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Foundation of Neutrosophic Crisp Probability Theory

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Abstract

This paper deals with the application of Neutrosophic Crisp sets (which is a generalization of Crisp sets) on the classical probability, from the construction of the Neutrosophic sample space to the Neutrosophic crisp events reaching the definition of Neutrosophic classical probability for these events. Then we offer some of the properties of this probability, in addition to some important theories related to it. We also come into the definition of conditional probability and Bayes theory according to the Neutrosophic Crisp sets, and eventually offer some important illustrative examples. This is the link between the concept of Neutrosophic for classical events and the neutrosophic concept of fuzzy events. These concepts can be applied in computer translators and decision-making theory.

Keywords

Neutrosophic logic; fuzzy logic; classical logic; classical probability; Neutrosophic Crisp sets.

1 Introduction

The Neutrosophic logic is non-classical and new logic founded by the philosopher and mathematical American Florentin Smarandache in 1999. In [6] Salama introduced the concept of neutrosophic crisp set Theory, to represent any event by a triple crisp structure. Moreover the work of Salama et al. [1-10] formed a starting point to construct new branches of neutrosophic mathematics and computer sci. Hence, Neutrosophic set theory turned out to be a generalization of both the classical and fuzzy counterparts. When he presented it as a generalization of the Fuzzy logic, and an extension of the Fuzzy Sets Theory [9] presented by Zadeh in 1965 Played an important role in expanding our scientific and practical approach and reducing the degree of randomization in data that helps us reach high-resolution results. An extension of that logic was introduced by A.A. Salama, the Neutrosophic crisp set theory as a generalization of classical set theory and Neutrosophic logic is a new branch that studies the origin, nature, and field of indeterminacy, as well as the interaction of all the different spectra imaginable in a case. This logic takes into account each idea with its antithesis with the indeterminacy spectrum. The main idea of Neutrosophic logic is to distinguish every logical statement in three dimensions[3.10] are truth in degrees (T) , false in degrees (F) and indeterminacy in degrees (I) we express it in form (T, I, F) and puts them under the field of study, which gives a more accurate description of the data of the phenomenon studied, as this reduces the degree of randomization in the data, which will reach high-resolution results contribute to the adoption of the most appropriate decisions among decision makers. The Neutrosophyis a word composed of two sections :Neutro (in French Neutre, in LatinNeuter) meaning Neutral, and SophyIt is a Greek word meaning wisdom and then the meaning ofthe word in its entirety (knowledge of neutral thought). We note that classical logic studies the situation with its opposite without acknowledging the state of indeterminacy, which is an explicit quantity in the logic of Neutrosophic and one of its components, which gives a more accurate description of the study and thus obtain more correct results. -In this paper we present a study of the application of the Neutrosophic logic to the classical possibilities ،from the occurrence of the experiment to the creation of probability and then to study its properties.

2 Terminologies

2.1 Neutrosophic Random Experiments

We know the importance of experiments in the fields of science and engineering. Experimentation is useful in use, assuming that experiments under close conditions will yield equal results.

In these circumstances, we will be able to determine the values of variables that affect the results of the experiment. In any case, in some experiments, we cannot determine the values of some variables and therefore the results will change from experiment to other.

However, most of the conditions remain as it is. These experiments are described as randomized trials. When we get an undetermined result in the experiment (indeterminacy) and we take and acknowledge this result, we have a neutrosophic experience.

2.2 Example

When throwing the dice, the result we will get from the experiment is one of the following results: {1, 2, 3, 4, 5, 6, i} Where i represents an indeterminacy result. We call this experience a Neutrosophic randomized experiment.

2.3 Sample Spaces and Events due to Neutrosophic

Group X consists of all possible results of a randomized experiment called the sample space. When these results include the result of the indeterminacy, we obtain the Neutrosophic sample space.

2.4 Neutrosophic events

The event: Is a subset A of the sample space X, that is, a set of possible outcomes. The Neutrosophic set of the sample space formed by all the different assemblies (which may or may not include indeterminacy) of the possible results these assemblies are called Neutrosophic. Salama and Hanafy et al. [12-14] introduced laws to calculate correlation coefficients and study regression lines for the new type of data; *a new concept of probability has been introduced for this kind of events. It is a generalization of the old events and the theory of the ancient possibilities. This is the link between the concept of Neutrosophic for classical events and the neutrosophic concept of fuzzy events. These concepts can be applied in computer translators and decision-making theory.*

2.5 The concept of Neutrosophic probability

We know that probability is a measure of the possibility of a particular event, and Smarandache presented the neutrosophic experimental probability, which is a generalization of the classical experimental probability as follows [2, 4]:

$$\left(\frac{number\ of\ times\ event\ A\ occurs}{total\ number\ of\ trials}, \frac{number\ of\ times\ indeterminacy\ occur}{total\ number\ of\ trials}, \frac{number\ of\ times\ event\ A\ does\ not\ occur}{total\ number\ of\ trials}\right)$$

If we had the neutrosophic event, $A = (A_1, A_2, A_3)$we define the neutrosophic probability (Which is marked with the symbol NP) for this event as follows:

$NP(A) = \big(P(A_1)\ , P(A_2)\ , P(A_3)\big) = (T\ , I\ , F\ )$, with:

$P(A_1)$ represents the probability of event A

$P(A_2)$ represents the probability of $indeterminacy$

$P(A_3)$ represents the probability that event A will not occur

According to the definition of classical probability:$P_1, P_2, P_3 \ \ \in [0,1]$

We therefore define the neutrosophic probability [2] in the form:

$NP: X \rightarrow\ [0,1]^3$, where X is a neutrosophic sample space.

The micro-space of the total group, which has a neutrosophic probability for each of its partial groups, calls it a neutrosophic classical probability space.

In [7, 13] the neutrosophic logic can distinguish between the absolutely sure event (the sure event in all possible worlds and its probabilistic value is 1+) and the relative sure event (the sure event in at least one world and not in all worlds its probability is 1) where $1 < 1^{+}$. Similarly, we distinguish between the absolutely impossible event (the impossible event in all possible worlds its probabilistic value is -0) and the relative impossible event (the impossible event in at least one world and not in all worlds its probabilistic value is 0) where -0 <0.

$^{-}0 = 0 - \varepsilon$ & $1^{+} = 1 + \varepsilon$ where ε is a very small positive number.

So, define components(T , I , F)on the non-standard domain]-0 , 1+ [.

For $A = (A_1, A_2, A_3)$neutrosophic classical event Then it is:

$$^{-}0 \leq P(A_1) + P(A_2) + P(A_3) \leq 3^{+}$$

For $A = (A_1, A_2, A_3)$neutrosophic crisp event of the first type Then :

$$0 \leq P(A_1) + P(A_2) + P(A_3) \leq 2$$

The probability of neutrosophic crisp event of the second type is a neutrosophic crisp event then :

$$^{-}0 \leq P(A_1) + P(A_2) + P(A_3) \leq 2^{+}$$

The probability of neutrosophic crisp event of the third type is a neutrosophic crisp event then:

$$^{-}0 \leq P(A_1) + P(A_2) + P(A_3) \leq 3^{+} \qquad \text{……….. [12]}$$

2.6 The Axioms of Neutrosophic probability

For $A = (A_1, A_2, A_3)$ neutrosophic crisp event on the X then :

$$NP(A) = \big(P(A_1)\,, P(A_2)\,, P(A_3)\big)$$

where:

$P(A_1) \geq 0$, $P(A_2) \geq 0$, $P(A_3) \geq 0$

The probability of neutrosophic crisp event $A = (A_1, A_2, A_3)$

$$NP(A) = \big(P(A_1)\,, P(A_2)\,, P(A_3)\big)$$

Where:

$0 \leq P(A_1) \leq 1$, $0 \leq P(A_2) \leq 1$, $0 \leq P(A_3) \leq 1$

For $A_1, A_2, \ldots$.. Inconsistent neutrosophic crisp events then :

$$\mathrm{NP(A)} = (A_1 \cup A_2 \cup \ldots \ldots) = \Big(\mathrm{P}(A_1) + \mathrm{P}(A_2) + \cdots \ldots \ldots , \mathrm{P}\big(i_{A_1 \cup A_2 \cup \ldots}\big) , \mathrm{p}(\overline{A_1 \cup A_2 \cup \ldots})\Big).$$

3 Some important theorems on the neutrosophic crisp probability

Theorem 1

If we have A, B two neutrosophic crisp events and $A \subseteq B$ then:

The first type:

$$NP(A) \leq NP(B) \quad \Leftrightarrow \quad P(A_1) \leq P(B_1) \ , \qquad P(A_2) \leq P(B_2) \ , \qquad P(A_3) \geq P(B_3)$$

The second type:

$$NP(A) \leq NP(B) \quad \Leftrightarrow \quad P(A_1) \leq P(B_1) \ , \qquad P(A_2) \geq P(B_2) \ , \qquad P(A_3) \geq P(B_3)$$

Theorem 2

Probability of the neutrosophic impossible event (symbolized by form $NP\ (\emptyset_N)$) we define it as four types:

The first type:

$$NP\ (\emptyset_N) \ = \big(P(\emptyset), P(\emptyset), P(\emptyset)\big) = (0,0,0) = 0_N$$

The second type:

$$NP\ (\emptyset_N) \ = \big(P(\emptyset), P(\emptyset), P(X)\big) = (0,0,1)$$

The third type:

$$NP\ (\emptyset_N) \ = \big(P(\emptyset), P(X), P(\emptyset)\big) = (0,1,0)$$

The fourth type:

$$NP\ (\emptyset_N) \ = \big(P(\emptyset), P(X), P(X)\big) = (0,1,1)$$

Theorem 3

Probability of the neutrosophic overall crisp event (symbolized by form $NP\ (X_N)$) we define it as four types:

The first type:

$$NP\ (X_N) \ = \big(P(X)\ , P(X)\ , P(X)\big) = (1,1,1) = 1_N$$

The second type:

$$NP\ (X_N) \ = \big(P(X)\ , P(X)\ , P(\emptyset)\big) = (1,1,0)$$

The third type:

$$NP\ (X_N) \ = \big(P(X)\ , P(\emptyset)\ , P(\emptyset)\big) = (1,0,0)$$

The fourth type:

$$NP\ (X_N) \ = \big(P(X)\ , P(\emptyset)\ , P(X)\big) = (1,0,1)$$

Theorem 4

If A^c represents the complement of the event A, then the probability of this event is given according to the following may be three types:

Where $A^c = (A_1^c, A_2^c, A_3^c)$

The first type:

$$NP\ (A^c) \ = \big(P(A_1^c)\ , P(A_2^c)\ , P(A_3^c)\big) = (\ 1 - p(A_1)\ \ , 1 - p(A_2)\ , 1 - p(A_3)\ )$$

The second type:

$$NP\ (A^c) = (\ P(A_3)\ , P(A_2)\ , P(A_1)\ )$$

The third type:

$$NP\ (A^c) = (\ P(A_3)\ , P(A_2^c)\ , P(A_1)\ )$$

Theorem 5

For A, B two neutrosophic crisp events

$$A = (A_1, A_2, A_3)$$
$$B = (B_1, B_2, B_3)$$

Then the probability of the intersection of these two events is given in the form:

$$NP(A \cap B) = (\ P(A_1 \cap\ B_1)\ , P(A_2 \cap\ B_2), P(A_3 \cup B_3)\ )$$

or

$$NP(A \cap B) = (\ P(A_1 \cap\ B_1)\ , P(A_2 \cup\ B_2), P(A_3 \cup B_3)\ )$$

In general if we have the neutrosophic crisp events A, B, C then:

$$NP(A \cap B \cap C) = (\ P(A_1 \cap\ B_1 \cap C_1)\ , P(A_2 \cap\ B_2 \cap C_2), P(A_3 \cup B_3 \cup C_3)\ )$$

Or

$$NP(A \cap B \cap C) = (\ P(A_1 \cap\ B_1 \cap C_1)\ , P(A_2 \cup\ B_2 \cup C_2), P(A_3 \cup B_3 \cup C_3)\ )$$

We can generalize on n of the neutrosophic crisp events.

Theorem 6

Under the same assumptions in theory (1-5) the union of these two neutrosophic crisp events will be: [28]

$$NP(A \cup B) = (\ P(A_1 \cup\ B_1)\ , P(A_2 \cup\ B_2), P(A_3 \cap B_3)\ )\ \text{Or}$$
$$NP(A \cup B) = (\ P(A_1 \cup\ B_1)\ , P(A_2 \cap\ B_2), P(A_3 \cap B_3)\ )$$

Theorem 7

If we have a neutrosophic crisp event that is about:

$$A = A_1 \cup A_2 \cup \ldots \ldots \ldots . \cup A_n$$

The neutrosophic crisp events $A_1, A_2, \ldots \ldots, A_n$ are In consistent then neutrosophic crisp event A we write it in the form:

$$A = (A_1, A_2, A_3)$$
$$= ((A_{11}, A_{12}, A_{13}) \cup (A_{21}, A_{22}, A_{23}) \cup \ldots \ldots . . \cup (A_{n1}, A_{n2}, A_{n3}))$$

Therefore:

$$NP(A) = NP(A_1) + NP(A_2) + \cdots \ldots + \ NP(A_n)$$

Theorem 8

If we have A neutrosophic crisp event and A^c It is an complement event on the whole set X then:

$A \cup A^c = X$ Therefore:

$$NP\ (A) + NP(A^c) = NP(X_N) = 1_N = (1,1,1)$$

4 Neutrosophic Crisp Conditional Probability

If we have A, B two neutrosophic crisp events

$B = (B_1, B_2, B_3) \qquad A = (A_1, A_2, A_3)$

Then the neutrosophic conditional probability is defined to occur A if B occurs in the form:

$$NP(A|B) = \Big(P(A|B)\ , P\big(indeter_{A|B}\big) , P(A^{c}|B)\Big)$$

IF: $P(B) > 0$ $\qquad = \left(\frac{p(A\cap B)}{P(B)}\ , P\big(indeter_{A|B}\big)\ , \frac{p(A^{c}\cap B)}{P(B)} \right)$

From it we conclude that:

$$NP(A|B) \neq NP(B|A)$$

- The conditional probability of complement the neutrosophic event A^c is conditioned by the occurrence of the event B.

  We distinguish it from the following types:

  The first type:

  $$NP(A^{c}|B) = \left(\frac{P(A_3 \cap B_1)}{P(B_1)} , \frac{P(A_2^c \cap B_2)}{P(B_2)} , \frac{P(A_1 \cap B_3)}{P(B_3)}\right)$$

  The second type:

  $$NP(A^{c}|B) = \left(\frac{P(A_3 \cap B_1)}{P(B_1)} , \frac{P(A_2 \cap B_2)}{P(B_2)} , \frac{P(A_1 \cap B_3)}{P(B_3)}\right)$$

- The rule of multiplication in neutrosophic crisp conditional probability:

  $$NP(A \cap B) = \left(P(A_1).P(B_1|A_1)\ , P(A_2).P(B_2|A_2) , P(A_3)\ .P(B_3^c|A_3) \right)$$

5 Independent Neutrosophic Events

We say of the neutrosophic events that they are independent if the occurrence of either does not affect the occurrence of the other.Then the neutrosophic conditional probability of the crisp event A condition of occurrence B is it neutrosophic crisp probability of A. We can verify independence of A, B if one of the following conditions is check:

$$NP(A|B) = NP(A),\ NP(B|A) = NP(B),\ NP(A \cap B) = NP(A).NP(B)$$

(We can easily validate the above conditions based on classical conditional probability) .

Equally:

If the two neutrosophic crisp events A, B are independent then:

A^c Independent of B

A Independent of B^c

A^c Independent of B^c

(Pronounced from the definition of a complementary event in Theorems 4).

6 The law of total probability and Bayes theorem via Neutrosophic crisp sets

6.1 The law of Neutrosophic crisp total probability

(1) We have a sample space consisting of then neutrosophic crisp comprehensive events $A_1, \mathrm{A}_2 \ldots\ldots, \mathrm{A}_\mathrm{n}$

$$A_1 \cup \mathrm{A}_2 \cup \ldots . \cup \mathrm{A}_\mathrm{n} = \mathrm{X}_\mathrm{N}$$

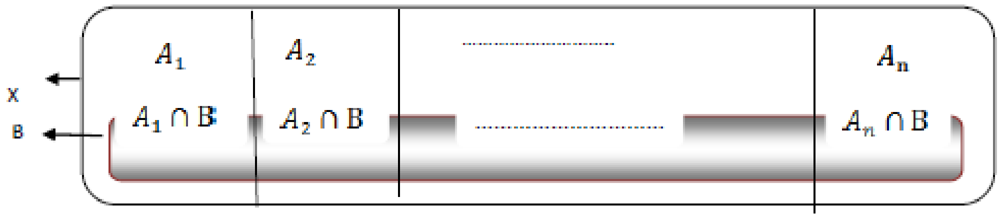
$$\left((A_{11}, A_{12}, A_{13}) \cup (A_{21}, A_{22}, A_{23}) \cup \ldots \ldots . . \cup (A_{n1}, A_{n2}, A_{n3})\right) = \mathrm{X}_\mathrm{N}$$

(2) The neutrosophic comprehensive events are inconsistent two at a time among them:

$$\mathrm{A_i} \cap \mathrm{A_j} = \emptyset \quad \forall\, i \neq j$$

(3) The neutrosophic crisp event B represents a common feature in all joint neutrosophic crisp events , note the following figure(1):

Figure (1)



We take the neutrosophic crisp probability for these events:

$$NP(A_1), \mathrm{NP}(\mathrm{A}_2), \ldots\ldots, \mathrm{NP}(\mathrm{A}_\mathrm{n})$$

From the graphic, we note that:

$$NP(B) = NP(A_1 \cap \mathrm{B}) + NP(A_2 \cap \mathrm{B}) + \cdots \ldots + NP(A_\mathrm{n} \cap \mathrm{B})$$

From the definition of neutrosophic crisp conditional probability:

$$NP(\mathrm{B} \cap A_\mathrm{i}) = (P(A_{\mathrm{i}\,1}).\mathrm{P}(\mathrm{B} \backslash A_{\mathrm{i}\,1}), P(A_{\mathrm{i}\,2}).\mathrm{P}(\mathrm{B} \backslash A_{\mathrm{i}\,2}), P(A_{\mathrm{i}\,3}).\mathrm{P}(\mathrm{B} \backslash A_{\mathrm{i}\,3}))$$

Therefore:

$$NP(B) = NP(B|A_1).NP(A_1) + NP(B|A_2).NP(A_2) + \ldots\ldots + NP(B|A_n).NP(A_n)$$

Which is equal to

$$= (p(A_{11}).P(B \backslash A_{11}), p(A_{12}).P(B \backslash A_{12}), p(A_{13}).P(B^c \backslash A_{13}) + p(A_{21}).P(B \backslash A_{21}), p(A_{22}).P(B \backslash A_{22}), p(A_{23}).P(B^c \backslash A_{23}) + + \cdots \ldots \ldots . . + (p(A_{n1}).P(B \backslash A_{n1}), p(A_{n2}).P(B \backslash A_{n2}), p(A_{n3}).P(B^c \backslash A_{n3})$$

6.2 Bayes theorem by Neutrosophic:

Taking advantage of the previous figure (1):

Neutrosophic total probability iff Probability of occurrence a common feature B.

Bayes theorem iff provided that the neutrosophic crisp event occur B,What is the probability of being from A_{i} (Item selected from B, What is the probability of being from A_{i})

Under the same assumptions that we have set in the definition of the law of neutrosophic crisp total probability, we reach the Bayes Law as follows:

$$NP(A_{\mathrm{i}} \backslash \mathrm{B}) = (\frac{\mathrm{P(B_1 \backslash A_{i1})p(A_{i1})}}{\mathrm{p(B_1)}}, \frac{\mathrm{P(B_2 \backslash A_{i2})p(A_{i2})}}{\mathrm{p(B_2)}}, \frac{\mathrm{P(B_3 \backslash A_{i3}^c)p(A_{i3}^c)}}{\mathrm{p(B_3)}})$$

6.3 Examples

Let us have the experience of throwing a dice stone and thus we have the neutrosophic sample space as: X= {1, 2, 3, 4, 5, 6, i}, where i represents the probability of getting indeterminacy.

We have the possibility of getting indeterminacy= 0.10

Then to calculate the following possibilities:

1- $NP(1) = \left(\frac{1-0.10}{6}, 0.10, 5.\frac{1-0.10}{6}\right)$

$= (0.15, 0.10, 0.75) = NP(2) = \cdots \ldots = NP(6)$

2- $NP(1^c) = (P(2,3,4,5), 0.10, P(1))$

$=(5(0.15), 0.10, 0.15) = (0.75, 0.10, 0.15)$

3- $NP(1 \ or \ 2) = \left(p(1) + p(2), 0.10, p(3,4,5,6)\right)$

$= \left(2(0.15), 0.10, 4(0.15)\right) = (0.30, 0.10, 0.60)$

But when we have B={2,3,4,5} , A={1,2,3 } then :

$NP(A \ or \ B) = (P(A) + P(B) - P(A \cap B), 0.10, P(A^c) \ and \ P(B^c))$

$= (3(0.15) + 4(0.15) - 2(0.15), 0.10, P\{4,5,6\} \ and \ P\{1, 6\})$

$= \left(0.75, 0.10, P(6)\right) = (0.75, 0.10, 0.15)$

4- $NP(\{1,2,3\}) = (P\{1,2,3\}, 0.10, P\{1,2,3\}^c)$

$= \left(p(1) + p(2) + p(3), 0.10, p(4) + p(5) + p(6)\right)$

$= (0.15 + 0.15 + 0.15, 0.10, 0.15 + 0.15 + 0.15)$

$= (0.45, 0.10, 0{,}45)$

I. Assuming we have a jar containing:

5 cards have a symbol A, 3cards have a symbol B

2 cards are not specified (The symbol is erased on them)

If A represents is getting the card A from the jar

B represents is getting the card B from the jar

Then

$NP(A) = (\frac{5}{10}, \frac{2}{10}, \frac{3}{10})$, $NP(B) = (\frac{3}{10}, \frac{2}{10}, \frac{5}{10})$

If card B is withdrawn from the jar then it will be:

$$NP\ (A\backslash B) = \left(\frac{P(B\backslash A).P(A)}{P(B)},\quad P\big(indeter_{\ A\backslash B}\big),\quad P(A^c\backslash B) \right)$$

$$= \left(\frac{(\frac{3}{9})\ (\frac{5}{9})}{\frac{3}{9}},\quad \frac{2}{9},\quad P(B\backslash B) = P(B) = \frac{2}{9} \right) = (\frac{5}{9}, \frac{2}{9}, \frac{2}{9})$$

If card A is withdrawn from the jar then it will be:

The same way we get: $NP(B\backslash A) = (\ \frac{3}{9}\ , \frac{2}{9}\ , \frac{4}{9}\ )$

Thus, Bayes theory according to neutrosophic be as:

$$NP(A\backslash B) = \Big(\ P(A\backslash B\ ), P\big(indeter_{\ A\backslash B}\big), P(A^c\backslash B)\Big)$$

$$= \frac{P(B\backslash A).P(A)}{P(B)}\ ,\ P\big(indeter_{\ A\backslash B}\big)\ , \frac{P(B\backslash A^c).P(A^c)}{P(B)}\ )$$

$$= \left(\frac{3}{9}\frac{\frac{5}{10}}{\frac{3}{10}},\quad \frac{2}{9}\quad, P(B\backslash B) \right) = (\frac{5}{9}, \frac{2}{9}, \frac{2}{9})$$

Let us have the X set X={ a ,b ,c ,d } and

A= ({a,b} , {c} , {d})

B= ({a}, {c}, {d,b})

Two neutrosophic events from the first type on X and we have:

$$U_1 = (\{\mathrm{a, b}\}, \{\mathrm{c, d}\}, \{\mathrm{a, d}\})$$

$$U_2 = (\{\mathrm{a, b}, c\}, \{\mathrm{c}\}, \{\mathrm{d}\})$$

Two neutrosophic events from the third type on X then:

The first type :

$$A \cap B = (\ \{a\}, \{c\}, \{d, b\})$$

$$NP(A \cap B) = (\ 0.25\ \ , 0.25\ , 0.50\ )$$

The second type:

$$A \cap B = (\ \{a\}, \{c\}, \{d, b\})$$

$$NP(A \cap B) = (\ 0.25\ \ , 0.25\ , 0.50\ )$$

The first type :

$$A \cup B = (\ \{a, b\}, \{c\}, \{d\})$$

$$NP(A \cup B) = (\ 0.50\ , 0.25\ , 0.25\ )$$

The second type:

$$A \cup B = (\ \{a, b\}, \{c\}, \{d\})$$

$$NP(A \cup B) = (\ 0.50\ , 0.25\ , 0.25\ )$$

The first type :

$$A^{\mathrm{c}} = (\{c, d\}, \{a, b, d\}, \{a, b, c\})$$

$$NP(A^{\mathrm{c}}) = (\ 0.50\ , 0.75\ , 0.75\ )$$

The second type:

$$A^{\mathrm{c}} = (\{d\}, \{c\}, \{a, b\})$$

$$NP(A^c) = (\ 0.25\ , 0.25\ , 0.50\ )$$

The third type:

$$A^c = (\{d\}, \{a, b, d\}, \{a, b\})$$

$$NP(A^c) = (\ 0.25\ , 0.75\ , 0.50\ )$$

The first type :

$$B^c = (\{b, c, d\}, \{a, b, d\}, \{a, b\})$$

$$NP(B^c) = (\ 0.75\ , 0.75\ , 0.50\ )$$

The second type

$$B^c = (\{b, d\}, \{c\}, \{a\})$$

The third type:

$$B^c = (\{b, d\}, \{a, b, d\}, \{a\})$$

$$NP(B^c) = (\ 0.50\ , 0.75\ , 0.25)$$

The first type

$$U_1 \cup \mathrm{U}_2 = (\{\ a, b, c\}, \{\ c, d\}, \{d\})$$

$$NP(U_1 \cup \mathrm{U}_2) = (\ 0.75\ , 0.50\ , 0.25\ )$$

The second type

$$U_1 \cup \mathrm{U}_2 = (\{\ a, b, c\}, \{\ c\}, \{d\})$$

$$NP(U_1 \cup \mathrm{U}_2) = (\ 0.75\ , 0.25\ \ , 0.25\ )$$

The first type

$$U_1 \cap \mathrm{U}_2 = (\{\ a, b\}, \{\ c\}, \{a, d\})$$

$$NP(U_1 \cap \mathrm{U}_2) = (\ 0.50\ , 0.25\ \ , 0.50\ )$$

The second type

$$U_1 \cap \mathrm{U}_2 = (\{\ a, b\}, \{\ c, d\}, \{a, d\})$$

$$NP(U_1 \cap \mathrm{U}_2) = (\ 0.50\ , 0.50\ \ , 0.50\ )$$

The first type:

$$U_1^c = (\{c, d\}, \{a, b\}, \{b, c\})$$

$$NP\ (U_1^c) = (0.50\ , 0.50\ , 0.50\ )$$

The second type:

$$U_1^c = (\{a, d\}, \{c, d\}, \{a, b\})$$

$$NP\ (U_1^c) = (0.50\ , 0.50\ , 0.50\ )$$

The third type

$$U_1^c = (\{a, d\}, \{a, b\}, \{a, d\})$$

$$NP\ (U_1^c) = (0.50\ , 0.50\ , 0.50\ )$$

The first type:

$$U_2^c = (\{d\}, \{a, b, d\}, \{a, b, c\})$$

$$NP\ (U_2^c) = (0.25\ , 0.75\ , 0.75\ )$$

The second type:

$$U_2^c = (\{d\}, \{c\}, \{a, b, c\})$$

9- $NP\ (U_2^c) = (0.25\ , 0.25\ , 0.75\ )$

The third type

$$U_2^c = (\{d\},\{a,b,d\},\{a,b,c\})$$
$$NP\ (U_2^c) = (0.25\ ,0.75\ ,0.75\ )$$

$NP\ (A)$ =(0.50, 0.25, 0.25)

NP(B) =(0.25, 0.25, 0.50)

$NP\ (U_1)$ =(0.50,0.50,0.50)

$NP\ (U_2)$ =(0.75,0.25,0.25)

$NP\ (U_1^c\ \ )$ =(0.50,0.50,0.50)

$NP\ (U_2^c\ \ )$ =(0.25,0.75,0.75)

10- $(A \cap B)^{\mathrm{c}} = (\{\mathrm{b,c,d}\},\{\mathrm{a,b,d}\},\{\mathrm{a,c}\})$

$$NP(A \cap B)^{\mathrm{c}} = (0.75\ \ ,0.75\ ,0.50\ )$$

11- $NP\ (A^c)\ \cap NP(B^c) = (\{c,d\},\{a,b,d\},\{a,b,c\})$

$= (\ 0.50\ ,0.75\ ,0.75\ )$

$NP\ (A^c)\ \cup NP(B^c) = (\{c,d,b\},\{a,b,d\},\{a,b,c\})$

$$= (0.75\ \ ,0.75\ ,0.75\ )$$

12- $A * B = \{(a,a),(b,a)\},\{(c,c)\},\{(d,d),(d,b)\}$

$$NP(A * B) = (\frac{2}{16}\ ,\frac{1}{16}\ ,\frac{2}{16}\ )$$

$$B * A = (\{(a,a),(a,b)\},\{c,c\},\{(d,d),(b,d)\}$$

$$NP(B * A) = (\frac{2}{16}\ ,\frac{1}{16}\ ,\frac{2}{16})$$

$$A * U_1 = \{(a,a),(a,b),(b,a\ ),(b,b)\},\{(c,c),(c,d)\},\{(d,a),(d,d\ )\}$$

$$NP(A * U_1) = (\frac{4}{16},\frac{2}{16},\frac{2}{16})$$

$\mathrm{U_1 * U_2}$

$= (\{\mathrm{(a,a),(a,b),(a,c),(b,a),(b,b),(b,c)}\},\{\mathrm{(c,c),(d,c)}\},\{\mathrm{(a,d),(d,d)}\}$

$$= (\frac{6}{16},\frac{2}{16},\frac{2}{16})$$

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