information such as decision making support, difficulties occur in maintaining and analyzing data warehouses. The amount of data that has to be processed in a data warehouse increases every day and turns into challenging tasks for administration and analysis. Next to the problem of high quantity, data from operational systems are often incomplete, vague or uncertain. This quality issue cannot be completely eliminated in the pre-processing stage of the data. Consequently, a certain amount of vagueness directly impacts the analysis and decision making that is based on the information of a data warehouse [4]. Data warehousing and on-line analytical processing (OLAP) are essential elements of decision support. "In [24-29] OLAP is computer processing that enables a user to easily and selectively extract and view data from different points of view. Data warehouse and OLAP tools provide an efficient framework for data mining. Besides, data from real world are often imperfect, either because they are uncertain, or because they are imprecise. To solve this problem, We have presented a structure that manages imprecision by means of

neutrosophic techniques [7, 8]. The use of neutrosophic set theory in systems enhances the understand ability of the discovered knowledge; this is the reason why we have proposed an approach to perform OLAP-based on neutrosophic concept. The ability to analyze large amounts of data for the extraction of valuable information presents a competitive advantage for any organization. . The managers need information about their business and insight into the existing data so as to make decision more efficiently without interrupting the daily work of an On-Line Transaction Processing (OLTP) system. The technologies of data warehousing, OLAP, and data classification support that ability. The data warehouse is a central data pool which integrates heterogeneous data sources and provides strategic information for analysis and decision support. The special needs of the OLAP technology was the main cause of the use of a multidimensional view of the data. On-Line Analytical Processing (OLAP) presents an approach to data analysis where data is consolidated and aggregated with respect to multiple dimensions of interest. The idea is to consolidate large amounts of data by summarizing and aggregating data elements for every cell of a data cube. Classification of data elements reduces an arbitrarily high number of data elements into an arbitrarily small set of classes, which highly reduces the granularity of data. In OLAP, classification is used for the consolidation of dimensional attributes." In Many complicated problems like, engineering problems, social, economic, computer science, medical science...etc, the data associated are not necessarily crisp, precise, and deterministic because of their vague nature. Most of these problems were solved by different theories. One of these theories was the fuzzy set theory discovered by Zadeh in [17-20] to handle vagueness , uncertainty and imprecision ,In fuzzy logics the two-point set of classical truth values  $\{0, 1\}$  is replaced by the real unit interval  $[0, 1]$  each real value in  $[0, 1]$  is intended to represent a different degree of truth, ranging from 0, corresponding to false in classical logic, to 1, corresponding to true. . A fuzzy set  $A$  in  $M$  can be represented as an ordered set of tuples  $\{(m, \mu_A(m))\}$ . But for some applications it is not enough to satisfy to consider only the membership-function supported by the evident but also have to consider the non-membership-function against by the evident Atanassov [6] introduced another type of fuzzy sets that is called Intuitionistic Fuzzy Set (IFS) which is more practical in real life situations. The main novelty of neutrosophic logic, as we shall see, is that we do not even assume that the incompleteness or "indeterminacy degree" is always given by  $1 - (t + f)$ . Smarandache and A.A.Salama [7, 8] introduced another concept of imprecise data called neutrosophic crisp sets. Neutrosophic set is a powerful general formal framework that has been recently proposed. Let  $N$  be a set defined as follows:  $N = \{(T, I, F) : T, I, F \in [0, 1]\}$ . Where (T) the Truth degree, (F) the falsehood degree and (I) the indeterminacy degree,  $I \in [0, 1]$  may represent not only indeterminacy but also vagueness, uncertainty, imprecision, error etc. Note also that T, I, F, called the neutrosophic components [9].Several researchers dealing with the concept of neutrosophic set such as M. Bhowmik and M.Pal in [10] and A.A.Salama in [11-15]. For more information on the application of neutrosophic theory, the readers can referes to [ 30-33] . In this paper we aim to construct a neutrosophic data warehouse. The key benefit of integrating neutrosophic logic in data warehouse it allows analysis of data in both classical and neutrosophic manners. The use of the proposed approach is demonstrated through a case study of a published housing for books. Finally we have presented an OLAP system that implements a neutrosophic multidimensional model to represent imprecision using neutrosophic concept in hierarchies and facts and achieving knowledge discovery from imperfect data.

## 2. Crisp Data Warehouse Concept

A data warehouse [1] is a database, which is kept separate from the organization's operational database. There is no frequent updating done in a data warehouse, it possesses consolidated historical data, which helps the organization to analyze, organize, understand, and use their data to take strategic decisions. This analytical view on data finally enables the enterprise to have a more global sight on its business environment than operational systems can provide. Therefore, data warehouses are often used as systems for decision making. The term "Data Warehouse" was first coined by Bill Inmon. he describe the data warehouse as "subject-oriented, integrated, non volatile, and time-variant collection of data in support of management's decision support. The components of his definition in the following way:

2.1. Subject-Oriented: Subject-Oriented means that the main objective of data warehouse is to facilitate decision process of a data company, and within any company data naturally concentrates around subject areas, so information gathering in warehouse is aiming for a specific subject rather than for the functions of a company.

2.2. Integrated: Being integrated means that the data is collected within the data warehouse, that can come from different tables, databases or even servers, but can be combined into one unit that is relevant and logical for convenience of making strategic decision.

2.3. Non-volatile: Non-volatile means the previous data is not erased when new data is added to it. A data warehouse is kept separate from the operational database and therefore frequent changes in operational database is not reflected in the data warehouse.

2.4. Time-variant: The content of the data warehouse grows over time, where on regular basis snapshot of current data is entered into the data pool. The key structure of the data warehouse always contains time.

### 3. Linguistic variables

Linguistic variables are collect elements into similar groups where we can deal with less precisely and hence we can handle more complex systems. it's is an important concept in fuzzy logic and plays a key role in its applications, especially in the fuzzy expert system Linguistic variable is a variable whose values are words in a natural language , For example, "speed" is a linguistic variable, which can take the values as "slow", "fast", "very fast" and so on. Zadeh developed on top of the fuzzy set theory a means for mathematically representing natural language [16]. Therefore, he defined a linguistic variable values [17, 18, 19] . The values of the linguistic variable called linguistic terms, are projected on a universe of discourse. Fuzzy sets are used to define the degree of membership with which a value might belong to a linguistic term. Zadeh defines a linguistic variable as follows:

**3.1. Definition (Linguistic variable [20]).** A linguistic variable is a quintuple  $(X; T(X); G;M; F)$  defined as follows:

X is the name of the linguistic variable,  $T(X)$  is the set linguistic terms of X , G represents a syntactic rule that generates the set of linguistic terms, M is the universe of discourse and F is a semantic rule that defines for each linguistic term its meaning in the sense of a fuzzy subset on U.

### 4. Concept of Neutrosophic Fuzzy Sets

The main idea of Neutrosophic Sets is to characterize each logical statement in a 3D Neutrosophic Space, where each dimension of the space represents respectively the truth (T), the falsehood (F), and the indeterminacy (I) of the statement. Neutrosophic Logic (NL) is a generalization of Zadeh's fuzzy logic (FL), and especially of Atanassov's intuitionistic fuzzy logic (IFL), and of other logics For example, suppose there are 10 voters during a voting process In time  $t_1$ , five vote \yes", three vote \no" and two are undecided, using neutrosophic notation, it can be expressed as  $\mathbf{x}_1(0.5, 0.2, 0.3)$  In time  $t_2$ , four vote \yes", two vote \no", and three are undecided, it then can be expressed as  $(0.4, 0.3, 0.2)$ . the notion of neutrosophic set is more general and overcomes the aforementioned issues. In neutrosophic set, indeterminacy is quantified explicitly and truth-membership, indeterminacy- membership and falsity-membership are independent. This assumption is very important in many applications such as information fusion in which we try to combine the data from different sensors. The neutrosophic set takes the value from real standard or non-standard subsets of  $] -0,1+[$ . So instead of  $] -0,1+[$  we need to take the interval  $[0,1]$  for technical applications, because  $] -0,1+[$  will be difficult to apply in the real applications such as in scientific and engineering problems. For software engineering proposals the classical unit interval  $[0, 1]$  is used. For single valued neutrosophic logic, the sum of the components is:

case (1)  $0 \leq t+i+f \leq 3$  when all three components are independent;

case (2)  $0 \leq t+i+f \leq 2$  when two components are dependent, while the third one is independent from them.

case (3)  $0 \leq t+i+f \leq 1$  when all three components are dependent.

### 5. Case Study

The case study discusses a The Publishing house for books , It currently offers a collection of books for purchase, Each customer is asked to rate the book when he read it, When the publishing house makes statistical survey To measure the performance of their business such that Publishing house for books analyzes the revenue of the books based on the age of the customers or stores or measure the performance based on rating of customers it is found that the proportion of persons did not give a specific answer (undecided). Their answer is not belong to a certain class or not belonging to this category, This percentage has not been taken into account for it found that there is ambiguity in the data became unclear ,for example the book "Scientific Miracles in the Holy Quran " some of people classify this book to scientific category and some of people classify it to religious category and others not decided(not sure) if this book belong to scientific category or religious category. so they didn't give a specific answer. Now we have three answers membership, Non Membership and Indeterminacy. Neutrosophic Sets to solve this ambiguity in the data and taking the opinion of indeterminacy into account and gave them the degree. for example: the Neutrosophic set " scientific books " might contain the following tuples:" scientific books " = {" Scientific Miracles in the Holy Quran " ,  $\langle 0.7,0.1,0.2 \rangle$  }  $\langle 0.7,0.1,0.2 \rangle$  which 0.7 is represented the membership degree of this book to scientific books genre, 0.2 is represented the non membership degree of this book to scientific books genre and 0.1 is represented the indeterminacy degree of this book to scientific books genre. so we must integration neutrosophic fuzzy concept to data warehouse.

### 6. Neutrosophic Fuzzy Data Warehouse

In order to create a neutrosophic data warehouse, a method is presented that can guide the transformation of a classical data warehouse into a neutrosophic data warehouse. The input to the method is a classical data warehouse and the output is a neutrosophic data warehouse. This approach allows integrating neutrosophic concepts without the need for redesigning the core of a data warehouse. By using this neutrosophic data warehousing approach, it is possible to extract and analyze the data simultaneously in a classical manner and in a neutrosophic manner.

For example, books might be classified into different genres. In the classical data warehouse, a book always belongs or not belong fully to one or more genres the numbers of the interval [0, 1] where 1 implies full belonging and 0 implies no belonging at all. In reality, books can often be categorized into several genres while belonging or not belonging more to one genre than to another with different degrees. A book “can be a scientific book, a religious book, a political book, a social book or a literary book and so on. In this classification it belonging at the same time to one or more genre but with different degrees and it not belonging with different degrees, and also has indeterminacy degree. In order to truly represent this ambiguity in classification the neutrosophic set theory can be applied therefore, the Publishing house classifies the books with a neutrosophic concept.

The following figure show convert classical data warehouse into neutrosophic data warehouse:

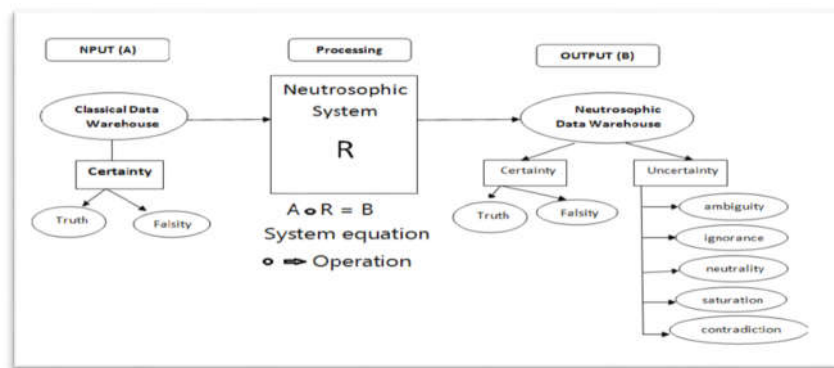


Figure 1: convert classical data warehouse into Neutrosophic Fuzzy Data warehouse

#### 6.1. Basic Definitions of Neutrosophic Fuzzy Data Warehouse

In this section we introduce and study the following definitions of Neutrosophic Data Warehouse.

##### 6.1.1 Definition (Neutrosophic fuzzy data warehouse (NDW)).

A neutrosophic fuzzy data warehouse model is a set of combination of four types of tables .these are (Dimension tables (D), Fact tables (F), Neutrosophic Classification Tables (NCT) and Neutrosophic Degree Tables (NDT)) and it is represented by NDW.  $NDW = \{D, F, NCT, NDT\}$

##### 6.1.2 Definition (Neutrosophic Fuzzy Table (NT) ).

Neutrosophic Table is the table which contain a neutrosophic target element and the table may be dimension table or Fact table.

##### 6.1.3 Definition (Neutrosophic Fuzzy Target Element (NTE)).

Neutrosophic Target Element (NTE) is the element may be in in a Fact table or a dimension table which required to be classified in neutrosophic.

##### 6.1.4 Definition (Class Neutrosophic Fuzzy Target Element (CNTE) ).

A class neutrosophic Target element (CNTE) for a neutrosophic Target element (NTE), it's all possible values (linguistic terms) for a neutrosophic Target element.

##### 6.1.5 Definition (Neutrosophic Fuzzy Degree (ND) ).

All values for neutrosophic Target element belong to a certain neutrosophic degree to a class neutrosophic which neutrosophic Target element belong . The degree of belonging to a value to class neutrosophic is called neutrosophic degree.

##### 6.1.7 Definition (Neutrosophic Classification Table (NCT)).

A table that holds linguistic terms (neutrosophic classes and it consists of two attribute (a primary key of the table and a class neutrosophic target element which can be classified neutrosophic),  $NCT = \{PK, CNTE\}$

##### 6.1.8 Definition (Neutrosophic Fuzzy Degree Table (NDT) ).

A table that stores the degree of each linguistic term is called neutrosophic degree table, it contains four attributes: (the primary key of the table, the foreign key of neutrosophic table which contains neutrosophic target, the foreign key of neutrosophic classification table and neutrosophic degree of each linguistic term for linguistic variable (neutrosophic target)).  $NDT = \{PK, FK\_NT, FK\_NCT, FK\_NDT\}$

### 6.2. Neutrosophic Date Warehouse Model

In addition to the classical analysis in a data warehouse, The Publishing house for books needs some features that are available using neutrosophic concepts. For integrating neutrosophic concepts into a data warehouse, one must first analyze which elements in the data warehouse should be classified neutrosophic. The element may be an element in the fact table or an element in a dimension table. An element that has to be classified neutrosophic is called the neutrosophic target element (NTE). The steps are the following:

- 1) First step: identify what should be classified to identify the neutrosophic target element.
- 2) Second step: identify the set of linguistic terms that are used for classifying the neutrosophic target element. Repeat this step for all neutrosophic target elements.
- 3) Third step: define a neutrosophic function for each linguistic term. Repeat this step for each linguistic term.
- 4) Fourth step: create Neutrosophic classification table which holds classes of neutrosophic target element (linguistic terms) and it contains two attributes one is the primary key of the table and second is the class neutrosophic target element.
- 5) Fifth step: create Neutrosophic degree table which holds neutrosophic degrees for each linguistic term and it contains four attributes which the first attribute is the primary key of the table, the second attribute is the foreign key of the neutrosophic table, the third attribute is the foreign key of Neutrosophic classification table and the fourth attribute is the neutrosophic degree (ND) attribute for the neutrosophic target element. The values of neutrosophic degree attribute are calculated by neutrosophic functions (represented by  $\langle T, I, F \rangle$  where  $T$  is the membership degree of element to a set  $A$ ,  $F$  is the non membership degree of element to a set  $A$  and  $I$  is the degree of indeterminacy element to a set  $A$ ).
- 6) Sixth step: Relate neutrosophic table with neutrosophic classification table and neutrosophic degree table with each other.

The following figure represented Neutrosophic Date Warehouse Model:

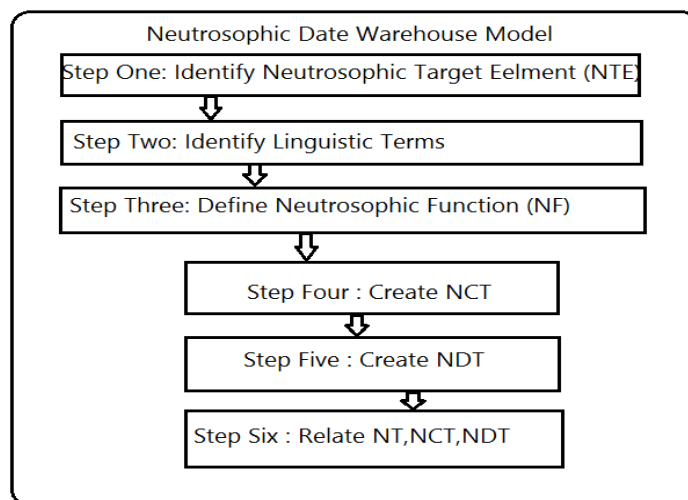


Figure 2: Neutrosophic fuzzy data warehouse Model

#### 6.2.1 Dimension Book

The books are classified into different genres. In the classical data warehouse, a book always belongs to interval  $[0,1]$  where 1 implies to belonging degree of the book fully to one or more genres and 0 implies to not belonging degree of the book to one or more genres. In reality, books can often be categorized into several genres while (belonging, indeterminacy, not belonging) more to one genre than to another at the same time. For Example, the book "Scientific Miracles in the Holy Quran" can be classified scientific and Religious, but more strongly scientific (membership), and can be classified not belonging to one or more genres by different degrees (non membership) and also the book has the indeterminacy degree. With the classification in the classical data warehouse approach, the published house cannot classify the books into different genres. Therefore, the published house classifies the books with a neutrosophic concept

- First the book genre is defined as neutrosophic target element.
  - The second step is to identify the linguistic terms. In this case, the linguistic terms are the different genres to which the books belong. These genres can be extracted from the dimension category in the book dimension.
  - In the third task is to identify the neutrosophic functions for each genre has to be defined
- After identifying neutrosophic target element, linguistic terms and their neutrosophic functions,
- forth step is to create neutrosophic classification table holds the genres as class neutrosophic element.
  - Fifth step is to create neutrosophic degree table contains neutrosophic degrees for each neutrosophic target element corresponding to class neutrosophic elements.
  - finally, sixth step is to relate the neutrosophic classification table, the neutrosophic degree table and the neutrosophic table to each other.

The following Figure show neutrosophic concept in book dimension:

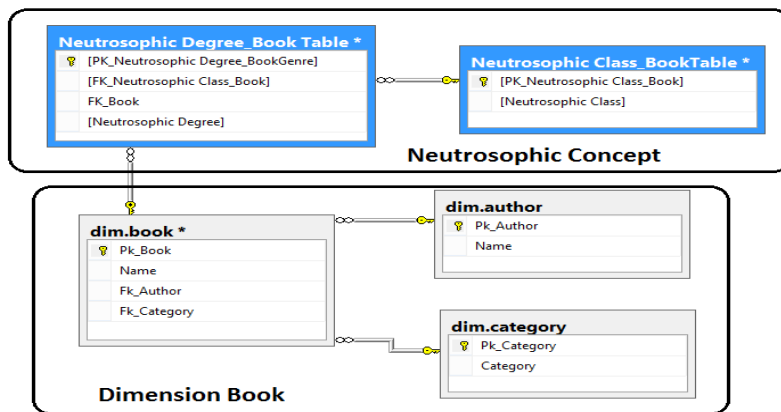


Figure 3: Neutrosophic Concept Book Genre

The following figure shows result sets of apply neutrosophic concept in book dimension

| Book ID | Name  | Neutrosophic Class | Membership Degree |
|---------|---|--------------------|-------------------|
| 1       | Scientific Miracles in the Holy Quran   | scientific         | < 0.6, 0.1, 0.3 > |
| 1       | Scientific Miracles in the Holy Quran   | religious          | < 0.5, 0.2, 0.3 > |
| 2       | Political Islam and the coming battle   | political          | < 0.7, 0.1, 0.2 > |
| 2       | Political Islam and the coming battle   | religious          | < 0.6, 0.1, 0.3 > |
| 3       | perhaps you laugh   | social             | < 0.4, 0.3, 0.3 > |
| 3       | perhaps you laugh   | Comic              | < 0.6, 0.2, 0.2 > |
| 5       | Soft hands  | romantic           | < 0.7, 0, 0.3 >   |
| 5       | Soft hands  | Comic              | < 0.3, 0.2, 0.5 > |
| 6       | Astronomical calculations and scientific applications in the service of Islamic law | scientific         | < 0.7, 0.1, 0.1 > |
| 6       | Astronomical calculations and scientific applications in the service of Islamic law | Fiction            | < 0.4, 0.1, 0.5 > |

Figure 4: Result Set of Apply Neutrosophic Concept Book Genre

**6.2.2 Dimension Customers** A data warehouse contains the dimension customer, each customer has the attributes name, address and birthday. From the attribute birthday, the age of the customer can be calculated using the function today birthday

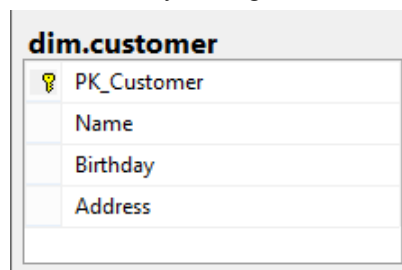


Figure 5: Dimension Customer



The published housing is interested in analyzing the revenue based on customers ages therefore, the publishing house classifies the customers ages with a neutrosophic concept in the following steps:

- The first step "The neutrosophic fuzzy target element is the customer age.
- The second step is to identify the linguistic terms for customer age. In this case, the linguistic terms for linguistic variable (customer age) are (old, middle, young) where old: customers more than 60

Middle: customers between 20 and 60, Young: customers less than 20  
 - The third step is to identify the neutrosophic functions for each linguistic term.

the publishing house defines the neutrosophic function that transform the age of customer into neutrosophic degrees by calculating neutrosophic function which represented by  $N = \langle \mu_A, \sigma_A, \nu_A \rangle$

Where  $\mu_A$  is the membership degree (belonging degree),  $\sigma_A$  is the indeterminacy degree and  $\nu_A$  is the non-membership degree (not belonging). The membership function depends on the customer age is the following

For example, if the customer age is 26 years old, it is transformed to term Young  $N_{young}(26) = \langle 0.4, 0.3, 0.3 \rangle$  and term Middle  $N_{Middle}(26) = \langle 0.6, 0.2, 0.2 \rangle$  and term Old  $N(26) = \langle 0, 0.1, 0.9 \rangle$

- membership degree of age 26 years old to Linguistic term young as fellow  $\mu_{young}(26) = 0.4$  and linguistic term Middle  $\mu_{middle}(26) = 0.6$  and linguistic term Old  $\mu_{old}(26) = 0.0$
- Non membership degree of age 26 years old to linguistic term young as fellow  $\nu_{young}(26) = 0.3$  and linguistic term Middle  $\nu_{middle}(26) = 0.2$  and linguistic term Old  $\nu_{old}(26) = 0.9$
- indeterminacy degree of age 26 years old to linguistic term young as fellow  $\sigma_{young}(26) = 0.3$

and linguistic term Middle  $\sigma_{middle}(26) = 0.2$  and linguistic term Old  $\sigma_{old}(26) = 0.1$

After identifying neutrosophic target element, linguistic terms and their neutrosophic functions, one neutrosophic classification table (NCT) which holds category of customers ages (set of linguistic terms (old , middle , young ) as class neutrosophic target element.

-Fifth step is to create neutrosophic degree table (NDT) contains neutrosophic degrees for each linguistic term corresponding to class neutrosophic target elements.

- Sixth step and final task, is to relate the neutrosophic classification table, the neutrosophic degree table and the neutrosophic table to each other.

The following figure gives dimension customer in neutrosophic concept

| BookID | Book Name   | Category   | Rating |
|--------|---|------------|--------|
| 1      | Scientific Miracles in the Holy Quran   | scientific | 9.00   |
| 2      | Political Islam and the coming battle   | political  | 7.00   |
| 3      | perhaps you laugh   | Comic      | 5.00   |
| 4      | Genius Omar   | religious  | 8.00   |
| 5      | Soft hands  | romantic   | 2.00   |
| 6      | Astronomical calculations and scientific applications in the service of Islamic law | scientific | 3.00   |

Figure 6: Neutrosophic Concept Customer age

The following figure show the input data in customer dimension in classical data warehouse

| Customer ID | Customer Name       | Age | Class"Old"         | Class"Middle"      | Class"Young"       |
|-------------|---------------------|-----|--------------------|--------------------|--------------------|
| 1           | Samy Ghareeb Bondok | 59  | < 0.9 , 0.1 , 0>   | < 0.1 , 0.2 , 0.7> | < 0 , 0.3 , 0.7>   |
| 2           | Walaa Samy Bondok   | 20  | < 0.0 , 0.1 , 0.9> | < 0 , 0.2 , 0.8>   | < 1 , 0 , 0>       |
| 3           | Marwa Fawzy Abbas   | 36  | < 0 , 0.5 , 0.5>   | < 1 , 0 , 0>       | < 0 , 0.3 , 0.7>   |
| 4           | Mohamed Samy Bondok | 25  | < 0 , 0.1 , 0.9>   | < 0.5 , 0.2 , 0.3> | < 0.5 , 0.3 , 0.2> |
| 5           | Rana Alaa Hamoda    | 66  | < 1 , 0 , 0>       | < 0 , 0.2 , 0.8>   | < 0 , 0.1 , 0.9>   |
| 6           | Ayoub Ahmed         | 6   | < 0 , 0.1 , 0.9>   | < 0 , 0.2 , 0.9>   | < 1 , 0 , 0>       |
| 7           | Omnia Hassan Osman  | 42  | < 0 , 0.5 , 0.5>   | < 1 , 0 , 0>       | < 0 , 0.3 , 0.7>   |
| 8           | Ali Mohsen Ali      | 33  | < 0 , 0.5 , 0.5>   | < 1 , 0 , 0>       | < 0 , 0.3 , 0.7>   |
| 9           | Brouj Ahmed         | 9   | < 0 , 0.1 , 0.9>   | < 0 , 0.2 , 0.8>   | < 1 , 0 , 0>       |

Figure 7: Input Date in Classical Data Warehouse in Dimension Customer  
 The following figure show how to construct neutrosophic analysis of dimension customer

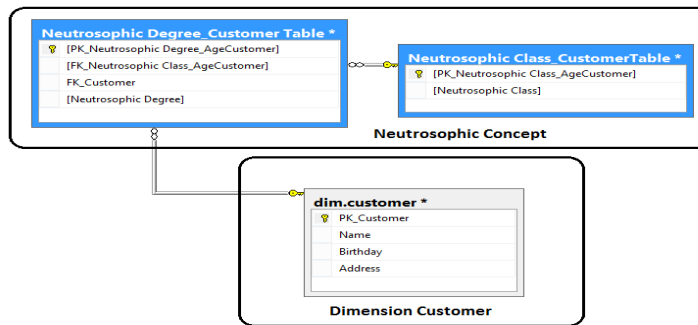


Figure 8: Result Set of Apply Neutrosophic Concept Customer Age

6.2.3 Fact

| Fact        |
|-------------|
| PK_Fact     |
| FK_Year     |
| FK_Book     |
| FK_Store    |
| FK_Customer |
| FK_Employee |
| FK_Type     |
| Rating      |
| Revenue     |

Figure 9: Fact Table

Customers are asked to rate every book when they read it. The rating of the book in the fact table As the rating is always between 0 and 10. The published housing for books uses this customer rating to evaluate the books into good, bad books. For this neutrosophic concept, the steps are the following:

The first step "The neutrosophic target element is the rating attribute in the fact table.

The second step is to identify the linguistic terms. In this case, the linguistic terms for customer rating are (good , bad) rate .

In the third task is to identify the neutrosophic functions for each linguistic term.

The publishing house defines the neutrosophic function as follows:

For example, if a customer rate a certain book 6 from 10 transformed to term good  $\mathcal{N}_{good} = < 0.6 , 0.1 , 0.3 >$  and term bad  $\mathcal{N}_{bad} = < 0.4 , 0.2 , 0.4 >$  and

Membership degree of rate (6) to Linguistic term good as fellow  $\mu_{good}(26) = 0.6$  and linguistic term bad  $\mu_{bad}(26) = 0.4$   
 Non membership degree of rate (6) to linguistic term good as fellow  $\vartheta_{good}(6) = 0.1$  and linguistic term bad  $\vartheta_{bad}(6) = 0.2$



Indeterminacy degree of rate (6) to linguistic term good as fellow  $\sigma_{good}(6) = 0.3$  and linguistic term bad  $\sigma_{bad}(6) = 0.4$  After identifying neutrosophic target element, linguistic terms and their neutrosophic functions, one neutrosophic classification table which holds the linguistic terms as class neutrosophic target element (good , bad ) is created then The neutrosophic degree table contains neutrosophic degrees for each neutrosophic target element corresponding to class neutrosophic elements. For final step, the neutrosophic classification table, the neutrosophic degree table and the neutrosophic table have to be related to each other . The following figure show how to apply neutrosophic concept in fact table:

| Book ID | Book Name   | Rating | Class"Good"     | Class"Bad"      |
|---------|---|--------|-----------------|-----------------|
| 1       | Scientific Miracles in the Holy Quran   | 9      | <1, 0, 0>       | <0, 0.1, 0.9>   |
| 2       | Political Islam and the coming battle   | 7      | <0.8, 0.1, 0.1> | <0.2, 0.2, 0.6> |
| 3       | perhaps you laugh   | 5      | <0.4, 0.1, 0.5> | <0.6, 0.2, 0.2> |
| 5       | Soft hands  | 2      | <0, 0.1, 0.9>   | <1, 0, 0>       |
| 6       | Astronomical calculations and scientific applications in the service of Islamic law | 3      | <0, 0.1, 0.9>   | <1, 0, 0>       |

Figure 10: Neutrosophic Concept Fact Rating

The following figure show input data in fact table in classical data warehouse

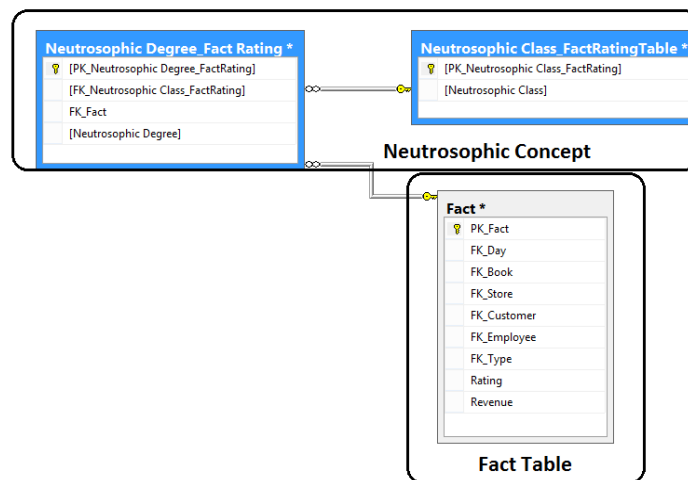


Figure 11: Input Data In Classical Data Warehouse in Fact Table

The following figure shows result set of apply neutrosophic concept in Fact table:

| Book ID | Book Name   | Rating | Class"Good"     | Class"Bad"      |
|---------|---|--------|-----------------|-----------------|
| 1       | Scientific Miracles in the Holy Quran   | 9      | <1, 0, 0>       | <0, 0.1, 0.9>   |
| 2       | Political Islam and the coming battle   | 7      | <0.8, 0.1, 0.1> | <0.2, 0.2, 0.6> |
| 3       | perhaps you laugh   | 5      | <0.4, 0.1, 0.5> | <0.6, 0.2, 0.2> |
| 5       | Soft hands  | 2      | <0, 0.1, 0.9>   | <1, 0, 0>       |
| 6       | Astronomical calculations and scientific applications in the service of Islamic law | 3      | <0, 0.1, 0.9>   | <1, 0, 0>       |

Figure 12: Result Set of Apply Neutrosophic Concept in fact rating

**7. Olap Operations in Crisp Data Warehouse:**

Codd, E. F. defined [8] the specialized OLAP operations drill-down, roll-up, slice and dice. According Codd and the OLAP Council [27] the OLAP operations called classical data warehouse operations, can be described shortly as follows:

- The Roll-up operation consolidates the values of a dimension hierarchy to a value on the next higher level.
- The Drill-down operation is used to navigate from top to bottom in a dimension. It is the opposite operation of the Roll-up operation.
- The Slice operation extracts a subset of values based on one or more dimensions using one dimension attribute to define the subset.
- Dice operation extracts a subset of values based on more than one dimension using more than one dimension attribute to define the subset.

Vassiliadis [12] proposed a notation of basic cube, cube and multidimensional database as follows :

**7.1 Definition (Basic Cube):**

A basic cube [27] is as a 3-tuple (D, L, R<sub>b</sub>); where D = (D<sub>1</sub>,.....,D<sub>n</sub>, M ) is a list of dimensions D and a measure M in a fact, L =(DL<sub>1</sub>,.....,DL<sub>n</sub>, ML ) is a list of dimension levels DL, aggregated measures ML and R<sub>b</sub> is a set of cells x = {x<sub>1</sub>,....., x<sub>n</sub>, m}, where x<sub>n</sub> ∈ dom(DL<sub>i</sub>) and m ∈ dom(ML) represents the instance values of the basic cube.

Example 1, The example considers a data warehouse with a fact table containing the fact revenue and three dimensions region, Product and time. The hierarchies for the dimensions are as follows:

- region: store → city → region
- Product: book
- time: day → month → year

a basic cube may take the following form: ( < region ,Book, time, revenue >,< city, book, day, aggregated revenue >, R ) where D is the dimensions region, Book, time, revenue > , L is the dimension levels < city, book, day, aggregated revenue > and R : set of cells represented by Figure 2.

| Book   | Region | Time  | Revenue |
|--------|--------|-------|---------|
| book A | cairo  | jan 1 | 620     |
| book A | cairo  | jan 2 | 420     |
| book A | Alex   | Feb 1 | 300     |
| book A | Alex   | Feb 2 | 450     |
| book B | cairo  | jan 1 | 310     |

Figure 13: R<sub>b</sub> of Basic Cube

**7.2 Definition (Cube):**

A cube [28] is a 4-tuple < D, L, C<sub>b</sub>, R > where D = < D<sub>1</sub>,...,D<sub>n</sub>, M > is a list of dimensions D and a measure M in a fact, L = < DL<sub>1</sub>,...,DL<sub>n</sub> , ML > is a list of dimension levels DL and aggregated measure ML, C<sub>b</sub> is a basic cube and R is a set of cells x = {x<sub>1</sub>,..., x<sub>n</sub>, m}, where x<sub>n</sub> ∈ dom(DL) and m ∈ dom(ML) represents the instance values of the cube. A cube can therefore be denoted as ( < region, product, time, revenue >, < city, book, month, aggregated revenue >, (< region, product, time, revenue >,< city, book, day, aggregated revenue > , R<sub>b</sub> ), R ) where R<sub>b</sub> is represented in Figure 12 and R in Figure 13.

| book   | Region | Time     | Revenue |
|--------|--------|----------|---------|
| book A | cairo  | January  | 1040    |
| book A | Alex   | February | 750     |
| book B | cairo  | January  | 310     |

Figure 14: R of Cube

The aim of defining a cube and a basic cube is the traceability of operations. Suppose that a data warehouse operation will calculate the average yearly revenue of books based on the cube with monthly revenues. It is necessary to go back to daily revenue, the lowest granularity, in order to give a meaningful result on yearly level. If the basic cube of the cube monthly revenue is not known, it is not possible to go to a lower level. No prediction can be made how the daily revenues have been aggregated to monthly revenues. Therefore, one can say that every cube representing a data collection in the data warehouse owns a basic cube which contain the lowest granularity of a dimension [23].

**7.3 Definition (Multidimensional database):**

A multidimensional database is a couple  $\langle D, C \rangle$  where  $D$  is a set of dimensions and  $C$  is the basic cube representing the lowest granularity [12].

Every OLAP operation defined below will have the following characteristics

- To identify the level  $l$  of the dimension  $d$  we will use  $d.l$  such as time.year
- The dimension and dimension level are merged using a dot notation into one variable in order to simplify the operation. Therefore, a dimension can be specified as  $D = \text{time}$  or with including a dimension level as  $D = \text{time.month}$ .
- The dot notation can be extended in order to integrate other category attributes. A full path to a dimensional attribute month in dimension level month of dimension time can be specified as  $D = \text{time.month.month}$ .

The definition of a cube is adapted as follows:

**7.4 Definition (cube (C)):**

A basic cube [13] is a 3-tuple  $\langle D, M, R \rangle$  where  $D = (D_1, \dots, D_n)$  is a list of dimensions, dimensions levels separated by a dot,  $M$  is an aggregated measure (in a fact) and  $R$  is a set of data tuples  $x = \{x_1, \dots, x_k, m\}$ , where  $x_k \in \text{dom}(D)$  and  $m \in \text{dom}(M)$  representing the instance values of the cube. the following figure represent a cube for three dimension year, books, city and measure revenue:

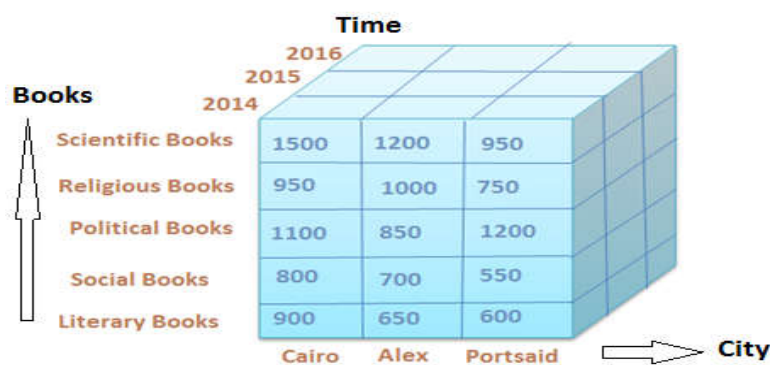


Figure 15: Cube

Once we have the structure of the multidimensional model, we need the operations to analyze the data in the Data Cube. Over this structure we have defined the normal operations of the multidimensional model such as Roll-up, Drill-down, Slice and Dice.

**7.5 Roll up Operation in Classical Data Warehouse:**

The roll-up operation is used to navigate a dimension upwards. Agrawal, Gupta and Sarawagi [29] defined a roll-up operation as a special case of their merge operation that it is executed on one dimension. For the dimension region a roll-up operation is executed when the dimension level store is aggregated to the next higher level city, and dimension level day is aggregated to the next higher level month. An example would be a cube C that is a subset representing the daily revenues, the revenues aggregated to the lowest hierarchy level of dimension time. Cube C can then be merged into a cube C' that is a data set representing the monthly revenues.

**7.5.1 Definition (Roll-up):**

Roll up  $(C, D', f_D, f_m) = C'$ ; where C is a basic cube, C' is anew cube after applying roll up operation, D' is the dimension of higher level,  $f_D$  is the dimension merge function and  $f_m$  is the measure aggregation function. The domain ( $dom_D$ ) of D  $dom_D(C)$  is a set containing the dimension instancesd,  $dom_D(C')$  is calculated by applying the function  $f_D$  on the dimension d of  $dom_D(C)$ .  $dom_D(C') = \{f_D(d) = d' | d \in dom_D(C)\}$ ,  $d \in D, \in d' \in D'$ . The measure  $m_C(d)$  of each instance d is calculated by applying the function  $f_m$  to each  $m_C(d)$  in regards to the aggregation function  $f_D$ .

$d = \{jan1, jan2, \dots, jan31\} \in D, D = \{d_1, d_2, d_3, \dots\}, d' = \{jan\} \in D', D' = \{jan, feb, mar, \dots\} = \{d'_1, d'_2, d'_3, \dots\}$   
 $m_C(d) = f_m(\{t | t = m_C(d) \wedge f_D(d) = d'\})$ .  $dom_{d'}(C') = dom_{time:month}(C')$  might be the set {January, February}, The instances of  $dom_{time:month}(C')$  are composed of instances of  $dom_d(C) = dom_{time:day}(C) = \{January 1, \dots, January 31, February 1, \dots, February 28\}$ , The function  $f_D$  defines  $\{January 1, \dots, January 31\} \rightarrow January$  and  $\{February 1, \dots, February 28\} \rightarrow February$ .

$C' = roll\ up(< time.day, revenue, S >, time.month, f_d, Sum\ Revenue)$  rollup the daily revenue to monthly revenue where C =  $< time.day, revenue, S >$ ,  $D' = time.month$ ,  $f_d$  = function on dimension time to transform lower level day to higher level month ex {January 1, ..., January 31}  $\rightarrow$  January and  $f_m$  = sum of revenue per month for example The revenue of January 1 might be  $m_C(January1) = 6000$  and January 31  $m_C(January31) = 5000$ , The function  $f_m$  is defined as summation the revenue of January is calculated as  $m_{C'}(January) = 6000 + 5000 = 11000$ .

**7.6 Drill-down Operation in Crisp Data Warehouse:**

With a Drill-down operation, values in a dimension level will be decomposed in values of the lower dimension level. This operation is used to reveal more detailed levels of data in a dimension. Drill-down is the opposite operation of roll-up. To be able to perform a drill-down, how the category attribute instances are compound from the lower hierarchy level instances must be known in advance. Considering the revenue of the year 2010 cannot drilled-down to monthly revenue, if the revenue of every month in 2010 is not known in advance. Otherwise, one can decompose the revenue of 2010 in infinite ways. A roll-up operation defining the aggregation functions and the value instances of the higher hierarchy level has to be executed before a drill-down operation [27].

Hence, the drill-down operation can be considered as a binary operation and formally be defined as:

**7.6.1 Definition (Drill-down).**

Drill down  $(C', D'', f_D^{-1}, f_m^{-1}) = C''$ ; where C'' is anew cube after applying the operation drill down, D'' is the dimension of lower,  $C' = roll\ up(C, D', f_D, f_m)$  and  $f_D^{-1}, f_m^{-1}$  are the inverse function of  $f_D$  respectively  $f_m$ . The following figure shows the roll-up and drill-down operation in a cube:

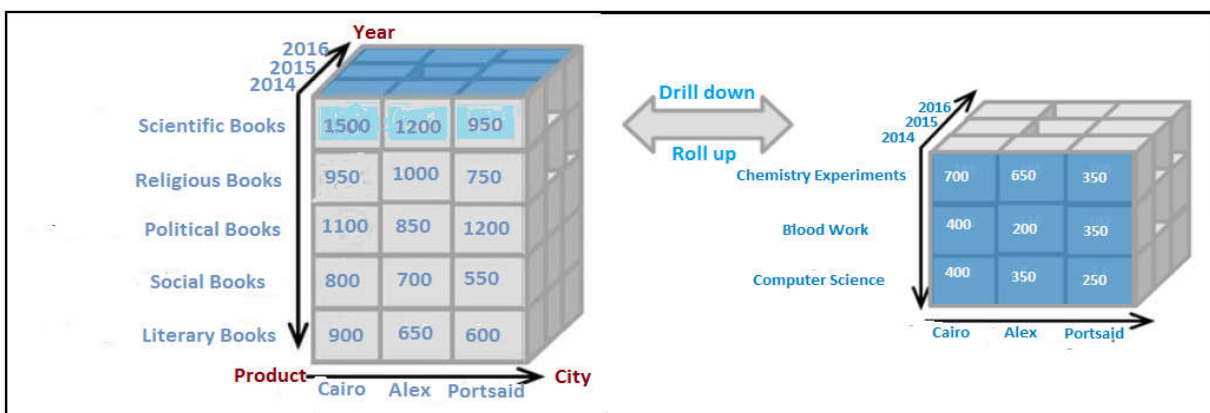


Figure 16: Roll-up and Drill-down Operation in a Cube

**7.7 Slice Operation in Crisp Data Warehouse:**

Slice is the act of picking a rectangular subset of a cube by choosing a single value for one of its dimensions. The slice operation cuts out a slice from a data cube in the multidimensional space of a data warehouse. For example, the cube  $C = \langle \text{time.year, product.book, region.city} \rangle, \text{revenue, } R$  can be sliced using the value 2014 for the dimensional attribute year. This will extract the revenue by all books category achieved in 2014 in each city .A slice operation can formally be defined as:

**7.7.1 Definition (Slice):**

$\text{slice}(C, d_m) = C'$ ; where  $C$  is a cube , $C'$  is anew cube after applying the slice operation and  $d_m \in \text{dom}(D_m)$  is the element instance that slices the cube. For extracting the revenue of all books category in all cities in 2014, the slice operation would be defined as follows:  $\text{slice} ( C = \langle \text{time.year, product.book, region.city} \rangle, \text{revenue, } S \rangle, d_m = \text{time.year.year} = "2014")$ . The following figure 6 shows the slice operation in a cube:

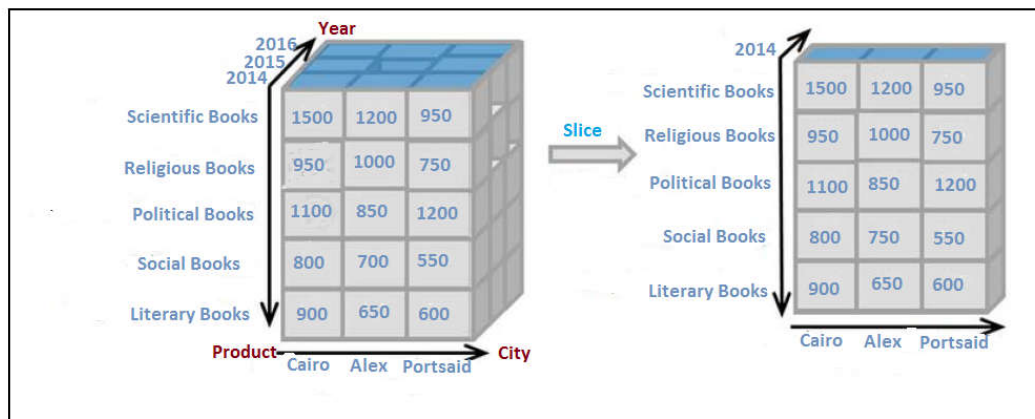


Figure 17: Slice Operation

**7.8 Dice Operation in Crisp Data Warehouse:**

The dice operation produces a sub cube by allowing the analyst to pick specific values of multiple dimensions. The Dice operation cuts out a dice from a data cube in the multidimensional space of a data warehouse. Slicers in a dice are combined using the logical operations AND, OR or NOT. The dice operation can formally be defined as follows:

**7.8.1 Definition (Dice).**

$\text{Dice}(C, \{d_m, \dots, d_k\}, \{f_m, \dots, f_{k-1}\}) = C'$  ; where  $C$  is the cube,  $C'$  is acube after applying the dice operation,  $\forall n \in \{m, \dots, k\} : d_n \in \text{dom}(D_n)$  are the element instances that slice the cube and  $\forall x \in m, \dots, k-1 : f_x \in \{\text{AND, OR, NOT}\}$  are the logical operator that combine the slicers in a way that  $d_m f_m f_{m+1}, \dots, d_{k-1} f_{k-1} d_k$ . As an example of a dice operation, the cube  $C = \langle \text{time.year, product.book, region.city} \rangle, \text{revenue, } R$  can be diced in order to show the revenue of category of books( scientific ,religious and political ) in city cairo and,  $\text{dice} = \langle \text{time.year, product.book, region.city} \rangle, \text{revenue, } S \rangle, \langle \text{product.book} = \text{"Scientific" or "religious" or "political", regon.city} = \text{"Cairo" or "Alex"} \rangle, \{\text{AND, AND}\}$ .The following figure 18 shows the dice operation:

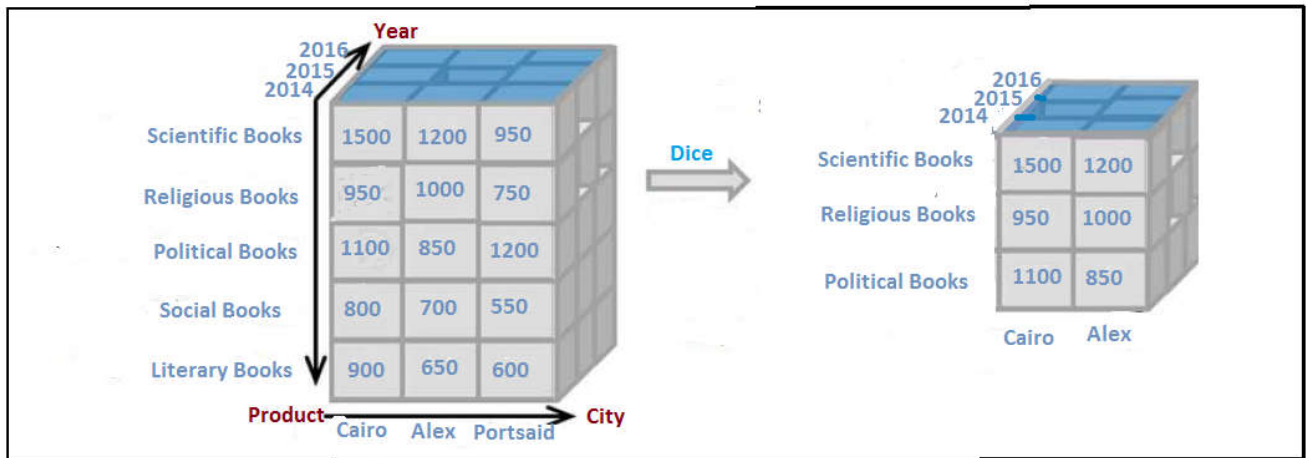


Figure 18: Dice Operation

**8. Aggregation of Neutrosophic Fuzzy Concepts:**

Aggregating of data in a data warehouse affects the neutrosophic concepts that classify data. Data is grouped together in a more dense view or split to reveal a more detailed view. The grouping is defined by the aggregation function that is often a summation, a maximization, a minimization, a count or an average function. This aggregation function is fundamental to the standard data warehouse operations. In order to be able to classify aggregated data, the neutrosophic concepts have to be aggregated too. In the next sections, two methods for aggregation of neutrosophic concepts are discussed. The first method is redefines the neutrosophic concept with the aggregated data instances as the new neutrosophic target element. for each dimension level, one neutrosophic degree table is created for the aggregate neutrosophic concepts. The class neutrosophic target elements are reused from the base neutrosophic neutrosophic concept containing the class neutrosophic target elements and a neutrosophic function is specified Therefore, this method is described not aggregating values. the second method is aggregates the neutrosophic degree instances of the neutrosophic concept to a more dense view. Each method is illustrated using an example.

**7.5.1 First Method "Redefine of Neutrosophic Fuzzy Concepts for Aggregation Data":**

A possible solution for aggregation of the neutrosophic concept onto another dimension hierarchy level. Redefining of a neutrosophic concept does not take the neutrosophic degree values into consideration. The linguistic terms and the neutrosophic functions are applied to a new hierarchy level. The neutrosophic degrees are recalculated based on the new neutrosophic target element. Neutrosophic degrees from the neutrosophic concept on the lower hierarchy level are not taken into consideration. for example , To redefine the neutrosophic concept store surface from dimension hierarchy level store, the neutrosophic concept is created on the level city. The neutrosophic classification table (NCT) is taken from the original neutrosophic concept. Whereas, the neutrosophic degree table (NDT) has to be newly created for the new neutrosophic concept. This is due to that new neutrosophic functions are calculating the neutrosophic degree based on the new neutrosophic target elements. These newly calculated neutrosophic degree are stored in the new NDT.

for example 2. To the dimension store a fact table is added. The fact table contains a measure revenue and the primary key of fact table and the foreign key relation to the store table (FK\_store). A neutrosophic concept is added having revenue as the neutrosophic target element. Store A and B earned multiple revenues of 2500 for store A and 4500 for store B. For every revenue a new instance is stored in the fact table. The total revenue of a city is the sum of all revenues earned by stores. Each revenue has a neutrosophic degree for each linguistic term in the neutrosophic concept revenue ( $\mathcal{N}_{high}$ ,  $\mathcal{N}_{middle}$ ,  $\mathcal{N}_{low}$ ). For the city hierarchy level the revenues are aggregated to the city Cairo and we want to classify revenues of level city. For do that the neutrosophic concept including the neutrosophic degree table must be defined on city level. The neutrosophic target element for this neutrosophic concept is the city revenue and neutrosophic class for store revenue is reused for city revenue



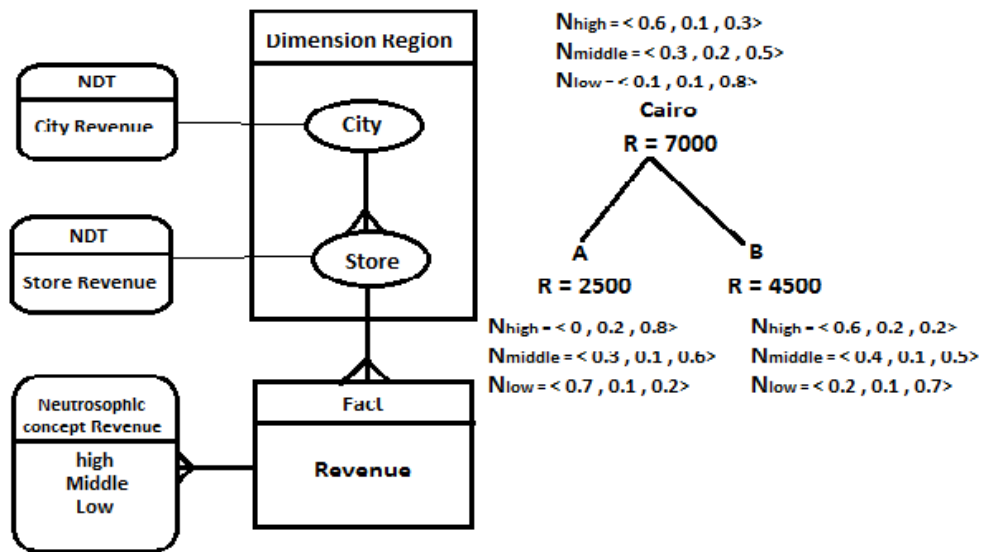


Figure 19: Dimension Store and Fact Revenue with Neutrosophic Concepts

the idea of reuse the neutrosophic classification table of the base concept to reduce the amount of extra tables and to limit the resources in the neutrosophic data

**7.5.2 Second method: Aggregation of Neutrosophic Concepts**

The second method is to aggregate the neutrosophic degree of each neutrosophic target element instance into a next higher hierarchy level. Each value of the next higher hierarchy level is composed of a set of value from the neutrosophic target element .a neutrosophic degree for each instance of the next higher hierarchy level can be considered as aggregation of the neutrosophic degree of the lower level. In order to illustrate the aggregation of neutrosophic concepts, consider the following Example 2. A data warehouse contains a dimension store with two category: store and city. All stores are aggregated to the corresponding city. For all stores their area is measured and added to the dimension as dimensional attributes on level store. Considering Lehner [16], dimensional attributes can be aggregated on higher hierarchy levels, similarly as it would happen for measures. Therefore, the store area can be aggregated to the level city. a neutrosophic concept with store area as neutrosophic target element is defined. The neutrosophic concept classifies the area as big, medium and small. In example 1, the average store area of a city can be calculated by aggregating the area of all stores in a city with an average function. In order to apply the neutrosophic concept store area on the level city, the neutrosophic degree have to be aggregated. By the foreign key relation of stores to cities, it is known which store areas are aggregated to a distinct city area. the neutrosophic degree on level store can be identified that aggregate to a neutrosophic degree on level city. An additional aggregation function can then be defined that aggregates the neutrosophic degree of the stores to the neutrosophic degree of the city. In the case of store area, the arithmetic average of the neutrosophic degree of each class neutrosophic can be used to generate the corresponding neutrosophic degree for the cities. The dimensional attribute area is aggregated using an average function and therefore an aggregation of the neutrosophic concept using the arithmetic average . In this case, no additional tables have to be created. For example: A city Cairo contain 2 store A and B, A neutrosophic concept apply in store area ,the area of store A is 90 meter square with neutrosophic degree  $\mathcal{N}_{big} = \langle 0, 0.4, 0.6 \rangle$  ,  $\mathcal{N}_{medium} = \langle 0, 0.6, 0.4 \rangle$  ,  $\mathcal{N}_{small} = \langle 1, 0, 0 \rangle$  and the area of store B is 270 meter square with neutrosophic degree  $\mathcal{N}_{big} = \langle 0.7, 0.1, 0.2 \rangle$  ,  $\mathcal{N}_{medium} = \langle 0.7, 0.2, 0.1 \rangle$  ,  $\mathcal{N}_{small} = \langle 0, 0.6, 0.4 \rangle$  , here need to do roll up operation from level store to a higher level city ,this is done by two steps:

- 1) the city (cairo) area is aggregated using an average function = (90+270) /2 = 180 meter square
- 2) the neutrosophic degree for city area is aggregation of the neutrosophic concept using the arithmetic average such as:

$$\mathcal{N}_{big} = \langle \frac{\mu_{big} * 90 + \mu_{big} * 270}{360}, \frac{\sigma_{big} * 90 + \sigma_{big} * 270}{360}, \frac{\vartheta_{big} * 90 + \vartheta_{big} * 270}{360} \rangle$$

$$= \langle \frac{0 * 90 + 0.7 * 270}{360}, \frac{0.4 * 90 + 0.1 * 270}{360}, \frac{0.6 * 90 + 0.2 * 270}{360} \rangle = \langle 0.525, 0.175, 0.3 \rangle$$



$$\begin{aligned}
 \mathcal{N}_{\text{medium}} &= \left\langle \frac{\mu_{\text{medium}}*90 + \mu_{\text{medium}}*270}{360}, \frac{\sigma_{\text{medium}}*90 + \sigma_{\text{medium}}*270}{360}, \frac{\vartheta_{\text{medium}}*90 + \vartheta_{\text{medium}}*270}{360} \right\rangle \\
 &= \left\langle \frac{0*90 + 0.7*270}{360}, \frac{0.6*90 + 0.2*270}{360}, \frac{0.4*90 + 0.1*270}{360} \right\rangle = \langle 0.525, 0.3, 0.175 \rangle \\
 \mathcal{N}_{\text{small}} &= \left\langle \frac{\mu_{\text{small}}*90 + \mu_{\text{small}}*270}{360}, \frac{\sigma_{\text{small}}*90 + \sigma_{\text{small}}*270}{360}, \frac{\vartheta_{\text{small}}*90 + \vartheta_{\text{small}}*270}{360} \right\rangle \\
 &= \left\langle \frac{1*90 + 0*270}{360}, \frac{0*90 + 0.6*270}{360}, \frac{0*90 + 0.4*270}{360} \right\rangle = \langle 0.25, 0.45, 0.3 \rangle
 \end{aligned}$$

The following figure 8 shows that the second method for aggregation neutrosophic concept in dimension store :

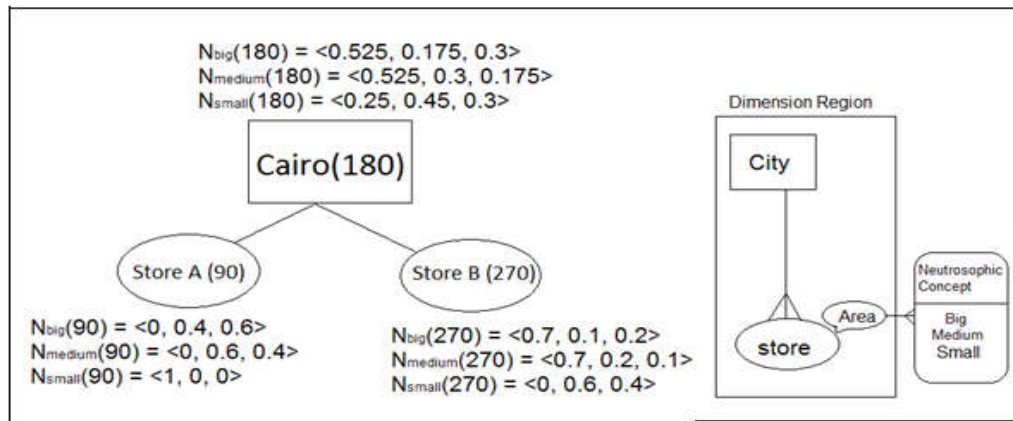


Figure20: Aggregation of a Neutrosophic Concept

**8. Olap Operations in Neutrosophic Data Warehouse :**

the classical data warehouse operation can be extended to support neutrosophic concept. neutrosophic concepts can be treated as dimensional attributes. Lehner [2] defines multidimensional objects that are capable of aggregating dimensional attributes and takes them into consideration as segments for slice and dice operations. it is possible to aggregate neutrosophic concepts and to use them as slicers for slice and dice operations. a fact can be aggregated over a dimension hierarchy, the neutrosophic concept with the fact as neutrosophic target can be aggregated . The neutrosophic concepts on facts can be considered as segmentation in slice and dice operation just as neutrosophic concepts on dimensional attributes. When segmenting a cube with a neutrosophic concept, the class neutrosophic degree are the delimiter of a segment. In order to discuss the classical operations, these characteristics of neutrosophic concepts have to be taken into consideration. The definition of crisp cube is adapted to integrate neutrosophic concepts.

**8.1 Definition ( Neutrosophic Fuzzy Cube (NC)):**

A cube in neutrosophic data warehouse is composed of 4-tuple (D,N,M,S ); where D = (D<sub>1</sub>, D<sub>2</sub>,.....,D<sub>n</sub>) is a list of dimensions, dimensions levels including the dimension attribute separated by a dot , N = (N<sub>1</sub>, N<sub>2</sub>,.....,N<sub>k</sub>) is a list of neutrosophic concepts with neutrosophic target that are either in (facts or dimension) or class neutrosophic target element of neutrosophic concept separated by a dot, M is a measure in a fact and S is a set of data tuples x = {x<sub>1</sub>,.....x<sub>n</sub>, n<sub>1</sub>,.....n<sub>k</sub> ,m}, where x<sub>n</sub> ∈ dom(D), ∀n<sub>k</sub> ∈ dom(N) and m ∈ dom(M) representing the instance values of the cube. For example: A neutrosophic concept is added to the fact revenue as neutrosophic target element. The neutrosophic concept revenue contains three classes “low”, “middle” and “high” revenue and the book decomposed into several genre such as (scientific, political, religious,.... so on ) and the dimension city with neutrosophic target area contain three neutrosophic class ( Big , Medium , Small).

A Neutrosophic Cube is a binary operation which can be involved two steps as follow:

- 1) select Crisp Dimensions.
- 2) apply the neutrosophic concept on the neutrosophic target element.

A neutrosophic cube can be: (< time.month, Region.city, Product.book>,< time.month.revenue, Region.city.area, Product.book.genre >, revenue ,S). The neutrosophic concept revenue is propagated on dimension time on level month and on dimension Region on level city and on dimension Product on level book. The result set of the neutrosophic cube are tuples containing the aggregated revenue as follow:

| Book Name                             | NC.Book          | Neutrosophic Degree Book | City  | NC.City Area | Neutrosophic Degree City Area | Month    | Revenue | NC.Revenue | Neutrosophic Degree Revenue |
|---------------------------------------|------------------|--------------------------|-------|--------------|-------------------------------|----------|---------|------------|-----------------------------|
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8 , 0.1 , 0.1 >      | Cairo | Big          | < 0.9 , 0.1 , 0 >             | February | 6200    | High       | < 0.7 , 0.1 , 0.2 >         |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8 , 0.1 , 0.1 >      | Cairo | Big          | < 0.9 , 0.1 , 0 >             | February | 6200    | Middle     | < 0.3 , 0.2 , 0.5 >         |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8 , 0.1 , 0.1 >      | Cairo | Big          | < 0.9 , 0.1 , 0 >             | February | 6200    | Low        | < 0 , 0.3 , 0.7 >           |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8 , 0.1 , 0.1 >      | Cairo | Medium       | < 0.1 , 0.2 , 0.7 >           | February | 6200    | High       | < 0.7 , 0.1 , 0.2 >         |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8 , 0.1 , 0.1 >      | Cairo | Medium       | < 0.1 , 0.2 , 0.7 >           | February | 6200    | Middle     | < 0.3 , 0.2 , 0.5 >         |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8 , 0.1 , 0.1 >      | Cairo | Medium       | < 0.1 , 0.2 , 0.7 >           | February | 6200    | Low        | < 0 , 0.3 , 0.7 >           |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8 , 0.1 , 0.1 >      | Cairo | Small        | < 0 , 0.2 , 0.8 >             | February | 6200    | High       | < 0.7 , 0.1 , 0.2 >         |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8 , 0.1 , 0.1 >      | Cairo | Small        | < 0 , 0.2 , 0.8 >             | February | 6200    | Middle     | < 0.3 , 0.2 , 0.5 >         |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8 , 0.1 , 0.1 >      | Cairo | Small        | < 0 , 0.2 , 0.8 >             | February | 6200    | Low        | < 0 , 0.3 , 0.7 >           |
| Scientific Miracles in the Holy Quran | Political Books  | < 0.3 , 0.2 , 0.5 >      | Cairo | Big          | < 0.9 , 0.1 , 0 >             | February | 6200    | High       | < 0.7 , 0.1 , 0.2 >         |
| Scientific Miracles in the Holy Quran | Political Books  | < 0.3 , 0.2 , 0.5 >      | Cairo | Big          | < 0.9 , 0.1 , 0 >             | February | 6200    | Middle     | < 0.3 , 0.2 , 0.5 >         |
| Scientific Miracles in the Holy Quran | Political Books  | < 0.3 , 0.2 , 0.5 >      | Cairo | Big          | < 0.9 , 0.1 , 0 >             | February | 6200    | Low        | < 0 , 0.3 , 0.7 >           |
| Scientific Miracles in the Holy Quran | Political Books  | < 0.3 , 0.2 , 0.5 >      | Cairo | Medium       | < 0.1 , 0.2 , 0.7 >           | February | 6200    | High       | < 0.7 , 0.1 , 0.2 >         |
| Scientific Miracles in the Holy Quran | Political Books  | < 0.3 , 0.2 , 0.5 >      | Cairo | Medium       | < 0.1 , 0.2 , 0.7 >           | February | 6200    | Middle     | < 0.3 , 0.2 , 0.5 >         |
| Scientific Miracles in the Holy Quran | Political Books  | < 0.3 , 0.2 , 0.5 >      | Cairo | Medium       | < 0.1 , 0.2 , 0.7 >           | February | 6200    | Low        | < 0 , 0.3 , 0.7 >           |
| Scientific Miracles in the Holy Quran | Political Books  | < 0.3 , 0.2 , 0.5 >      | Cairo | Small        | < 0 , 0.2 , 0.8 >             | February | 6200    | High       | < 0.7 , 0.1 , 0.2 >         |
| Scientific Miracles in the Holy Quran | Political Books  | < 0.3 , 0.2 , 0.5 >      | Cairo | Small        | < 0 , 0.2 , 0.8 >             | February | 6200    | Middle     | < 0.3 , 0.2 , 0.5 >         |
| Scientific Miracles in the Holy Quran | Political Books  | < 0.3 , 0.2 , 0.5 >      | Cairo | Small        | < 0 , 0.2 , 0.8 >             | February | 6200    | Low        | < 0 , 0.3 , 0.7 >           |

Figure 21: S of Neutrosophic Cube

If N contains a class neutrosophic target element, By applying the third step (class neutrosophic target element ) in the binary neutrosophic cube. A Neutrosophic Cube is a trinary which can be involved three steps as follow :

- 1) Select Crisp Dimensions.
- 2) Apply the neutrosophic concept on the neutrosophic target element.
- 3) Apply the class neutrosophic target element.

For example : (< time.month, Region.city , Product.book>,< time.month.revenue, Region.city.area.big, Product.book.genre.scientific >, revenue ,S )

The following figure 10 shows the neutrosophic cube with class neutrosophic target (big city area) :

| Book Name                             | NC.Book          | Neutrosophic Degree Book | City  | NC.City Area | Neutrosophic Degree City Area | Month    | Revenue | NC.Revenue | Neutrosophic Degree Revenue |
|---------------------------------------|------------------|--------------------------|-------|--------------|-------------------------------|----------|---------|------------|-----------------------------|
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8 , 0.1 , 0.1 >      | Cairo | Big          | < 0.9 , 0.1 , 0 >             | February | 6200    | High       | < 0.7 , 0.1 , 0.2 >         |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8 , 0.1 , 0.1 >      | Cairo | Big          | < 0.9 , 0.1 , 0 >             | February | 6200    | Middle     | < 0.3 , 0.2 , 0.5 >         |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8 , 0.1 , 0.1 >      | Cairo | Big          | < 0.9 , 0.1 , 0 >             | February | 6200    | Low        | < 0 , 0.3 , 0.7 >           |

Figure 22: Neutrosophic Cube with Class Neutrosophic Target.

**8.2 Roll-up in Neutrosophic Data Warehouse:**

A roll-up operation can be applied to a neutrosophic cube, the roll up operation on a neutrosophic data warehouse involving neutrosophic concepts can be defined as:

**8.2.1 Definition (Roll-up involving Neutrosophic Concept (RNC):**

RNC = roll-up ( NC, DH, NH, f<sub>D</sub>, f<sub>m</sub>, f<sub>N</sub>); where NC is a Neutrosophic cube , DH is the dimension of higher level, NH is the neutrosophic concept of higher level, f<sub>D</sub> is the dimension merge function, f<sub>m</sub> is the measure aggregation function , f<sub>N</sub> is the method how to aggregate neutrosophic concept on the next level and RNC is the result cube on the higher level after applying roll up operation. This roll-up operation is a binary operation that first aggregates the crisp fact revenue. In the second step, it applies the new neutrosophic concept to the data collection with the following steps:

- 1) roll-up of the crisp cube.
- 2) apply the neutrosophic concept on the new dimensional level.

For example: roll up (NC, < time.month >, < time.month.revenue >, revenue, S >, time.year, time.year.revenue, f<sub>time.year</sub>, Revenue, f<sub>N</sub>) = (< time.year >, <time.year.revenue>, revenue, NS >. where DH is the dimension of higher level year = time.year, NH is the neutrosophic concept of higher level year = time.year.revenue, f<sub>a</sub> is the dimension merge function such as { Jan<sub>2015</sub>,Feb<sub>2015</sub>,.....,Dec<sub>2015</sub>} → 2015, f<sub>m</sub> is the aggregation revenue per year and RNS is the result set of apply the roll up operation on neutrosophic cube. the following figure 11, 12 shows the roll-up operation in neutrosophic cube :

| Book Name                             | NC.Book          | Neutrosophic Degree Book | City  | NC.City Area | Neutrosophic Degree City Area | year | Revenue | NC.Revenue | Neutrosophic Degree Revenue |
|---------------------------------------|------------------|--------------------------|-------|--------------|-------------------------------|------|---------|------------|-----------------------------|
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8, 0.1, 0.1 >        | Cairo | Big          | < 0.9, 0.1, 0 >               | 2015 | 60500   | High       | < 0.8, 0.1, 0.1 >           |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8, 0.1, 0.1 >        | Cairo | Big          | < 0.9, 0.1, 0 >               | 2015 | 60500   | Middle     | < 0.2, 0.2, 0.6 >           |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8, 0.1, 0.1 >        | Cairo | Big          | < 0.9, 0.1, 0 >               | 2015 | 60500   | Low        | < 0, 0.3, 0.7 >             |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8, 0.1, 0.1 >        | Cairo | Medium       | < 0.1, 0.2, 0.7 >             | 2015 | 60500   | High       | < 0.8, 0.1, 0.1 >           |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8, 0.1, 0.1 >        | Cairo | Medium       | < 0.1, 0.2, 0.7 >             | 2015 | 60500   | Middle     | < 0.2, 0.2, 0.6 >           |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8, 0.1, 0.1 >        | Cairo | Medium       | < 0.1, 0.2, 0.7 >             | 2015 | 60500   | Low        | < 0, 0.3, 0.7 >             |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8, 0.1, 0.1 >        | Cairo | Small        | < 0, 0.2, 0.8 >               | 2015 | 60500   | High       | < 0.8, 0.1, 0.1 >           |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8, 0.1, 0.1 >        | Cairo | Small        | < 0, 0.2, 0.8 >               | 2015 | 60500   | Middle     | < 0.2, 0.2, 0.6 >           |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8, 0.1, 0.1 >        | Cairo | Small        | < 0, 0.2, 0.8 >               | 2015 | 60500   | Low        | < 0, 0.3, 0.7 >             |
| Scientific Miracles in the Holy Quran | Political Books  | < 0.3, 0.2, 0.5 >        | Cairo | Big          | < 0.9, 0.1, 0 >               | 2016 | 40000   | High       | < 0.3, 0.1, 0.6 >           |
| Scientific Miracles in the Holy Quran | Political Books  | < 0.3, 0.2, 0.5 >        | Cairo | Big          | < 0.9, 0.1, 0 >               | 2016 | 40000   | Middle     | < 0.4, 0.3, 0.3 >           |
| Scientific Miracles in the Holy Quran | Political Books  | < 0.3, 0.2, 0.5 >        | Cairo | Big          | < 0.9, 0.1, 0 >               | 2016 | 40000   | Low        | < 0, 0.2, 0.8 >             |
| Scientific Miracles in the Holy Quran | Political Books  | < 0.3, 0.2, 0.5 >        | Cairo | Medium       | < 0.1, 0.2, 0.7 >             | 2016 | 40000   | High       | < 0.3, 0.1, 0.6 >           |
| Scientific Miracles in the Holy Quran | Political Books  | < 0.3, 0.2, 0.5 >        | Cairo | Medium       | < 0.1, 0.2, 0.7 >             | 2016 | 40000   | Middle     | < 0.4, 0.3, 0.3 >           |
| Scientific Miracles in the Holy Quran | Political Books  | < 0.3, 0.2, 0.5 >        | Cairo | Medium       | < 0.1, 0.2, 0.7 >             | 2016 | 40000   | Low        | < 0, 0.2, 0.8 >             |
| Scientific Miracles in the Holy Quran | Political Books  | < 0.3, 0.2, 0.5 >        | Cairo | Small        | < 0, 0.2, 0.8 >               | 2016 | 40000   | High       | < 0.3, 0.1, 0.6 >           |
| Scientific Miracles in the Holy Quran | Political Books  | < 0.3, 0.2, 0.5 >        | Cairo | Small        | < 0, 0.2, 0.8 >               | 2016 | 40000   | Middle     | < 0.4, 0.3, 0.3 >           |
| Scientific Miracles in the Holy Quran | Political Books  | < 0.3, 0.2, 0.5 >        | Cairo | Small        | < 0, 0.2, 0.8 >               | 2016 | 40000   | Low        | < 0, 0.2, 0.8 >             |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8, 0.1, 0.1 >        | Alex  | Big          | < 0.7, 0.2, 0.1 >             | 2015 | 50800   | High       | < 0.7, 0.1, 0.2 >           |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8, 0.1, 0.1 >        | Alex  | Big          | < 0.7, 0.2, 0.1 >             | 2015 | 50800   | Middle     | < 0.6, 0.2, 0.2 >           |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8, 0.1, 0.1 >        | Alex  | Big          | < 0.7, 0.2, 0.1 >             | 2015 | 50800   | Low        | < 0.1, 0.2, 0.7 >           |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8, 0.1, 0.1 >        | Alex  | Medium       | < 0.5, 0.1, 0.4 >             | 2015 | 50800   | High       | < 0.7, 0.1, 0.2 >           |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8, 0.1, 0.1 >        | Alex  | Medium       | < 0.5, 0.1, 0.4 >             | 2015 | 50800   | Middle     | < 0.6, 0.2, 0.2 >           |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8, 0.1, 0.1 >        | Alex  | Medium       | < 0.5, 0.1, 0.4 >             | 2015 | 50800   | Low        | < 0.1, 0.2, 0.7 >           |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8, 0.1, 0.1 >        | Alex  | Small        | < 0.3, 0.2, 0.5 >             | 2015 | 50800   | High       | < 0.7, 0.1, 0.2 >           |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8, 0.1, 0.1 >        | Alex  | Small        | < 0.3, 0.2, 0.5 >             | 2015 | 50800   | Middle     | < 0.6, 0.2, 0.2 >           |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8, 0.1, 0.1 >        | Alex  | Small        | < 0.3, 0.2, 0.5 >             | 2015 | 50800   | Low        | < 0.1, 0.2, 0.7 >           |
| Scientific Miracles in the Holy Quran | Political Books  | < 0.3, 0.2, 0.5 >        | Alex  | Big          | < 0.7, 0.2, 0.1 >             | 2016 | 28000   | High       | < 0.1, 0.2, 0.7 >           |
| Scientific Miracles in the Holy Quran | Political Books  | < 0.3, 0.2, 0.5 >        | Alex  | Big          | < 0.7, 0.2, 0.1 >             | 2016 | 28000   | Middle     | < 0.4, 0.3, 0.3 >           |
| Scientific Miracles in the Holy Quran | Political Books  | < 0.3, 0.2, 0.5 >        | Alex  | Big          | < 0.7, 0.2, 0.1 >             | 2016 | 28000   | Low        | < 0.8, 0.1, 0.1 >           |
| Scientific Miracles in the Holy Quran | Political Books  | < 0.3, 0.2, 0.5 >        | Alex  | Medium       | < 0.5, 0.1, 0.4 >             | 2016 | 28000   | High       | < 0.1, 0.2, 0.7 >           |
| Scientific Miracles in the Holy Quran | Political Books  | < 0.3, 0.2, 0.5 >        | Alex  | Medium       | < 0.5, 0.1, 0.4 >             | 2016 | 28000   | Middle     | < 0.4, 0.3, 0.3 >           |
| Scientific Miracles in the Holy Quran | Political Books  | < 0.3, 0.2, 0.5 >        | Alex  | Medium       | < 0.5, 0.1, 0.4 >             | 2016 | 28000   | Low        | < 0.8, 0.1, 0.1 >           |
| Scientific Miracles in the Holy Quran | Political Books  | < 0.3, 0.2, 0.5 >        | Alex  | Small        | < 0.3, 0.2, 0.5 >             | 2016 | 28000   | High       | < 0.1, 0.2, 0.7 >           |
| Scientific Miracles in the Holy Quran | Political Books  | < 0.3, 0.2, 0.5 >        | Alex  | Small        | < 0.3, 0.2, 0.5 >             | 2016 | 28000   | Middle     | < 0.4, 0.3, 0.3 >           |
| Scientific Miracles in the Holy Quran | Political Books  | < 0.3, 0.2, 0.5 >        | Alex  | Small        | < 0.3, 0.2, 0.5 >             | 2016 | 28000   | Low        | < 0.8, 0.1, 0.1 >           |

Figure 23: Roll-up Operation in Neutrosophic Cube

| Year | Cairo  | Alex   | Portsaid   |
|------|--|--|--|
| 2015 | 60500  | 31000  | 19000  |
| ND   | Nhigh < 0.8, 0.1, 0.1 ><br>Nmiddle < 0.2, 0.2, 0.6 ><br>Nlow < 0, 0.3, 0.7 > | Nhigh < 0.7, 0.1, 0.2 ><br>Nmiddle < 0.5, 0.1, 0.4 ><br>Nlow < 0, 0.3, 0.7 >   | Nhigh < 0.4, 0.2, 0.4 ><br>Nmiddle < 0.3, 0.3, 0.4 ><br>Nlow < 0.7, 0.1, 0.2 > |
| 2016 | 40000  | 10000  | 8000   |
| ND   | Nhigh < 0.3, 0.1, 0.6 ><br>Nmiddle < 0.4, 0.3, 0.3 ><br>Nlow < 0, 0.2, 0.8 > | Nhigh < 0.3, 0.2, 0.5 ><br>Nmiddle < 0.3, 0.1, 0.6 ><br>Nlow < 0.8, 0.1, 0.1 > | Nhigh < 0.1, 0.1, 0.8 ><br>Nmiddle < 0.2, 0.2, 0.6 ><br>Nlow < 0.8, 0.1, 0.1 > |

Figure 24: Roll-up Operation from month to year

If N contains a class neutrosophic target element, the third step is apply class neutrosophic target element to above steps in binary operation ,the roll up operation is a trinary operation in neutrosophic concept:

- 1) roll-up of the crisp cube
- 2) apply the neutrosophic concept on the new dimensional level.
- 3) select the class neutrosophic target element.

roll up ( $\langle \text{time.month} \rangle$ ,  $\langle \text{time.month.revenue} \rangle$ , revenue, S  $\rangle$ , time.year, time.year.revenue.middle,  $f_{\text{time.year}}$ , Revenue) = ( $\langle \text{time.year} \rangle$ ,  $\langle \text{time.year.revenue} \rangle$ , revenue , NS  $\rangle$

The following figure 13 , 14 shows the roll-up operation in neutrosophic cube with neutrosophic target element (middle revenue):

| Book Name                             | NC.Book          | Neutrosophic Degree Book        | City  | NC.City Area | Neutrosophic Degree City Area   | year | Revenue | NC.Revenue | Neutrosophic Degree Revenue     |
|---------------------------------------|------------------|---------------------------------|-------|--------------|---------------------------------|------|---------|------------|---------------------------------|
| Scientific Miracles in the Holy Quran | Scientific Books | $\langle 0.8, 0.1, 0.1 \rangle$ | Cairo | Big          | $\langle 0.9, 0.1, 0 \rangle$   | 2015 | 60500   | Middle     | $\langle 0.2, 0.2, 0.6 \rangle$ |
| Scientific Miracles in the Holy Quran | Scientific Books | $\langle 0.8, 0.1, 0.1 \rangle$ | Cairo | Medium       | $\langle 0.1, 0.2, 0.7 \rangle$ | 2015 | 60500   | Middle     | $\langle 0.6, 0.1, 0.3 \rangle$ |
| Scientific Miracles in the Holy Quran | Scientific Books | $\langle 0.3, 0.2, 0.5 \rangle$ | Cairo | Small        | $\langle 0, 0.2, 0.8 \rangle$   | 2015 | 60500   | Middle     | $\langle 0.3, 0.2, 0.5 \rangle$ |
| Scientific Miracles in the Holy Quran | Scientific Books | $\langle 0.8, 0.1, 0.1 \rangle$ | Cairo | Big          | $\langle 0.9, 0.1, 0 \rangle$   | 2016 | 40000   | Middle     | $\langle 0.4, 0.3, 0.3 \rangle$ |
| Scientific Miracles in the Holy Quran | Scientific Books | $\langle 0.8, 0.1, 0.1 \rangle$ | Cairo | Medium       | $\langle 0.1, 0.2, 0.7 \rangle$ | 2016 | 40000   | Middle     | $\langle 0.4, 0.3, 0.3 \rangle$ |
| Scientific Miracles in the Holy Quran | Scientific Books | $\langle 0.3, 0.2, 0.5 \rangle$ | Cairo | Small        | $\langle 0, 0.2, 0.8 \rangle$   | 2016 | 40000   | Middle     | $\langle 0.4, 0.3, 0.3 \rangle$ |

Figure 25: Roll-up Operation with Class Neutrosophic Target Element

| Year | Cairo                                   | Alex                                    | Portsaid                                |
|------|---|---|---|
| 2015 | 60500                                   | 31000                                   | 19000                                   |
| ND   | Nmiddle $\langle 0.2, 0.2, 0.6 \rangle$ | Nmiddle $\langle 0.5, 0.1, 0.4 \rangle$ | Nmiddle $\langle 0.3, 0.3, 0.4 \rangle$ |
| 2016 | 40000                                   | 10000                                   | 8000                                    |
| ND   | Nmiddle $\langle 0.4, 0.3, 0.3 \rangle$ | Nmiddle $\langle 0.3, 0.1, 0.6 \rangle$ | Nmiddle $\langle 0.2, 0.2, 0.6 \rangle$ |

Figure 26: Roll-up Operation with Class Neutrosophic Target Element

The original cube contains the aggregated revenue of month in all cities ordered by the neutrosophic concept revenue. The cube resulting from the roll-up operation contains the aggregated revenue of year in all cities ordered by the neutrosophic concept revenue

**8.3 Drill-down in Neutrosophic Data Warehouse:**

A drill-down operation is the opposite operation of a roll-up. It is not a valid operation if the roll-up operation is not defined in an earlier step. Therefore, a drill-down operation in the neutrosophic data warehouse can be defined as given in § 6.3.1.

**6.3.1 Definition (Drill-down Involving Neutrosophic Concepts (DRNC)):**

DRNC = drill down (RNC, DL, NL,  $f^1_D$ ,  $f^1_m$ ,  $f^1_N$ ); where RNC is ROLL UP (NC, DH, NH,  $f_d$ ,  $f_m$ ,  $f_N$ ), DL is the dimension of level lower ,NL is the neutrosophic concept of lower level,  $f^1_D$  is the inverse dimension merge function of  $f_D$ ,  $f^1_m$  is the inverse measure aggregation function of  $f_m$  and  $f^1_N$  is the inverse of  $f_N$ , DRNC = result cube on the lower level after applying the drill down operation .The following figure 15 shows that the function and inverse function on dimension:



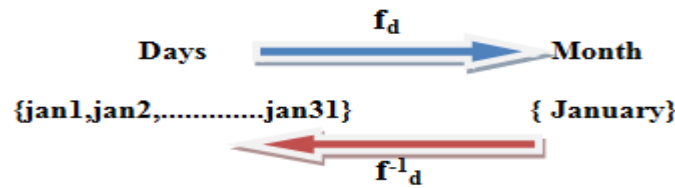


Figure 27.: Function merge on Dimension

The following example shows a drill-down operation with a neutrosophic cube:

DRNC = drill down ( $\langle \langle \text{time.year, region.city} \rangle, \langle \text{time.year.revenue, region.city.area} \rangle, \text{revenue}, S \rangle, \langle \text{time.month, region.store} \rangle, \langle \text{time.month.revenue, region.store.area} \rangle, f^1_d, f^1_m$ ); where  $DL = \langle \text{time.month, region.store} \rangle$ ,  $NL = \langle \text{time.month.revenue, region.store.area} \rangle$ ,  $f^1_d$  is the inverse function from year to month and from city to store and,  $f^1_m$  is the aggregate revenue per month and RNC = rollup ( $\langle \text{time.month, region.store} \rangle, \langle \text{time.month.revenue, region.store.area} \rangle, \text{revenue}, S \rangle, \langle \text{time.year.revenue, Region.city.area} \rangle, f_D, \text{Revenue}$ )= ( $\langle \text{time.year, region.city} \rangle, \langle \text{time.year.revenue, region.city.area} \rangle, \text{revenue}, NS \rangle$ ). The following figure16 shows the drill down operation in neutrosophic cube:

| Year | Month               | Cairo   |                     |                     | Alex                |                     | Portsaid            |
|------|---------------------|---|---------------------|---------------------|---------------------|---------------------|---------------------|
|      |                     | A   | B                   | C                   | D                   | E                   | F                   |
| 2015 | January             | 3200  | 4100                | 3900                | 3800                | 2300                | 3600                |
|      | ND                  | Nhigh < 0.7, 0.1, 0.2><br>Nmiddle < 0.5, 0.2, 0.3><br>Nlow < 0.2, 0.2, 0.6> | Neutrosophic Degree | Neutrosophic Degree | Neutrosophic Degree | Neutrosophic Degree | Neutrosophic Degree |
|      | February            | 4200  | 3800                | 3200                | 4100                | 3200                | 4200                |
|      | ND                  | Neutrosophic Degree   | Neutrosophic Degree | Neutrosophic Degree | Neutrosophic Degree | Neutrosophic Degree | Neutrosophic Degree |
|      | .....               | 9900  | 8500                | 8700                | 5300                | 6200                | 7100                |
|      | ND                  | Neutrosophic Degree   | Neutrosophic Degree | Neutrosophic Degree | Neutrosophic Degree | Neutrosophic Degree | Neutrosophic Degree |
|      | December            | 3600  | 3800                | 3600                | 3700                | 2400                | 4100                |
| ND   | Neutrosophic Degree | Neutrosophic Degree   | Neutrosophic Degree | Neutrosophic Degree | Neutrosophic Degree | Neutrosophic Degree |                     |
| 2016 | January             | 3200  | 3700                | 2900                | 1100                | 900                 | 2100                |
|      | ND                  | Neutrosophic Degree   | Neutrosophic Degree | Neutrosophic Degree | Neutrosophic Degree | Neutrosophic Degree | Neutrosophic Degree |
|      | February            | 3000  | 2500                | 2100                | 1200                | 850                 | 1500                |
|      | ND                  | Neutrosophic Degree   | Neutrosophic Degree | Neutrosophic Degree | Neutrosophic Degree | Neutrosophic Degree | Neutrosophic Degree |
|      | .....               | 6700  | 4600                | 3800                | 2000                | 1600                | 3100                |
|      | ND                  | Neutrosophic Degree   | Neutrosophic Degree | Neutrosophic Degree | Neutrosophic Degree | Neutrosophic Degree | Neutrosophic Degree |
|      | December            | 2300  | 2700                | 2500                | 1200                | 1150                | 1300                |
| ND   | Neutrosophic Degree | Neutrosophic Degree   | Neutrosophic Degree | Neutrosophic Degree | Neutrosophic Degree | Neutrosophic Degree |                     |

Figure 28: Drill-down Operation in Neutrosophic Cube

The cube resulting from the drill down operation contains the aggregated revenue per month in all stores ordered by the neutrosophic concept revenue.

**9.4 Slice in Neutrosophic Data Warehouse:**

The classical slice operation extracts a subset of values of a neutrosophic cube depend on select one dimension. A definition of a slice operation in the neutrosophic data warehouse is as follows:

**9.4.1 Definition (Slice in Neutrosophic Concepts(SNC)):**

SNC= slice (NC, s) where NC is neutrosophic cube and s is the element instance that slices cube NC

A slice operation of a neutrosophic cube will always result in a neutrosophic cube. A slice operation on a crisp cube will also always result in a crisp cube. In order to illustrate, the following operation slices a neutrosophic cube according its class neutrosophic target element:

slice( $\langle \langle \text{time.year} \rangle, \langle \text{time.year.revenue} \rangle, \text{revenue}, R \rangle, \text{time.year.year} = "2015" ) = \langle \langle \text{time.year} \rangle, \langle \text{time.year.year} \rangle, \text{revenue}, S \rangle$ .The following figure 17 show the slice operation in neutrosophic cube:

| Book Name                             | NC.Book          | NC.Books            | City  | NC.City Area | osophic Degree City | year | Revenue | NC.Revenue | Neutrosophic Degree Revenue |
|---------------------------------------|------------------|---------------------|-------|--------------|---------------------|------|---------|------------|-----------------------------|
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8 , 0.1 , 0.1 > | Cairo | Big          | < 0.9 , 0.1 , 0 >   | 2015 | 60500   | High       | < 0.8 , 0.1 , 0.1 >         |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8 , 0.1 , 0.1 > | Cairo | Big          | < 0.9 , 0.1 , 0 >   | 2015 | 60500   | Middle     | < 0.2 , 0.2 , 0.6 >         |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8 , 0.1 , 0.1 > | Cairo | Big          | < 0.9 , 0.1 , 0 >   | 2015 | 60500   | Low        | < 0 , 0.3 , 0.7 >           |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8 , 0.1 , 0.1 > | Cairo | Medium       | < 0.1 , 0.2 , 0.7 > | 2015 | 60500   | High       | < 0.8 , 0.1 , 0.1 >         |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8 , 0.1 , 0.1 > | Cairo | Medium       | < 0.1 , 0.2 , 0.7 > | 2015 | 60500   | Middle     | < 0.2 , 0.2 , 0.6 >         |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8 , 0.1 , 0.1 > | Cairo | Medium       | < 0.1 , 0.2 , 0.7 > | 2015 | 60500   | Low        | < 0 , 0.3 , 0.7 >           |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8 , 0.1 , 0.1 > | Cairo | Small        | < 0 , 0.2 , 0.8 >   | 2015 | 60500   | High       | < 0.8 , 0.1 , 0.1 >         |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8 , 0.1 , 0.1 > | Cairo | Small        | < 0 , 0.2 , 0.8 >   | 2015 | 60500   | Middle     | < 0.2 , 0.2 , 0.6 >         |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8 , 0.1 , 0.1 > | Cairo | Small        | < 0 , 0.2 , 0.8 >   | 2015 | 60500   | Low        | < 0 , 0.3 , 0.7 >           |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8 , 0.1 , 0.1 > | Alex  | Big          | < 0.7 , 0.2 , 0.1 > | 2015 | 50800   | High       | < 0.7 , 0.1 , 0.2 >         |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8 , 0.1 , 0.1 > | Alex  | Big          | < 0.7 , 0.2 , 0.1 > | 2015 | 50800   | Middle     | < 0.6 , 0.2 , 0.2 >         |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8 , 0.1 , 0.1 > | Alex  | Big          | < 0.7 , 0.2 , 0.1 > | 2015 | 50800   | Low        | < 0.1 , 0.2 , 0.7 >         |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8 , 0.1 , 0.1 > | Alex  | Medium       | < 0.5 , 0.1 , 0.4 > | 2015 | 50800   | High       | < 0.7 , 0.1 , 0.2 >         |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8 , 0.1 , 0.1 > | Alex  | Medium       | < 0.5 , 0.1 , 0.4 > | 2015 | 50800   | Middle     | < 0.6 , 0.2 , 0.2 >         |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8 , 0.1 , 0.1 > | Alex  | Medium       | < 0.5 , 0.1 , 0.4 > | 2015 | 50800   | Low        | < 0.1 , 0.2 , 0.7 >         |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8 , 0.1 , 0.1 > | Alex  | Small        | < 0.3 , 0.2 , 0.5 > | 2015 | 50800   | High       | < 0.7 , 0.1 , 0.2 >         |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8 , 0.1 , 0.1 > | Alex  | Small        | < 0.3 , 0.2 , 0.5 > | 2015 | 50800   | Middle     | < 0.6 , 0.2 , 0.2 >         |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8 , 0.1 , 0.1 > | Alex  | Small        | < 0.3 , 0.2 , 0.5 > | 2015 | 50800   | Low        | < 0.1 , 0.2 , 0.7 >         |

Figure 29: Slice Operation in Neutrosophic Cube

In the resulting cube contain all revenue in city for year 2015 and the year 2016 and others not found

**9.5 Dice Operation in Neutrosophic Data Warehouse:**

A dice operation in a neutrosophic can restrict neutrosophic concepts and dimensions. Furthermore, The slice operation can be extended to a dice definition by including logical operators to combine multiple slicers.

**9.5.1 Definition (Dice involving neutrosophic fuzzy concepts (DNC)):**

DNC = Dice (NC, {s<sub>m</sub>,....., s<sub>k</sub>}, {f<sub>m</sub>,....., f<sub>k-1</sub>}); where NC is a neutrosophic cubes, s<sub>m</sub>,....., s<sub>k</sub> ∈ dom (D), f<sub>m</sub>,....., f<sub>k-1</sub> ∈ {AND, OR,NOT} are the logical operators that combine the slicers in a way that combines s<sub>m</sub> with s<sub>m+1</sub>. The dice operation works similarly and performs a selection on two or more dimensions. For example: A dice operation on a neutrosophic cube with two neutrosophic concepts is illustrated as follows: dice (<< time.year , region.city > , < time.year.revenue, region.city.area > , revenue , S > , {time.year.year = "2015", region.city.area = "big"}, {AND} ). The following figure 18 shows that the dice operation in neutrosophic cube.

| Book Name                             | NC.Book          | NC.Books            | City  | NC.City Area | osophic Degree City | year | Revenue | NC.Revenue | Neutrosophic Degree Revenue |
|---------------------------------------|------------------|---------------------|-------|--------------|---------------------|------|---------|------------|-----------------------------|
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8 , 0.1 , 0.1 > | Cairo | Big          | < 0.9 , 0.1 , 0 >   | 2015 | 60500   | High       | < 0.8 , 0.1 , 0.1 >         |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8 , 0.1 , 0.1 > | Cairo | Big          | < 0.9 , 0.1 , 0 >   | 2015 | 60500   | Middle     | < 0.2 , 0.2 , 0.6 >         |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8 , 0.1 , 0.1 > | Cairo | Big          | < 0.9 , 0.1 , 0 >   | 2015 | 60500   | Low        | < 0 , 0.3 , 0.7 >           |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8 , 0.1 , 0.1 > | Alex  | Big          | < 0.7 , 0.2 , 0.1 > | 2015 | 50800   | High       | < 0.7 , 0.1 , 0.2 >         |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8 , 0.1 , 0.1 > | Alex  | Big          | < 0.7 , 0.2 , 0.1 > | 2015 | 50800   | Middle     | < 0.6 , 0.2 , 0.2 >         |
| Scientific Miracles in the Holy Quran | Scientific Books | < 0.8 , 0.1 , 0.1 > | Alex  | Big          | < 0.7 , 0.2 , 0.1 > | 2015 | 50800   | Low        | < 0.1 , 0.2 , 0.7 >         |

Figure 30: Dice Operation in Neutrosophic Cube

The resulting cube shows only the revenue of the year2015” that belong to city class "big"

**Conclusion**

Using a neutrosophic approach in data warehouse concepts improves information quality for the business process. this approach include neutrosophic concept into structure of dimensions or into fact tables of the data warehouse model, then we construct truth degree, falsity degree and indeterminacy degree which close to natural language. Added , We have presented a structure that manages imprecision by means of neutrosophic logic. Most of the methods previously documented give a neutrosophic set as a result. we have presented an OLAP system that implements a neutrosophic multidimensional model to achieve knowledge discovery from imperfect data and to enhance the system performance.

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