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Numerical investigation of natural convection phenomena in a uniformly heated circular cylinder immersed in square enclosure filled with air at different vertical locations☆

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ABSTRACT

In this model, a numerical study of two dimensional steady natural convection is performed for a uniform heat source applied on the inner circular cylinder in a square air ($Pr = 0.7$) filled enclosure in which all boundaries are assumed to be isothermal (at a constant low temperature). The developed mathematical model is governed by the coupled equations of continuity, momentum and energy and is solved by finite volume method. The effects of vertical cylinder locations and Rayleigh numbers on fluid flow and heat transfer performance are investigated. Rayleigh number is varied from 10^3 to 10^6 and the location of the inner cylinder is changed vertically along the centerline of the enclosure from −0.25 L to 0.25 L upward and downward, respectively. It is found that at small Rayleigh numbers does not have much influence on the flow field while at high Rayleigh numbers have considerable effect on the flow pattern. In addition, the numerical solutions yield a two cellular flow field between the inner cylinder and the enclosure. Also, the total average Nusselt number behaves nonlinearly as a function of locations. Results are presented in terms of the streamlines, isotherms, local and average Nusselt numbers. Detailed results of the numerical has been compared with literature ones, and it gives a reliable agreement.

1. Introduction

Natural convection in enclosures is encountered in many engineering systems such as convection in buildings, fluid movement in solar energy collectors, cooling of electronic circuits, and cooling of nuclear reactors, etc. The main advantage of natural convection is the reliability, because the air movement is simply generated by local density gradients in the presence of the gravitational field, without the need for prime movers as pumps or fans. Buoyancy driven flow and heat transfer between a cylinder and its surrounding medium has been a problem of considerable importance. This problem has a wide range of applications. Energy storage devices, crude oil storage tanks, heat exchangers, spent fuel storage of nuclear power plants are a few to name. Considerable studies have been devoted to the problems of the aspect. Karim et al. [1] have experimentally demonstrated the influence of horizontal confinement on heat transfer around a cylinder for Rayleigh numbers ranging from 10^3 to 10^5. These authors found that the heat flux around the cylinder increases with decreasing distance between the cylinder and the enclosure wall. Ghaddar [2] studied natural convection from a uniformly heated horizontal cylinder placed in a rectangular enclosure filled with air. He found a relationship of the average Nusselt number as a function of Rayleigh number. Fu et al. [3] used a finite element method to investigate enhancement of natural convection of an enclosure by a rotating circular cylinder near a hot wall. They concluded that the direction of the rotating cylinder played a significant role in enhancing natural convection heat transfer in the enclosure.

The laminar convection flow around a heated horizontal square cylinder rotating slowly within a concentric circular enclosure has been numerically examined by Yang and Farouk [4]. Moukalled and Acharya [5] studied the thermal and flow fields between the low temperature outer square enclosure and high temperature inner circular cylinder. The results showed that when the Rayleigh number was constant, the convection contribution to the total heat transfer decreases with an increasing aspect ratio value. Lacroix and Joyeux [6] conducted a numerical study of natural convection heat transfer from two horizontal heated cylinders confined to a rectangular enclosure having finite wall conductance. They indicated that wall heat conduction reduced the average temperature differences across the cavity, partially stabilized the flow and decreased natural convection heat transfer around the cylinders. Nguyen et al. [7] investigated numerically the heat transfer from a rotating circular cylinder immersed in a spatially uniform, time-dependent convective environment including the effects of buoyancy forces. The results showed that vortex shedding was promoted by the cylinder rotation but it was

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Nomenclature

g	Gravitational acceleration, (m/s^2)
Gr	Grashof number
k	Thermal conductivity of fluid, (W/m °C)
L	Length or width of the square enclosure, (m)
n	Normal direction to the wall
$\overline{Nu}$	Average Nusselt number
P	Dimensionless pressure
p	Pressure, (N/m^2)
Pr	Prandtl number
q	Uniform heat flux per unit area, (W/m^2)
r	Radius of circular cylinder, (m)
Ra	Rayleigh number
s	Distance along the square enclosure,(m)
T	Temperature, (°C)
T_c	Temperature of the cold surface, (°C)
T_h	Temperature of the hot surface, (°C)
U	Dimensionless velocity component in x-direction
u	Velocity component in x-direction, (m/s)
V	Dimensionless velocity component in y-direction
v	Velocity component in y-direction, (m/s)
W	Surface area of wall, (m^2)
X	Dimensionless coordinate in horizontal direction
x	Cartesian coordinate in horizontal direction, (m)
Y	Dimensionless coordinate in vertical direction
y	Cartesian coordinate in vertical direction, (m)

Greek symbols

α	Thermal diffusivity, (m^2/s)
β	Volumetric coefficient of thermal expansion, (K^{-1})
θ	Dimensionless temperature
ν	Kinematic viscosity of the fluid, (m^2/s)
ρ	Density of the fluid, (kg/m^3)
δ	distance from center of square cylinder to circular cylinder center, (m)

vanished because of buoyancy forces. Koizumi and Hosokawa [8] investigated a natural convection flow around a cylinder, vertically confined by a ceiling, and reported on chaotic and oscillatory movements of the air above the cylinder. The results gave a classification of the nature of the flow above the cylinder, the classification depending on the Rayleigh number and on the distance separating the ceiling from the cylinder. Yoo [9] studied numerically the mixed convection in a horizontal concentric annulus with Prandtl number equal to 0.7. The inner cylinder was hotter than the outer cylinder. He concluded that the overall heat transfer at the wall was rapidly decreased as Re approached the transitional value between two- and one-eddy flows. Sasaguchi et al. [10] reported the numerical results of the effect of the position of a cooled cylinder in a rectangular cavity on the cooling process of water around the cylinder. The initial water temperature is varied at 4, 6, 8 and 12 °C, while the temperature of the cylinder surface is fixed at 0 °C. They observed that the changes in the position of the cylinder and the initial water temperature largely affect fluid flows. Also, they found that the average Nusselt number over the cylinder surface and averaged water temperature vary in a complicated manner with time for initial temperature greater than 4 °C. The cooling rate of water is largely affected by the change of the position of the cylinder. Cesini et al. [11] analysed the convective heat transfer generated by a horizontal cylinder located in a rectangular cavity. This study was carried out through both experimental analysis and a numerical study. The results indicated that the average heat transfer coefficients increases with increasing Rayleigh number. Asan [12] analysed the convection in an annulus between two isothermal concentric square ducts. He showed that dimension ratio and Rayleigh number have a clear effect on the thermal and flow fields. Shu et al. [13] investigated computationally the natural convection between an outer square enclosure and inner circular cylinder. The results showed that the circulation, flow separation and the top space between the square outer enclosure and circular inner cylinder have an important influence on the flow and thermal fields. Ha et al. [14] considered natural convection problem in a horizontal layer of fluid with a periodic array of square cylinder in the interior, in which they concluded that the transition of the flow from quasi-steady up to unsteady convection depends on the presence of bodies and aspect ratio effect of the cell. Atmane et al. [15] investigated the effect of vertical confinement on the natural convection flow and heat transfer around a horizontal heated cylinder. The flow characteristics and the time-resolved heat transfer have been measured respectively above and around the cylinder. The relationship between the flow pattern and the heat transfer characteristics at the cylinder surface was studied and the origin of the oscillation discussed. Misirlioglu [16] numerically investigated the heat transfer in a square cavity filled with clear fluid or porous medium. To change the heat transfer in the cavity a rotating circular cylinder was placed at the centre of the cavity. The results showed that rotation was more effective in the forced convection regime than in mixed and natural convection regimes, and at high spin velocities the heat transfer was almost independent on the Darcy number. Laskowski et al. [17] studied both experimentally and numerically heat transfer to and from a circular cylinder in a cross-flow of water at low Reynolds number. The results explained that, when the lower surface was unheated, the temperatures of the lower surface and water upstream of the cylinder were maintained approximately equal and the flow was laminar. Ben-Nakhi and Chamkha [18] performed a numerical study of steady, laminar, conjugate natural convection around a finned pipe placed in the center of a square enclosure with uniform internal heat generation. The results for the local and average Nusselt numbers are presented and discussed for various parametric conditions. Saha et al. [19], studied numerically natural convection in two-dimensional square enclosure containing adiabatic cylinder centered within using finite element method. The Grashof number was varied from 10^3 to 10^6 and Prandtl number is taken as 0.71. The effect of heat source length on the fluid flow and heat transfer process in the enclosure are analyzed. Results are presented in the form of streamline and isotherm plots. Kim et al. [20] numerically investigated a two-dimensional natural convection problem in a cooled square enclosure with an inner heated circular cylinder. A detailed analysis for the distribution of streamlines, isotherms and Nusselt number was carried out to investigate the effect of the vertical locations of the heated inner cylinder on the fluid flow and heat transfer in the cooled square enclosure for different Rayleigh numbers in the range of $10^3 \leq Ra \leq 10^6$. They found that the number, size and formation of the cell strongly depend on the Rayleigh number and the position of the inner circular cylinder. Xu et al. [21] made a numerical simulation to investigate the steady laminar natural convective heat transfer for air within the horizontal annulus between a heated triangular cylinder and its circular cylindrical enclosure. The results showed that at constant radius ratio, inclination angles of the inner triangular cylinder are found to have negligible effects on the average Nusselt number. Shih et al. [22], studied the periodic laminar flow and heat transfer due to an insulated or various isothermal rotating objects (circle, square, and equilateral triangle) placed in the center of the square cavity. Transient variations of the average Nusselt number of the respective systems show that for high Re numbers, a quasi-periodic behavior while for low Re numbers, periodicity of the system is clearly observed. Very recently, Costa and Raimundo [23] studied the problem of mixed convection in a square enclosure with a rotating cylinder centered within. Results clearly show how the rotating cylinder affects the thermal performance of the enclosure, and how the thermo-physical properties of the cylinder are important in the overall heat transfer process across the enclosure. They have also concluded that, for small rotating velocities, the highest Nusselt numbers are obtained for the smallest values of the thermal conductivity and thermal capacity of the cylinder.

From the above survey, it can be seen numerous researches have studied the natural convection heat transfer and flow in an enclosure with internal circular cylinder which is either rotating or stationary subjecting to various boundary condition (insulated, conductive, isothermal or isoflux) but there is not special research that focuses on the effect of vertical locations of the isoflux circular solid cylinder enclosed in a square enclosure on the natural convection heat transfer and flow. Therefore, the present work is based on the configuration of the previous work by Kim et al. [20] but in their work attention has been considered to investigate the effect of inner heated cylinder (T_h) with vertical location placed inside a cooled square enclosure on the heat transfer and fluid flow. The present work deals with same problem but in this situation, a uniform heat flux per unit area (q) enters through the inner circular cylinder instead of kept it at a constant high temperature of T_h. Numerical solutions are obtained over a wide range of Rayleigh numbers and various vertical locations. The finite volume technique is used to solve the Navier–Stokes and the energy equations.

2. Problem definition and governing equations

Fig. 1 shows the geometry of the present work. The problem deals with a circular cylinder with a radius R (=0.2 L) located inside a square enclosure with sides of length L and moves along the vertical centerline of it in the domain from −0.25 L to 0.25 L. The walls of the square enclosure were kept at a constant low temperature of T_c, and a uniform heat flux per unit area (q) enters through the inner circular cylinder. In the present work the Prandtl number, Pr, and R have been taken to be 0.7 and 0.2, respectively. The Rayleigh number, Ra, varies in the range of 10^3–10^6. The dimensionless horizontal distance δ, which represents the position of the inner cylinder along the vertical centerline of the square enclosure, varies in the range of −0.25 L to 0.25 L. The fluid properties are also assumed constant, and the Boussinesq approximation is applied to model the buoyancy effect. The acceleration due to gravity acts in the negative y-direction. The fluid is assumed Newtonian while viscous dissipation effects are negligible. The governing equations inside the square enclosure with a heated circular cylinder are described by the Navier–Stokes and the energy equations, respectively. The governing equations are transformed into dimensionless forms under the following non-dimensional variables [19]:

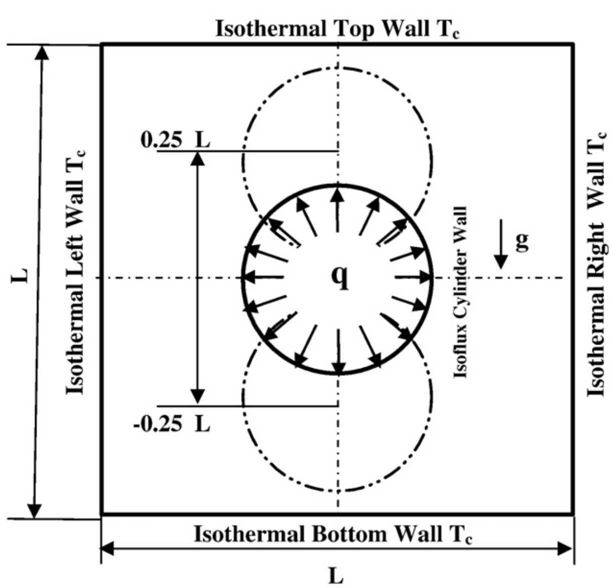

Fig. 1. Schematic diagram of the square enclosure with inner circular cylinder with coordinate system along with boundary conditions.

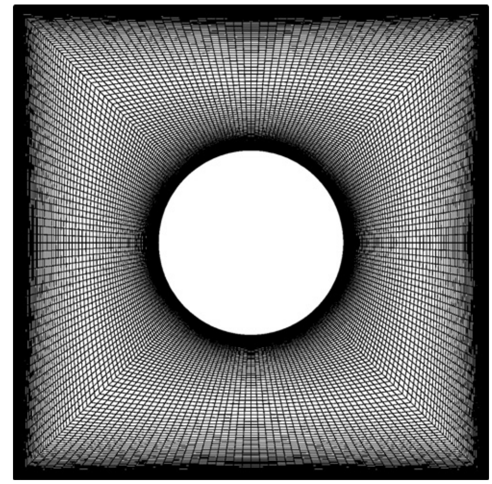

Fig. 2. A typical grid distribution (200×100) with non-uniform and non-orthogonal distributions for $\delta = 0$.

$$\theta = \frac{T - T_c}{q\frac{L}{k}}, X = \frac{x}{L}, Y = \frac{y}{L}, U = \frac{uL}{\alpha},$$
$$V = \frac{vL}{\alpha}, P = \frac{pL^2}{\rho\alpha^2}, Pr = \frac{\upsilon}{\alpha} \text{ and } \mathrm{Ra} = \frac{g\beta qL^4}{k v\alpha} \tag{1}$$

The dimensionless forms of the governing equations under steady state condition are expressed in the following forms [19]:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{2}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + Pr\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \tag{3}$$

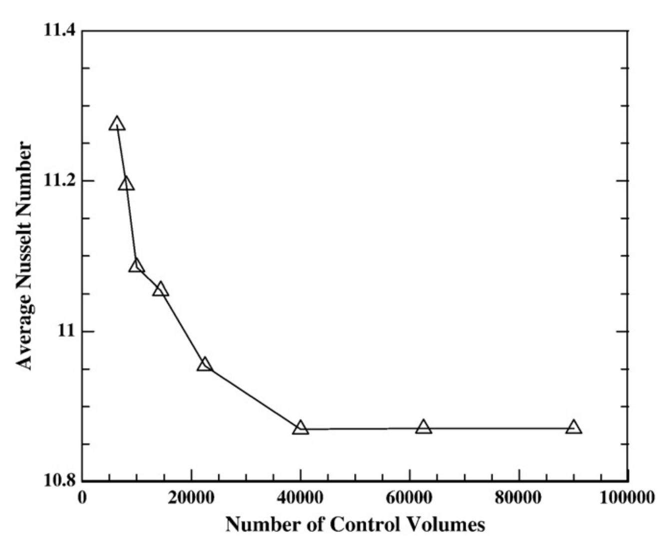

Fig. 3. Convergence of average Nusselt number along the inner hot cylinder wall with grid refinement for $\mathrm{Ra} = 10^6$, $Pr = 0.7$, and $\delta = 0$.

Table 1
Comparison of present surface-averaged Nusselt number with those of previous numerical studies.

Ra	Mean Nusselt number at the hot wall			Error (%)
	Present study	Kim et al. [20]	Moukalled and Acharya [5]	
10^4	3.4047	3.414	3.331	−2.2125
10^5	5.12893	5.1385	5.08	−0.96318
10^6	9.38875	9.39	9.374	−0.1573
10^7	15.6995	15.665	15.79	0.57314

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+Pr\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+\mathrm{Ra}Pr\theta \tag{4}$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \tag{5}$$

The rate of heat transfer is computed at each wall and is expressed in terms of local surface Nusselt number (Nu) and surface-averaged Nusselt number $(\overline{Nu})$ [8] as:

$$Nu=\frac{\partial\theta}{\partial n}|_{wall}\,,\ \overline{Nu}=\frac{1}{w}\int_0^w Nuds \tag{6}$$

Where n is the normal direction with respect to the walls, w is the surface area of walls.

3. Numerical procedure

The coupled governing Eqs. (2)–(5) are transformed into sets of algebraic equations using finite volume method. The above equations are integrated over each control volume to obtain a set of discretized linear algebraic equations of the form:

$$a_p\emptyset_p=\sum a_{nb}\emptyset_{nb}+S \tag{7}$$

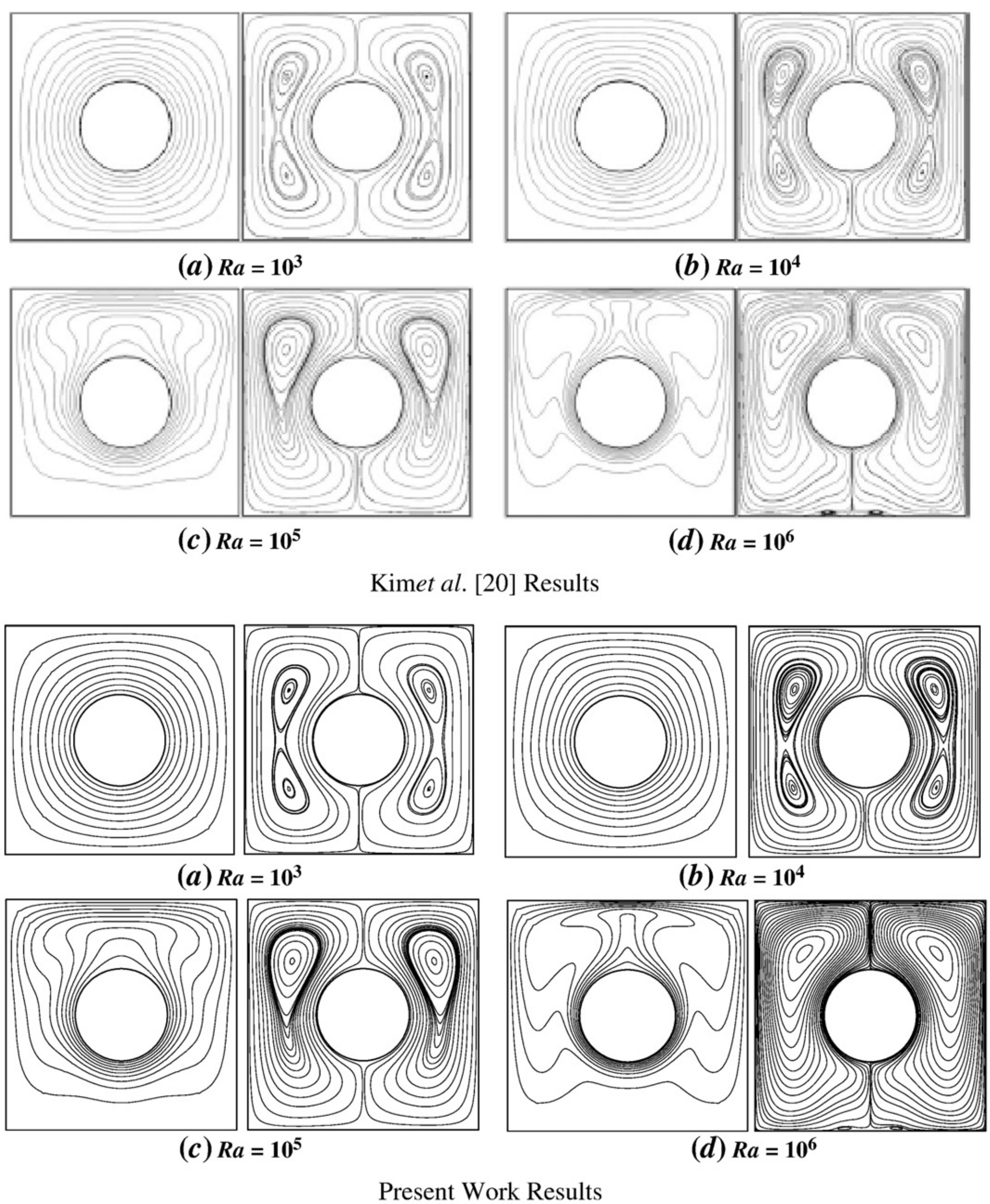

Fig. 4. Comparison of the temperature contours and streamlines between the present work and that of Kim et al. [20], at $\delta=0.0$ for four different Rayleigh numbers of (a) 10^3, (b) 10^4, (c) 10^5 and (d) 10^6.

The convective terms were approximated by the power scheme and the diffusive terms with the central differencing scheme. The SIMPLE algorithm of Patankar [24] was used to couple continuity and momentum equations. The computational procedure is based on a collocated mesh (Fig. 2), and the details can be found in the text by Ferziger and Peric [25]. The systems of algebraic equations are solved using the strongly implicit procedure (SIP) of Stone [26]. The computation is terminated when the residuals for the continuity and momentum equations get below 10^{-6} and the residual for the energy equation gets below 10^{-8}.

4. Grid refinement check

In order to obtain grid independent solution, a grid refinement study is performed for a square enclosure with isoflux inner circular cylinder at $Ra=10^6$, $Pr=0.7$ and $\delta=0$ (Fig. 3). In the present work, eight combinations (80×80, 90×90, 100×100, 120×120, 150×150, 200×200, 250×250 and 300×300) of control volumes are used to test the effect of grid size on the accuracy of the predicted results. Fig. 3 shows the convergence of the average Nusselt number $(\overline{Nu})$, at the inner hot cylinder wall with grid refinement. It is observed that grid independence is achieved with combination of (200×200) control volumes where there is insignificant change in the average Nusselt number $(\overline{Nu})$ with the improvement of finer grid. The agreement is found to be excellent which validates the present computations indirectly.

5. Numerical results verification

For the purpose of the present numerical algorithm validation, the natural convection problem for a low temperature outer square enclosure and high temperature inner circular cylinder was tested. The calculated surface-averaged Nusselt numbers for the test case are compared with the benchmark values by Kim et al. [20] and Moukalled and Acharya [5] as shown in Table 1. Further validation was performed by using the present numerical algorithm to investigate the same problem considered by Kim et al. [20] using the same flow conditions and geometries, which were reported for laminar natural convection heat transfer using the same boundary conditions but the numerical scheme is different. The comparison is made using the following dimensionless parameters: $Pr=0.7$, $Ra=10^3$–10^6 and $\delta=0$. Excellent agreement was achieved between Kim et al. [20] and the present numerical scheme for both the stream lines and temperature contours inside the square enclosure with the inner cylinder along the horizontal centerline at $\delta=0$ as shown in Fig. 4. These validations make a good confidence in the present numerical model to deal with the other vertical centerline locations (δs).

6. Results and discussion

6.1. Basic features of isotherms and streamlines

Fig. 5 explains the temperature contours and streamlines when the distance from the center of square cylinder to circular cylinder center equals zero for $Ra=10^3$, 10^4, 10^5 and 10^6, respectively. The figure shows that the hot air adjacent to the inner cylinder surface due to the heat flux source moves upward and after that it hits the isothermal cold surface of the square enclosure. After hitting the cold surface of the enclosure it changes its direction downward producing rotating symmetrical vortices. With respect to isotherms, when the Rayleigh numbers are low, the isotherms are approximately parallel and the heat is transferred due to conduction. The temperature contours and streamlines become more confused and the vortices intensity increases as Rayleigh number increases. In this case the flow strongly hits the top of the square enclosure, which leads to produce a thin thermal boundary layer in this place. The shape of the rotating vortices increase and the isotherms are move upward and the heat is transferred due to convection.

6.2. Isotherms and streamlines as a function of vertical locations (δs)

Figs. 6–9 show the streamlines and isotherms for $Ra=10^3$, 10^4, 10^5 and 10^6 respectively and ten locations of the inner cylinder which are changed vertically upward and downward along the square enclosure centerline. When Rayleigh number values are low (i.e., $Ra=10^3$ and $Ra=10^4$) as shown in Figs. 6 and 7 respectively, as the cylinder moves downward in the direction of the bottom wall (i.e., the values of δ decrease), the upper symmetrical rotating vortices are increased in size and shape, also the distance between the temperature contours becomes gradually very small. It is observed that the two minor vortices which are noticed at the lower zone of the enclosure are

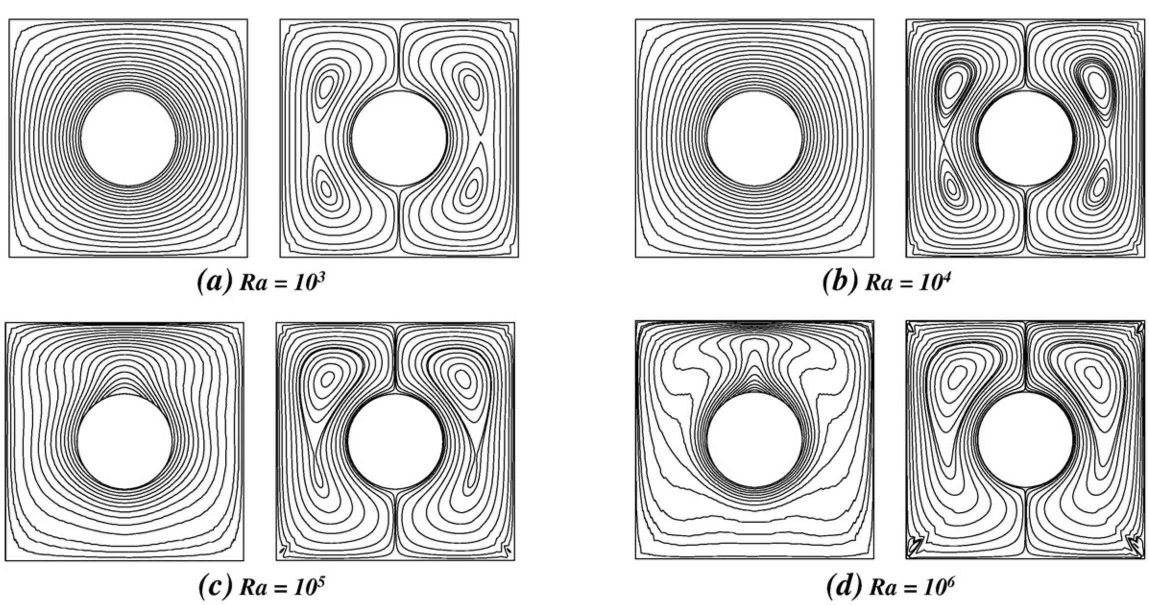

Fig. 5. The temperature contours and streamlines at $\delta=0.0$ for four different Rayleigh numbers of (a) 10^3, (b) 10^4, (c) 10^5 and (d) 10^6.

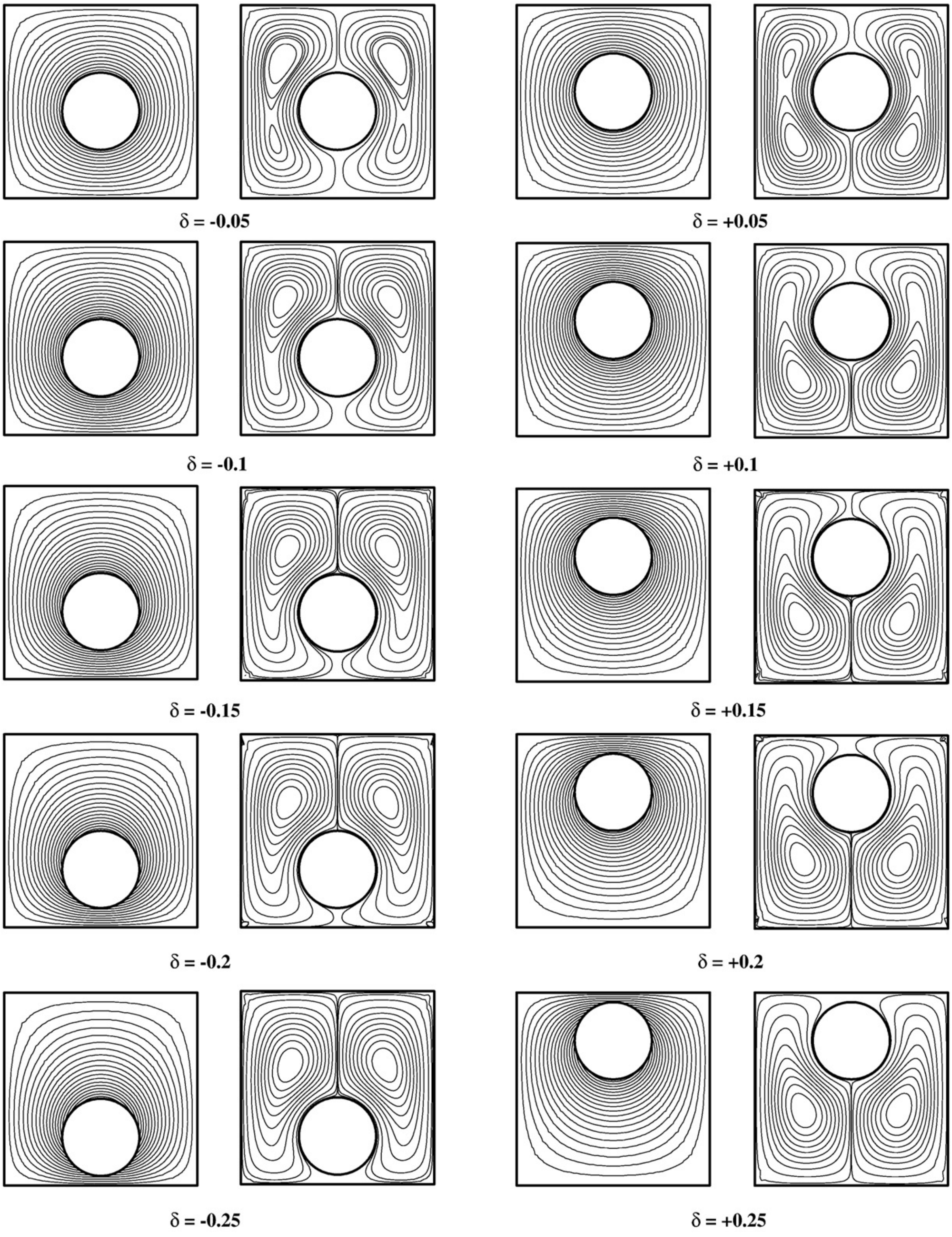

Fig. 6. Variations of isotherms and streamlines for different δs at $Ra = 10^3$.

converge from each other when values of δ decrease and meets to each other at $\delta = -0.2$ when $Ra = 10^3$.While when $Ra = 10^4$, it meets earlier at $\delta = -0.1$. A reverse behaviour can be noticed in the streamlines and isotherms when the cylinder moves upward in the direction of the upper wall (i.e., the values of δ increase) since the space between the inner cylinder and the bottom wall of the square enclosure increases in size. In this case, when the hot cylinder moves upward, the two minor vortices which are noticed at the upper zone of the enclosure are converge from each other when values of δ increase and meets to each other at $\delta = 0.15$ when $Ra = 10^3$.While when $Ra = 10^4$, it meets earlier at $\delta = 0.05$.The reason of this phenomena is due to significant effect of natural convection where the bulk of the rotating vortices are observed in the lower zone of the enclosure. When Rayleigh number values are large (i.e., $Ra = 10^5$ and $Ra = 10^6$) as shown in Figs. 8 and 9 respectively. The buoyancy force effect becomes more dominant with increasing Rayleigh number and both major and minor vortices becomes large in size and clear in comparison when $Ra = 10^3$ and $Ra = 10^4$ due to high convection

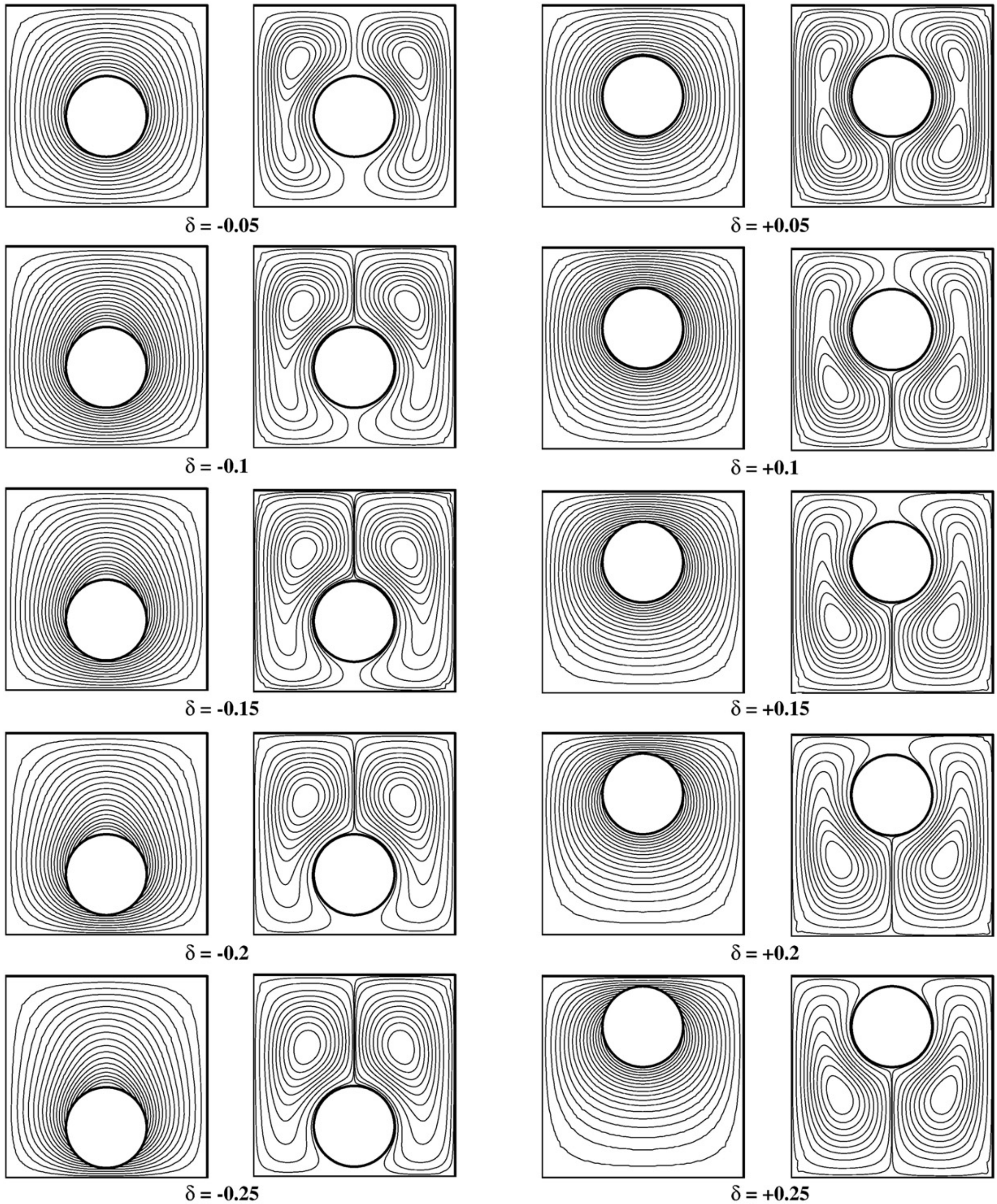

Fig. 7. Variations of isotherms and streamlines for different δs at $Ra = 10^4$.

effect. At higher Rayleigh number when the intensity of convection increases significantly, the core of rotating vortices moves up and isotherm contours change clearly indicating that the natural convection is the dominant heat transfer mechanism. As the cylinder moves upward in the direction of the upper wall (i.e., the values of δ increase) the temperature contours near the upper wall of square enclosure are accumulated, compressed and become more confused. From the other hand, the symmetrical major vortices down the bottom of the inner cylinder become large in size as the inner cylinder moves upward. Additional minor vortices on the inner cylinder surface can be observed in this case. Also, a small minor vortices can be detected near the bottom wall of the square enclosure, where the formation of this vortices are increased when the inner hot cylinder moves upward from $\delta = 0.05$ to $\delta = 0.25$.In general, the size of minor vortices which are generated near the bottom wall are small in comparison with the corresponding vortices which are observed in Kim et al. [20] work due to the heat flux effect. A reverse behaviour can be noticed in the streamlines and isotherms when the cylinder moves downward in the direction of the bottom wall (i.e., the values of δ decrease) since the space between the inner cylinder and the bottom wall of the square enclosure decrease in size. In this case, when the inner hot cylinder moves downward from $\delta = -0.05$ to $\delta = -0.25$. The heat transfer is modeled by conduction where δ decreases gradually, since the distance below the bottom of inner hot cylinder becomes near the

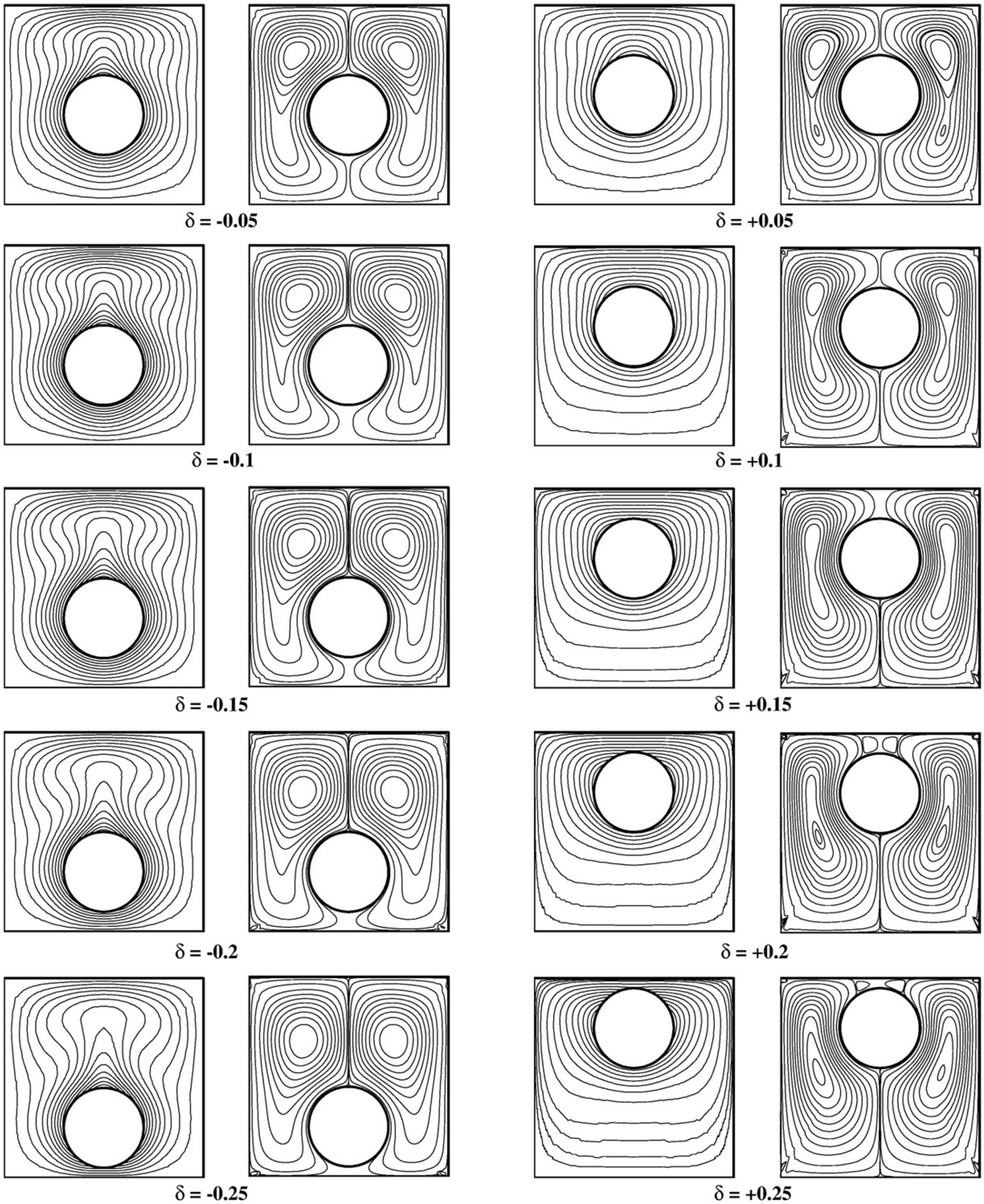

Fig. 8. Variations of isotherms and streamlines for different δs at $Ra = 10^5$.

vertical direction of flow motion. In this case, the minor vortices which are located below the hot inner cylinder are disappear gradually until it finishes at $\delta = -0.25$. At the same time, a small minor vortices can be detected near the bottom wall of the square enclosure while, the major two rotating vortices which are located above the hot inner cylinder are increased in size and take most of the enclosure region. The same phenomena can be observed when $Ra = 10^6$ where a small minor vortices can be noticed near the bottom edges of the enclosure, but the intensity of vortices become more severe due to high convection influence. In Fig. 9, as $Ra = 10^6$, the convection effect becomes significant causes the temperature contours to be sharper and more confused while the thermal boundary layer can be observed on the square enclosure and the cylinder surfaces. From the other hand, the two symmetrical vortices are constructed on the bottom surface of the inner cylinder when the inner cylinder moves upward, while the two symmetrical vortices are constructed on the upper surface of the inner cylinder when the inner cylinder moves downward. The intensity of these vortices becomes larger than the previous case. This is due to the high convection effects when the Rayleigh number increased.

6.3. The effect of local Nusselt number

The variations of the local Nusselt number with the distance along the surfaces of the square enclosure (i.e., A, B, C and D) and with

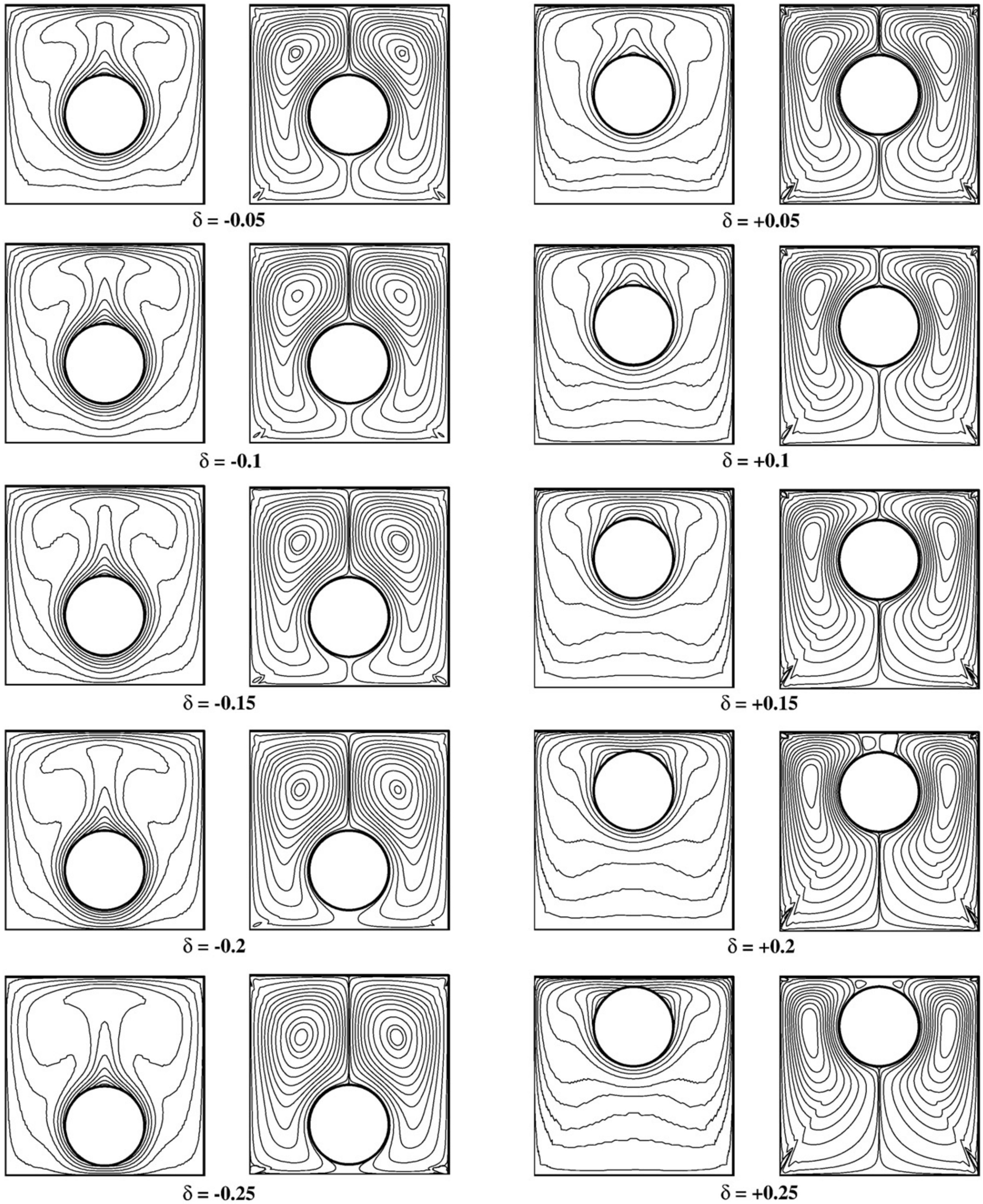

Fig. 9. Variations of isotherms and streamlines for different δs at $Ra = 10^6$.

different upward and downward locations of the inner cylinder are presented in Fig. 10. In general a uniform heat source applied on the inner circular cylinder which is enclosed in a square enclosure causes a decrease in the local Nusselt number values, comparing with Kim et al. [20] work whereas the cylinder was kept at a constant high temperature (T_h) indicating that the heat source size has a significant effect on the heat transfer rate. When Rayleigh number values are low (i.e., $Ra = 10^3$ and $Ra = 10^4$) as shown in Fig. 10a and b respectively, a similar behaviour can be observed between these figures. The reason of this behaviour is due to small effect of convection and the isotherms are approximately parallel since heat is transferred due to conduction in this range of Rayleigh number. When the inner cylinder moves upward (i.e., the values of δ increase) and downward (i.e., the values of δ decrease), the local Nusselt number starts with a maximum value due to heat source effect and drops along the surfaces of enclosure until it reaches a minimum value near point B. After this point the local Nusselt number increases gradually until it reaches a maximum value in the midway between points B and C. Then the local Nusselt number increases secondly until it reaches point D. When Rayleigh number values are high (i.e., $Ra = 10^5$ and $Ra = 10^6$) as shown in Fig. 10c and d respectively, the local Nusselt number values are increased due to significant effect of thermal convection. It begins

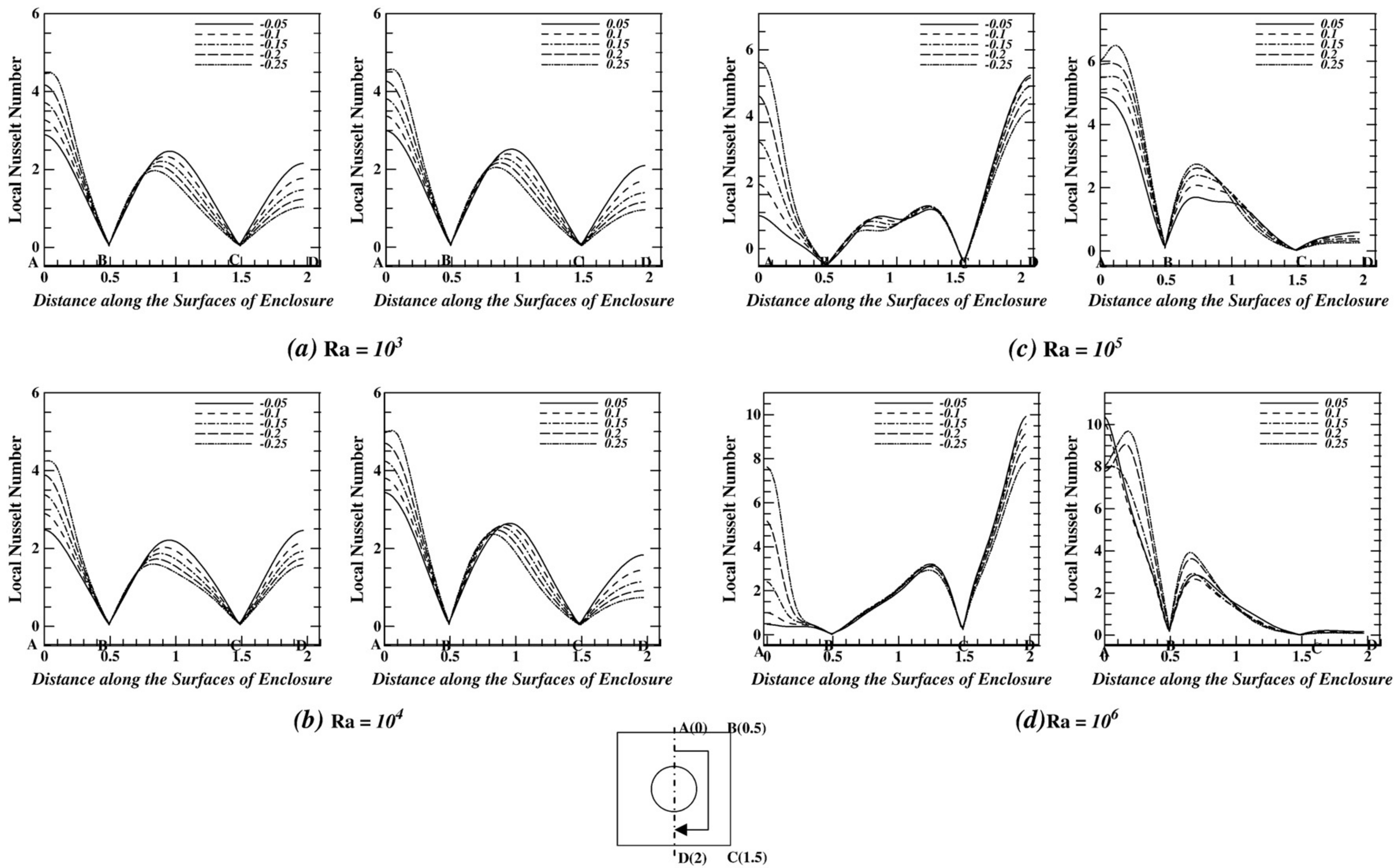

Fig. 10. Local Nusselt number distribution along the surfaces (A–B–C–D) of the square enclosure at different positions of the inner circular cylinder for different Rayleigh numbers.

from a maximum value due to heat source effect and the existence of minor vortices on the inner cylinder surface and drops along the surfaces of enclosure until it reaches a minimum value near point B. Then, the local Nusselt number increases gradually and then decreases unit it reaches point C. Then the local Nusselt number jumps immediately at the zone from point C to point D when the inner cylinder moves downward. The reason of this behaviour, because the distance between the bottom wall and the surface of the inner cylinder decreases when the inner cylinder moves downward and the temperature contours are very concentrated in this small region leading to increase the local Nusselt number values. When the inner cylinder moves upward (i.e., the values of δ increase) a similar behaviour can be observed except in the zone from point C to point D. Since in this case, the distance between the bottom wall and the surface of the inner cylinder increases, when the inner cylinder moves upward and the temperature contours are become more diverge causing to make local Nusselt number values invariant. Also, in the case when the inner cylinder moves upward, the local Nusselt number increases when the distance from the center of square cylinder to circular cylinder center increases since the zone between the top wall and the inner cylinder surface decreases causing the accumulation of the temperature contours which leading to increase the temperature gradient in this zone.

6.4. The effect of average Nusselt number on the inner hot cylinder and the cold square enclosure

Figs. 11 and 12 explain the variations of the average Nusselt number on the inner hot cylinder and the cold square enclosure for $Ra = 10^3$, 10^4, 10^5 and 10^6 with the distance from the center of square cylinder to circular cylinder center respectively. In Fig. 11, average Nusselt number values are increased with increasing of Rayleigh number due to significant effect of thermal convection. Also, the average Nusselt number values begin from a maximum value due to heat source effect and decreases with increasing the distance from the center of square cylinder to circular cylinder center. After that, the average Nusselt number values begin with slight increase, since the distance between the top wall and the surface of the inner cylinder decreases when the inner cylinder moves upward and the temperature contours are very concentrated in this small space leading to increase the average Nusselt number values. In Fig. 12, At $Ra = 10^5$ and $Ra = 10^6$, the average Nusselt number values begin from a maximum value and decreases gradually with increasing the distance from the center of square cylinder to circular cylinder center. After that, the average Nusselt number begins to increase due to the dominating influence of the space between top wall and the surface of inner cylinder in the existence of the strong minor vortices. At $Ra = 10^3$ and $Ra = 10^4$, the average Nusselt number values are increased very slightly with the increasing of the distance from the center of square cylinder to circular cylinder center, because the isotherms have a small effect and almost symmetrical leading to make the average Nusselt number values invariant with increasing the distance from the center of square cylinder to circular cylinder center.

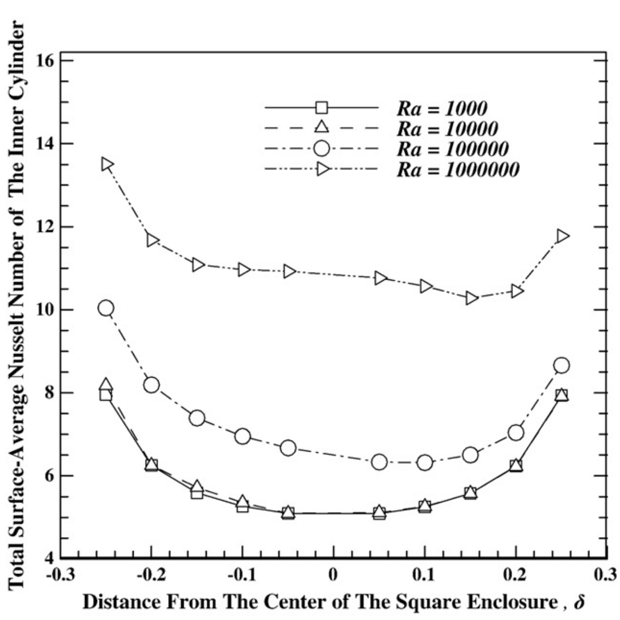

Fig. 11. Total surfaces-averaged Nusselt number of the inner hot cylinder along the δ for different Rayleigh numbers.

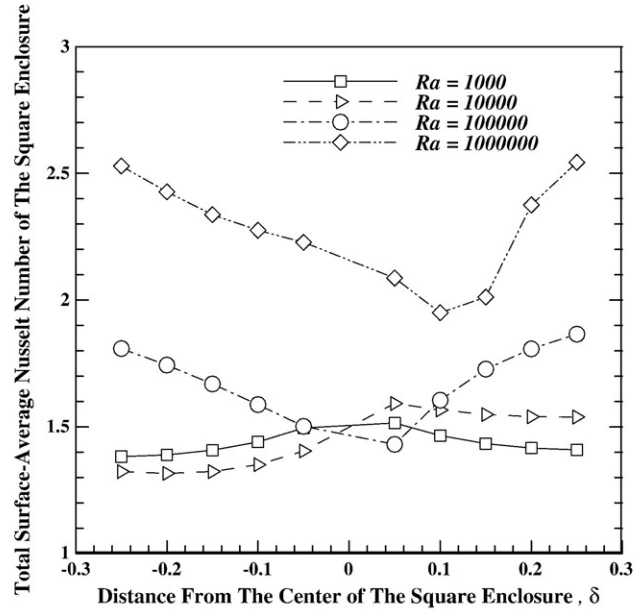

Fig. 12. Total surfaces-averaged Nusselt number of the cold square enclosure along the δ for the different Rayleigh numbers.

7. Conclusions

A numerical investigation of natural convection in a square enclosure has isothermal walls (T_c) and is heated by a concentric internal circular isoflux boundary has been presented. Two dimensional conservation equations of mass, momentum and energy with Boussinesq approximation have been solved using the finite volume method. The governing parameters are: $10^3 \le Ra \le 10^6$, and $-0.25\,L \le \delta \le 0.25\,L$. In view of the obtained results, the following findings have been summarized:

1. The average and local Nusselt number values increase with different upward and downward locations of the inner cylinder with increasing Rayleigh number, due to the significant influence of thermal convection.
2. Values of the average Nusselt number of the cold square enclosure is smaller than the corresponding values of the hot inner cylinder for all distances from the center of square cylinder to circular cylinder center because the isotherms have a slight effect and almost symmetrical.
3. Isotherms are approximately parallel and symmetrical, when the Rayleigh numbers are small and heat conduction is dominant. As Rayleigh number increases, the isotherms become more confused, and heat convection is dominant.
4. A thin thermal boundary layer can be observed at high Rayleigh numbers when the flow strongly hits the top and bottom of the square enclosure.

5. Vortices down the bottom and up the top of the inner cylinder can be noticed as the inner cylinder move upward and downward. Minor vortices on the inner cylinder surface can be also observed.

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