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Research Article

On Fekete-Szegö Problems for Certain Subclasses Defined by q-Derivative

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We derive the Fekete-Szegö theorem for new subclasses of analytic functions which are q-analogue of well-known classes introduced before.

1. Introduction

Denote by $\mathscr{A}$ the class of all analytic functions of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k \tag{1}$$

in the open unit disk $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$.

For two analytic functions f and g in $\mathbb{U}$, the subordination between them is written as $f \prec g$. Frankly, the function $f(z)$ is subordinate to $g(z)$ if there is a Schwarz function w with $w(0) = 0$, $|w(z)| < 1$, for all $z \in \mathbb{U}$, such that $f(z) = g(w(z))$ for all $z \in \mathbb{U}$. Note that, if g is univalent, then $f \prec g$ if and only if $f(0) = g(0)$ and $f(\mathbb{U}) \subseteq g(\mathbb{U})$.

In [1, 2], Jackson defined the q-derivative operator D_q of a function as follows:

$$D_q f(z) = \frac{f(qz) - f(z)}{(q-1)z} \quad (z \neq 0,\ q \neq 0) \tag{2}$$

and $D_q f(z) = f'(0)$. In case $f(z) = z^k$ for k is a positive integer, the q-derivative of $f(z)$ is given by

$$D_q z^k = \frac{z^k - (zq)^k}{z(1-q)} = [k]_q z^{k-1}. \tag{3}$$

As $q \to 1^-$ and $k \in \mathbb{N}$, we have

$$[k]_q = \frac{1-q^k}{1-q} = 1 + q + \cdots + q^k \longrightarrow k. \tag{4}$$

Quite a number of great mathematicians studied the concepts of q-derivative, for example, by Gasper and Rahman [3], Aral et al. [4], Li et al. [5], and many others (see [6–15]).

Making use of the q-derivative, we define the subclasses $\mathscr{S}_q^*(\alpha)$ and $\mathscr{C}_q(\alpha)$ of the class $\mathscr{A}$ for $0 \leq \alpha < 1$ by

$$\mathscr{S}_q^*(\alpha) = \left\{ f \in \mathscr{A} : \operatorname{Re}\left(\frac{zD_q(f(z))}{f(z)}\right) > \alpha,\ z \in \mathbb{U} \right\},$$

$$\mathscr{C}_q(\alpha) = \left\{ f \in \mathscr{A} : \operatorname{Re}\left(1 + \frac{zqD_q(D_q(f(z)))}{D_q f(z)}\right) > \alpha,\ z \in \mathbb{U} \right\}. \tag{5}$$

These classes are also studied and introduced by Seoudy and Aouf [16].

Noting that

$$f \in \mathscr{C}_q(\alpha) \Longleftrightarrow$$
$$zD_q f \in \mathscr{S}_q^*(\alpha),$$
$$\lim_{q\to 1}\mathscr{S}_q^*(\alpha) = \left\{ f \in \mathscr{A} : \lim_{q\to 1} \operatorname{Re}\left(\frac{zD_q(f(z))}{f(z)}\right) > \alpha,\ z \in \mathbb{U}\right\} = \mathscr{S}^*(\alpha),$$
$$\lim_{q\to 1}\mathscr{C}_q(\alpha) = \left\{ f \in \mathscr{A} : \lim_{q\to 1} \operatorname{Re}\left(1 + \frac{zqD_q(D_q(f(z)))}{D_q f(z)}\right) > \alpha,\ z \in \mathbb{U}\right\} = \mathscr{C}(\alpha), \tag{6}$$

where $\mathscr{S}^*(\alpha)$ and $\mathscr{C}(\alpha)$ are, respectively, the classes of starlike of order α and convex of order α in $\mathbb{U}$ ([17, 18]).

Next, we state the q-analogue of Ruscheweyh operator given by Aldweby and Darus [8] that will be used throughout.

Definition 1 (see [8]). Let $f \in \mathscr{A}$. Denote by $\mathscr{R}_q^\lambda$ the q-analogue of Ruscheweyh operator defined by

$$\mathscr{R}_q^\lambda f(z) = z + \sum_{k=2}^{\infty} \frac{[k+\lambda-1]_q!}{[\lambda]_q!\,[k-1]_q!} a_k z^k, \tag{7}$$

where $[k]_q!$ given by is as follows:

$$[k]_q! = \begin{cases} [k]_q [k-1]_q \cdots [1]_q, & k = 1, 2, \ldots; \\ 1, & k = 0. \end{cases} \tag{8}$$

From the definition we observe that if $q \to 1$, we have

$$\begin{aligned} \lim_{q\to 1} \mathscr{R}_q^\lambda f(z) &= z + \lim_{q\to 1}\left[\sum_{k=2}^{\infty} \frac{[k+\lambda-1]_q!}{[\lambda]_q!\,[k-1]_q!} a_k z^k\right] \\ &= z + \sum_{k=2}^{\infty} \frac{(k+\lambda-1)!}{(\lambda)!\,(k-1)!} a_k z^k = \mathscr{R}^\lambda f(z), \end{aligned} \tag{9}$$

where $\mathscr{R}^\lambda$ is Ruscheweyh differential operator defined in [19].

Using the principle of subordination and q-derivative, we define the classes of q-starlike and q-convex analytic functions as follows.

Definition 2. For $\varphi \in P$ and $\lambda > -1$, the class $\mathscr{S}_{q,\lambda}^*(\varphi)$ which consists of all analytic functions $f \in \mathscr{A}$ satisfies

$$\frac{zD_q(\mathscr{R}_q^\lambda(f(z)))}{\mathscr{R}_q^\lambda(f(z))} \prec \varphi(z), \quad |z| < 1. \tag{10}$$

Definition 3. For $\varphi \in P$ and $\lambda > -1$, the class $\mathscr{C}_{q,\lambda}(\varphi)$ which consists of all analytic functions $f \in \mathscr{A}$ satisfies

$$1 + \frac{zqD_q(D_q(\mathscr{R}_q^\lambda f(z)))}{D_q(\mathscr{R}_q^\lambda f(z))} \prec \varphi(z), \quad |z| < 1,\ 0 < q < 1. \tag{11}$$

To prove our results, we need the following.

Lemma 4 (see [18]). *If $p(z) = 1 + c_1 z + c_2 z^2 + \cdots \in P$ of positive real part is in $\mathbb{U}$ and μ is a complex number, then*

$$\left|c_2 - \mu c_1^2\right| \le 2\max\{1; |2\mu - 1|\}. \tag{12}$$

The result is sharp given by

$$p(z) = \frac{1+z}{1-z}, \qquad p(z) = \frac{1+z^2}{1-z^2}. \tag{13}$$

Lemma 5 (see [18]). *If $p(z) = 1 + c_1 z + c_2 z^2 + \cdots$ is a function with positive real part, then*

$$\left|c_2 - \nu c_1^2\right| \le \begin{cases} -4\nu + 2, & \text{if } \nu \le 0; \\ 2, & \text{if } 0 \le \nu \le 1; \\ 4\nu - 2, & \text{if } \nu \ge 1. \end{cases} \tag{14}$$

2. Main Results

Now is our theorem using similar methods studied by Seoudy and Aouf in [16].

Theorem 6. *Let $\varphi(z) = 1 + B_1 z + B_2 z^2 + \cdots \in P$. If f given by (1) is in the class $\mathscr{S}_{q,\lambda}^*(\varphi)$ and μ is a complex number, then*

$$\left|a_3 - \mu a_2^2\right| \le \frac{B_1}{q\left([\lambda]_q^2 + q^{2\lambda}(1+q)\left([\lambda]_q^2 + 1\right)\right)} \cdot \max\left\{1, \left|\frac{B_2}{B_1} + \frac{[\lambda]_q + q^\lambda - \mu\left([\lambda]_q + q^\lambda(1+q)\right)}{q\left([\lambda]_q + q^\lambda\right)} B_1\right|\right\}. \tag{15}$$

The result is sharp.

Proof. If $f \in \mathscr{S}_{q,\lambda}^*(\varphi)$, then there is a function $w(z)$ in $\mathbb{U}$ with $w(0) = 0$ and $|w(z)| < 1$ in $\mathbb{U}$ such that

$$\frac{zD_q(\mathscr{R}_q^\lambda(f(z)))}{\mathscr{R}_q^\lambda(f(z))} = \varphi(w(z)). \tag{16}$$

Define the function $p(z)$ by

$$p(z) = \frac{1+w(z)}{1-w(z)} = 1 + p_1 z + p_2 z^2 + \cdots. \tag{17}$$

Since $w(z)$ is a Schwarz function, immediately $\operatorname{Re}(p(z)) > 0$ and $p(0) = 1$. Let

$$g(z) = \frac{zD_q\left(\mathscr{R}_q^{\lambda}(f(z))\right)}{\mathscr{R}_q^{\lambda}(f(z))} = 1 + d_1 z + d_2 z^2 + \cdots. \quad (18)$$

Then from (16), (17), and (18), obtain

$$g(z) = \varphi\left(\frac{p(z)-1}{p(z)+1}\right). \quad (19)$$

Since

$$\frac{p(z)-1}{p(z)+1} = \frac{1}{2}\left[p_1 z + \left(p_2 - \frac{p_1^2}{2}\right)z^2 + \left(p_3 + \frac{p_1^3}{4} - p_1 p_2\right)z^3 + \cdots\right] \quad (20)$$

we have

$$\varphi\left(\frac{p(z)-1}{p(z)+1}\right) = 1 + \frac{1}{2}B_1 p_1 z + \left[\frac{1}{2}B_1\left(p_2 - \frac{p_1^2}{2}\right) + \frac{1}{4}B_2 p_1^2\right]z^2 + \cdots. \quad (21)$$

From the last equation and (18), we obtain

$$d_1 = \frac{1}{2}B_1 p_1, \qquad d_2 = \frac{1}{2}B_1\left(p_2 - \frac{p_1^2}{2}\right) + \frac{1}{4}B_2 p_1^2. \quad (22)$$

A simple computation in (18) and knowing that $[n]_q - 1 = q[n-1]_q$, we obtain

$$\frac{zD_q\left(\mathscr{R}_q^{\lambda}(f(z))\right)}{\mathscr{R}_q^{\lambda}(f(z))} = 1 + q[\lambda+1]_q a_2 z + \left\{q[\lambda+1]_q[\lambda+2]_q a_3 - q[\lambda+1]_q^2 a_2^2\right\}z^2 + \cdots. \quad (23)$$

Then, from last equation and (18), we see that

$$d_1 = q[\lambda+1]_q a_2, \qquad d_2 = q[\lambda+1]_q[\lambda+2]_q a_3 - q[\lambda+1]_q^2 a_2^2, \quad (24)$$

or equivalently, we have

$$a_2 = \frac{B_1 p_1}{2q[\lambda+1]_q}, \qquad a_3 = \frac{B_1}{2q[\lambda+1]_q[\lambda+2]_q}\left(p_2 - \frac{p_1^2}{2}\right) + \frac{B_2 p_1^2}{4q[\lambda+1]_q[\lambda+2]_q} + \frac{B_1^2 p_1^2}{8q^2[\lambda+1]_q[\lambda+2]_q}. \quad (25)$$

Therefore

$$a_3 - \mu a_2^2 = \frac{B_1}{2q[\lambda+1]_q[\lambda+2]_q}\left\{p_2 - \nu p_1^2\right\}, \quad (26)$$

where

$$\nu = \frac{1}{2}\left[1 - \frac{B_2}{B_1} - \frac{[\lambda]_q + q^{\lambda} - \mu\left([\lambda]_q + q^{\lambda}(1+q)\right)}{q\left([\lambda]_q + q^{\lambda}\right)}B_1\right]. \quad (27)$$

By an application of Lemma 4, our result follows. Again by Lemma 4, the equality in (15) is gained for

$$p(z) = \frac{1+z}{1-z} \quad \text{or} \quad p(z) = \frac{1+z^2}{1-z^2}. \quad (28)$$

Thus Theorem 6 is complete. □

Similarly, we can prove for the class $\mathscr{C}_{q,\lambda}(\varphi)$. We omit the proofs.

Theorem 7. *Let $\varphi(z) = 1 + B_1 z + B_2 z^2 + \cdots \in P$. If f given by (1) is in the class $\mathscr{C}_{q,\lambda}(\varphi)$ and μ is a complex number, then*

$$\left|a_3 - \mu a_2^2\right| \leq \frac{B_1}{2q[3]_q[\lambda+1]_q[\lambda+2]_q} \cdot \max\left\{1, \left|\frac{B_2}{B_1} + \frac{[2]_q^2 - \mu[3]_q[\lambda+2]_q}{q[2]^2}B_1\right|\right\}. \quad (29)$$

The result is sharp.

Taking $\lambda = 0$ in Theorem 6, we have the corollary for the class $\mathcal{S}_q^*(\varphi)$ as follows.

Corollary 8. *Let $\varphi(z) = 1 + B_1 z + B_2 z^2 + \cdots \in P$. If f given by (1) is in the class $\mathcal{S}_q^*(\varphi)$ and μ is a complex number, then*

$$\left|a_3 - \mu a_2^2\right| \leq \frac{B_1}{q(1+q)}\max\left\{1, \left|\frac{B_2}{B_1} + \frac{1-\mu(1+q)}{q}B_1\right|\right\}. \quad (30)$$

The result is sharp.

Taking $q \to 1$ and $\lambda = 0$ in Theorem 6, we obtain the following.

Corollary 9. *Let* $\varphi(z) = 1 + B_1 z + B_2 z^2 + \cdots$, $B_1 \in P$. *If* f *given by (1) is in the class* $\mathcal{S}^*_{q,\lambda}(\varphi)$ *and* μ *is a complex number, then*

$$\left|a_3 - \mu a_2^2\right| \le \frac{B_1}{2} \max\left\{1, \left|\frac{B_2}{B_1} + \frac{1-2\mu}{1} B_1\right|\right\}. \quad (31)$$

By using Lemma 4, we have the following theorem.

Theorem 10. *Let* $\varphi(z) = 1 + B_1 z + B_2 z^2 + \cdots$ *with* $B_1 > 0$ *and* $B_2 \ge 0$. *Let*

$$\sigma_1 = \frac{\left([\lambda]_q + q^\lambda\right)\left(B_1^2 + q\left(B_2 - B_1\right)\right)}{\left([\lambda]_q + q^\lambda [2]_q\right) B_1^2},$$
$$\sigma_2 = \frac{\left([\lambda]_q + q^\lambda\right)\left(B_1^2 + q\left(B_2 + B_1\right)\right)}{\left([\lambda]_q + q^\lambda [2]_q\right) B_1^2}. \quad (32)$$

Let f *given by (1) be in the class* $\mathcal{S}^*_{q,\lambda}(\varphi)$. *Then*

$$\left|a_3 - \mu a_2^2\right| \le \begin{cases} \dfrac{B_2}{q[\lambda+1]_q[\lambda+2]_q} + \dfrac{B_1^2}{q[\lambda+1]_q[\lambda+2]_q}\left(\dfrac{[\lambda]_q + q^\lambda - \left([\lambda]_q + q^\lambda[2]_q\right)\mu}{q\left([\lambda]_q + q^\lambda\right)}\right), & \text{if } \mu \le \sigma_1; \\ \dfrac{B_1}{q[\lambda+1]_q[\lambda+2]_q}, & \text{if } \sigma_1 \le \mu \le \sigma_2; \\ -\dfrac{B_2}{q[\lambda+1]_q[\lambda+2]_q} - \dfrac{B_1^2}{q[\lambda+1]_q[\lambda+2]_q}\left(\dfrac{[\lambda]_q + q^\lambda - \left([\lambda]_q + q^\lambda[2]_q\right)\mu}{q\left([\lambda]_q + q^\lambda\right)}\right), & \text{if } \mu \ge \sigma_2. \end{cases} \quad (33)$$

Proof. First, let $\mu \le \sigma_1$

$$\begin{aligned}\left|a_3 - \mu a_2^2\right| &\le \frac{B_1}{2q[\lambda+1]_q[\lambda+2]_q}[-4\nu + 2] \\ &\le \frac{B_2}{q[\lambda+1]_q[\lambda+2]_q} \\ &+ \frac{B_1^2}{q[\lambda+1]_q[\lambda+2]_q}\left(\frac{[\lambda]_q + q^\lambda - \left([\lambda]_q + q^\lambda[2]_q\right)\mu}{q\left([\lambda]_q + q^\lambda\right)}\right).\end{aligned} \quad (34)$$

Now, let $\sigma_1 \le \mu \le \sigma_2$; then using the above calculation, we obtain

$$\left|a_3 - \mu a_2^2\right| \le \frac{B_1}{q[\lambda+1]_q[\lambda+2]_q}. \quad (35)$$

Finally, if $\mu \ge \sigma_2$, then

$$\begin{aligned}\left|a_3 - \mu a_2^2\right| &\le \frac{B_1}{q[\lambda+1]_q[\lambda+2]_q}[4\nu - 2] \\ &- \frac{B_2}{q[\lambda+1]_q[\lambda+2]_q} \\ &- \frac{B_1^2}{q[\lambda+1]_q[\lambda+2]_q}\left(\frac{[\lambda]_q + q^\lambda - \left([\lambda]_q + q^\lambda[2]_q\right)\mu}{q\left([\lambda]_q + q^\lambda\right)}\right).\end{aligned} \quad (36)$$

□

Similarly, we can prove for the class $\mathscr{C}_{q,\lambda}(\varphi)$ as follows.

Theorem 11. *Let* $\varphi(z) = 1 + B_1 z + B_2 z^2 + \cdots$ *with* $B_1 > 0$ *and* $B_2 \ge 0$. *Let*

$$\varrho_1 = \frac{[2]_q\left(B_1^2 + q\left(B_2 - B_1\right)\right)}{[3]_q B_1^2},$$
$$\varrho_2 = \frac{[2]_q\left(B_1^2 + q\left(B_2 + B_1\right)\right)}{[3]_q B_1^2}. \quad (37)$$

If f *given by (1) is in the class* $\mathscr{C}_{q,\lambda}(\varphi)$, *then*

$$\left|a_3 - \mu a_2^2\right| \le \begin{cases} \dfrac{B_1}{2q[3]_q[\lambda+1]_q[\lambda+2]_q}\left[\dfrac{B_2}{B_1} + \left(\dfrac{[2]_q^2 - [3]_q[\lambda+2]_q\mu}{q[2]_q^2}\right)B_1\right], & \text{if } \mu \le \varrho_1; \\ \dfrac{B_1}{2q[3]_q[\lambda+1]_q[\lambda+2]_q}, & \text{if } \varrho_1 \le \mu \le \varrho_2; \\ \dfrac{B_1}{2q[3]_q[\lambda+1]_q[\lambda+2]_q}\left[-\dfrac{B_2}{B_1} - \left(\dfrac{[2]_q^2 - [3]_q[\lambda+2]_q\mu}{q[2]_q^2}\right)B_1\right], & \text{if } \mu \ge \varrho_2. \end{cases} \quad (38)$$

Taking $\lambda = 0$ in Theorem 10, we obtain next result for the class $\mathcal{S}_q^*(\varphi)$.

Corollary 12. *Let $\varphi(z) = 1 + B_1 z + B_2 z^2 + \cdots$ with $B_1 > 0$ and $B_2 \geq 0$. Let*

$$\sigma_1 = \frac{B_1^2 + q(B_2 - B_1)}{[2]_q B_1^2}, \qquad \sigma_2 = \frac{B_1^2 + q(B_2 + B_1)}{[2]_q B_1^2}. \tag{39}$$

If f given by (1) is in the class $\mathcal{S}_q^*(\varphi)$, then

$$\left|a_3 - \mu a_2^2\right| \leq \begin{cases} \dfrac{B_2}{q[2]_q} + \dfrac{B_1^2}{q[2]_q}\left(\dfrac{1-[2]_q \mu}{q}\right), & \text{if } \mu \leq \sigma_1; \\ \dfrac{B_1}{q[2]_q}, & \text{if } \sigma_1 \leq \mu \leq \sigma_2; \\ -\dfrac{B_2}{q[2]_q} - \dfrac{B_1^2}{q[2]_q}\left(\dfrac{1-[2]_q \mu}{q}\right), & \text{if } \mu \geq \sigma_2. \end{cases} \tag{40}$$

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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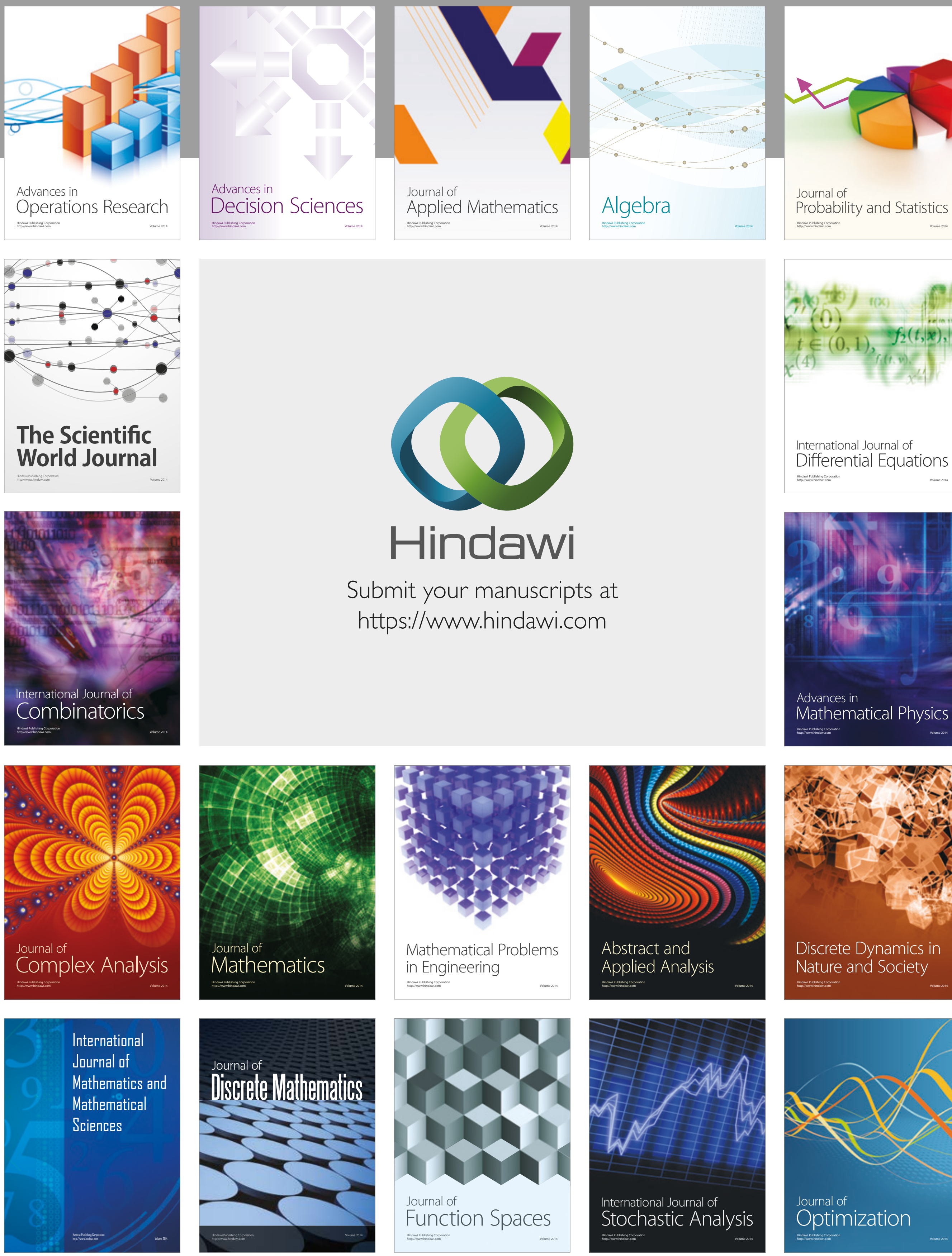
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