# Fuzzy Scheduling Problem on Two Machines

SawsanJawad Kadhum[#1], AsmaaAbdAswhad[*2], Niran Sabah Jasim[#3]
[#1,*2,#3]*Department of Mathematics, College of Education for pure Science/ Ibn-Al-Haitham, University of Baghdad, Iraq*

***Abstract***
*The main objective of this paper is to compute the total optimized penalty cost and to be minimizing in the fuzzy scheduling problem of the jobs on two machines. This total optimized penalty cost is composed of the total earliness and the total tardiness cost.*

**Keywords:**
*Two Machines; fuzzy scheduling; total optimized penalty cost; total earliness cost; total tardiness cost.*

## 1. INTRODUCTION

The concept of penalizing both earliness and tardiness has spawned a new and rapidly developing line of research in the scheduling field. Because the use of both earliness and tardiness penalties in fuzzy environment give rise to a non-regular performance measure, it has led to new methodological issues in the design of solution procedures, [1].

The study of earliness and tardiness penalties in scheduling models is a relatively recent area of inquiry. For many years scheduling research focused on single performance measures. Most of the literature deals with regular measure such as mean flow time mean lateness, percentage of jobs tardy, mean tardiness etc. in deterministic time but the environment in modern society is neither fixed nor probabilistic. So, here we are considering fuzzy environment i.e. the processing time of each job is in indeterminist environment, [2].

This paper is aimed to develop a heuristic algorithm to minimize the total optimized penalty cost due to earliness or lateness of sequence jobs in fuzzy environment on two machines by using Average High Ranking (AHR) for the processing time when the processing time is given in triangular fuzzy numberand the Graded Mean Integration Representation when the processing time is given in trapezoidal fuzzy number.Also we justified this algorithm by numerical examples.

Finally we can take advantage of this research and then apply it in laboratories and production plants so as to reduce the time and increase production.

## 2. FUZZY PROCESSING TIME

The processing time of a job can vary in many ways, may be due to environmental factor or due to the different work places. We find that when a contractor takes the work from a department, he calculates total expenditure at the time of allotment. But due to many factors like non available of labor, weather not favorable, or sometimes abnormal conditions, cost may vary. Hence due to these reasons work can be completed late and creates due date problem i.e. order can't be delivered on time, on the other hand if the work completes before the due time it arises the inventory problem, [3].

In this paper processing time of a job considered in in triangular fuzzy numberand trapezoidal fuzzy number.

## 3. TRIANGULAR FUZZY NUMBER

Triangular fuzzy number is a fuzzy number represented with three situations as $\tilde{A} = (a_1, a_2, a_3)$ where $a_1$ and $a_3$ denote the lower and upper limits of support of a fuzzy set $\tilde{A}$. The membership valueof the x denoted by $\mu_{\tilde{A}}(x)$, $x \in R^+$, can be calculated according to the following formula

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x \le a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & a_1 < x < a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & a_2 < x < a_3 \\ 0 & x \ge a_3 \end{cases}$$

The following figure represents a triangular fuzzy number, [4]

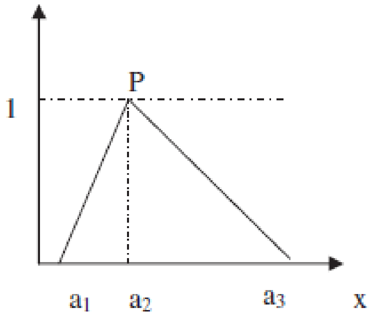


Fig.1 TriangularFuzzy Number

## 4. AVERAGE HIGH RANKING (AHR)

The processing time of the jobs are calculated by using Yager'sAverage High Ranking formula (AHR) when the processing time is given in triangular fuzzy number ($a_1,a_2,a_3$) where $a$ in favorable (High) condition, $b$ Normal (Medium) condition and $c$ in worse (Bad) condition, [5]

$$(\text{AHR}) = \frac{3a_2 + a_3 - a_1}{3}$$

## 5. TRAPEZOIDAL FUZZY NUMBER

Trapezoidal fuzzy number is a fuzzy number represented with four situations as $\tilde{A} = (a_1,a_2,a_3,a_4)$ The membership value of the x denoted by $\mu_{\tilde{A}}(x)$, x ∈ $R^+$, can be calculated according to the following formula

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x < a_1,\ x > a_4 \\ \dfrac{x - a_1}{a_2 - a_1} & a_1 \le x \le a_2 \\ 1 & a_2 \le x \le a_3 \\ \dfrac{a_4 - x}{a_4 - a_3} & a_3 \le x \le a_4 \end{cases}$$

The following figure represents a trapezoidal fuzzy number, [6]

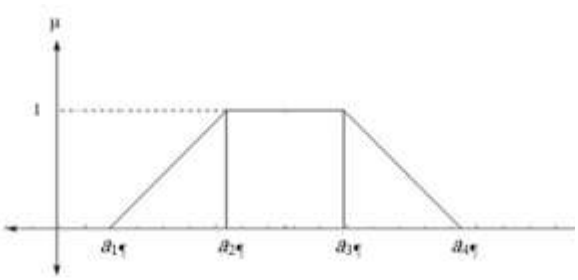


Fig.2 TrapezoidalFuzzy Number

## 6. GRADED MEAN INTEGRATION REPRESENTATION(GMIR)

The graded mean integration representation method based on the integral value of graded mean h level of generalized fuzzy number for fuzzifyinggeneralized fuzzy member. The generalized fuzzy number is defined as follows:

Suppose A is a generalized fuzzy number. It is described as any fuzzy subset of the real line R, whose membership function $\mu A$ satisfies the following conditions.

- ❖ $\mu_{\tilde{A}}(x)$ is a continuous mapping from R to [0,1],
- ❖ $\mu_{\tilde{A}}(x) = 0, -\infty < x \le \cong a_1$,
- ❖ $\mu_{\tilde{A}}(x) = L(x)$ is strictly increasing on [$a_1,a_2$],
- ❖ $\mu_{\tilde{A}}(x) = w_A$, $a_2 \le x \le a_3$,
- ❖ $\mu_{\tilde{A}}(x) = R(x)$ is strictly decreasing on [$a_3,a_4$],
- ❖ $\mu_{\tilde{A}}(x) = 0$, $a_4 \le x < \infty$,

Where $0 < w_A \le 1$ and $a_1$, $a_2$, $a_3$ and $a_4$ are real numbers.

Generalized fuzzy numbers are denoted as A = $(a_1,a_2,a_3,a_4;\ w_A)_{LR}$. The graded mean h-level value of generalized fuzzy number A = $(a_1,a_2,a_3,a_4;\ w_A)_{LR}$ is given by

$\frac{h}{2}\{L^{-1}(h) + R^{-1}(h)\}$ . Then the graded Mean Integration Representation of $P(\tilde{A})$ with grade $\mu_{\tilde{A}}(x)$ , where

$$P(\tilde{A}) = \frac{\int_0^{w_A} \frac{h}{2}\{L^{-1}(h) + R^{-1}(h)\}dh}{\int_0^{w_A} h\,dh}$$

Where $0 < h \le w_A$ and $0 < w_A \le 1$.

Let A be a trapezoidal fuzzy number and denoted as A = $(a_1,a_2,a_3,a_4)$. Then we can get the graded mean Integration Representation of A by the formula, [7]

$$(\text{GMIR}) = P(\tilde{A}) = \frac{\int_0^1 \frac{h}{2}[(a_1 + a_4) + h(a_2 - a_1 - a_4 + a_3)]dh}{\int_0^{w_A} h\,dh}$$

$$= \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}$$

## 7. ASSUMPTION AND NOTATION

- AHR: Average high ranking of the processing time ($a_1$,$a_2$,$a_3$).
- GMIR:Graded mean integration representation of the processing time ($a_1$,$a_2$,$a_3$,$a_4$).
- $P_i$:Processing time.
- $A_{i1}$:Average high ranking of the processing time of i[th] job on 1[st] machine.
- $A_{i2}$:Average high ranking of the processing time of i[th] job on 2[nd] machine.
- $G_{i1}$:Graded mean integration representation of the processing time of i[th] job on1[st] machine.
- $G_{i2}$:Graded mean integration representation of the processing time of i[th] job on 2[nd] machine.
- $t_i$:Transportation time from first machine to second machine.
- $d_i$: Due date for the job i.
- $c_i$: Completion time of job i.
- $T_i$: Max.{0, $c_i - d_i$}.
- $E_i$: Max.{0, $d_i - c_i$}.
- $sl_i$: Slack time of job i.
- $e_i$: penalty per unit time for the earliness of job i.
- $l_i$: penalty per unit time for the tardiness of job i.

An important special case in the family of E/T problems involves minimizing the sum of absolute deviations of job completion time form a DDD having processing time in fuzzy environment. In particular, the objective function can be written as

$$f(s) = \sum |c_i - d_i| = \sum (E_i + T_i)$$

When we write the objective function in this form, it is clear that earliness and tardiness are penalized at the rate $e_i$and $l_i$ for all the jobs.

## 8. ALGORITHM

**Step 1:** Find average high ranking (AHR) of the fuzzy processing time of all the jobs in the triangular fuzzy number ($a_1$,$a_2$,$a_3$) by the formula [3$a_2$ + $a_3$– $a_1$]/3, we denoted it by $A_{i1}$.

Find graded mean integration representation (GMIR) of the fuzzy processing time of all the jobs in the trapezoidal fuzzy number ($a_1$,$a_2$,$a_3$,$a_4$) by the formula[$a_1$+2$a_2$ + 2$a_3$+$a_4$]/6, we denoted it by $G_{i1}$.

**Step 2:**Add transportation time as follows: $A'_{i1} = A_{i1} + t_i$ and $A'_{i2} = A_{i2} + t_i$ if the processing time in the triangular fuzzy number, and $G'_{i1} = G_{i1} + t_i$ and $G'_{i2} = G_{i2} + t_i$ if the processing time in the triangular fuzzy number.

**Step 3:**Arrange the sequence in non decreasing order of $A'_{i1}$ if the processing time in the triangular fuzzy number, and $G'_{i1}$ if the processing time in the triangular fuzzy number.

**Step 4:** Find the processing time $P_i$ of each job i.

**Step 5:**Find the slack time of all the jobs as the formula $sl_i = |P_i - d_i|$.

**Step 6:** Arrange the jobs in increasing order of their slack time. If two jobs have the same slack time then considers the jobs of lowest processing time at the earlier position.

**Step 7:** Using the sequence obtained in step 6to find the total optimized penalty of all the jobs using earliness ($e_i$) and lateness ($l_i$) penalty cost to find

## 9. NUMERICAL EXAMPLES

**Example (1):**

Consider 5-jobs with processing time in triangular fuzzy number with fixed transportation time are given from first machine to second machine and distinct due date on the machines. Penalty cost ($e_i$) for earliness and ($l_i$) lateness is also given in the following table

| Job | $M_1$ | $t_i$ | $M_1$ | $d_i$ | $e_i$ | $l_i$ |
|---|---|---|---|---|---|---|
| 1 | (8,9,10) | 4 | (8,9,10) | 19 | 2 | 3 |
| 2 | (15,14,17) | 2 | (16,17,18) | 18 | 2 | 3 |
| 3 | (8,9,10) | 5 | (18,19,20) | 17 | 2 | 3 |
| 4 | (5,6,7) | 3 | (7,5,9) | 11 | 2 | 3 |
| 5 | (9,10,11) | 6 | (8,12,10) | 19 | 2 | 3 |

**Solution:**

**Step 1:** Find (AHR) of the processing time of each job as define in step 1 of the algorithm

| Job | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $A_1$ | 29/3 | 30/3 | 20/3 | 14/3 | 22/3 |
| $A_2$ | 20/3 | 12 | 40/3 | 4 | 26/3 |

**Step 2:** Add transportation time as define in step 2 of the algorithm

| Job | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $A'_1$ | 41/3 | 36/3 | 35/3 | 23/3 | 40/3 |

| Job | $M_1$ | $t_i$ | $M_1$ | di | $e_i$ | $l_i$ |
|---|---|---|---|---|---|---|
| 1 | (8,9,11,13) | 5 | (8,10,11,12) | 18 | 2 | 3 |
| 2 | (8,9,10,11) | 3 | (10,11,12,13) | 17 | 2 | 3 |
| 3 | (6,8,9,10) | 4 | (15,18,20,22) | 15 | 2 | 3 |
| 4 | (9,10,12,14) | 2 | (6,8,9,12) | 18 | 2 | 3 |
| 5 | (10,12,13,14) | 6 | (6,7,9,11) | 19 | 2 | 3 |

| | | | | | |
|---|---|---|---|---|---|
| $A'_2$ | 32/3 | 14 | 55/3 | 7 | 44/3 |

**Step 3:** The non decreasing sequence in which jobs are processed is **4-3-2-5-1**

**Step 4:** Processing time for each job is as follows

| Job | $M_1$ | | $M_2$ | |
|---|---|---|---|---|
| | Time In | Time Out | Time In | Time Out |
| 4 | 0 | 23/3 | 23/3 | 44/3 |
| 3 | 23/3 | 58/3 | 44/3 | 99/3 |
| 2 | 58/3 | 94/3 | 99/3 | 141/3 |
| 5 | 94/3 | 134/3 | 141/3 | 185/3 |
| 1 | 134/3 | 175/3 | 185/3 | 217/3 |

| Job | $P_i$ | $d_i$ | $sl_i$ | $e_i$ | $l_i$ |
|---|---|---|---|---|---|
| 1 | 217/3 | 19 | 160/3 | 2 | 3 |
| 2 | 141/3 | 18 | 87/3 | 2 | 3 |
| 3 | 99/3 | 17 | 48/3 | 2 | 3 |
| 4 | 44/3 | 11 | 11/3 | 2 | 3 |
| 5 | 185/3 | 19 | 128/3 | 2 | 3 |

Hence $P_1$= 217/3, $P_2$= 141/3, $P_3$= 99/3, $P_4$= 44/3, $P_5$= 185/3.

| Job | $P_i$ | $d_i$ | $sl_i$ | Cost |
|---|---|---|---|---|
| 4 | 0 – 44/3 | 11 | 11/3 | (11/3)(2) = 22/3 |
| 3 | 44/3 – 143/3 | 17 | 92/3 | (92/3)(3) = 92 |
| 2 | 143/3 – 284/3 | 18 | 230/3 | (230/3)(3) = 230 |
| 5 | 284/3 – 469/3 | 19 | 412/3 | (412/3)(3) = 412 |
| 1 | 469/3 – 686/3 | 19 | 629/3 | (629/3)(3) = 629 |

**Step 5:**The slack time for each job is as follows

**Step 6:** The optimal sequence is **4-3-2-5-1**.

**Step 7:** The total flow time of the system and the total optimized penalty cost due to earliness/tardiness of the jobs.

Total optimized penalty cost = 1370.3

**Example (2):**

Consider 5-jobs with processing time in trapezoidal fuzzy number with fixed transportation time are given from first machine to second machine and distinct due date on the machines. Penalty cost ($e_i$) for earliness and ($l_i$) lateness is also given in the following table

**Solution:**

**Step 1:** Find (GMIR) of the processing time of each job as define in step 1 of the algorithm

| Job | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $G_1$ | 61/6 | 57/6 | 50/6 | 67/6 | 74/6 |
| $G_2$ | 74/6 | 69/6 | 113/6 | 52/6 | 49/6 |

**Step 2:** Add transportation time as define in step 2 of the algorithm

| Job | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $G'_1$ | 91/6 | 75/6 | 74/6 | 79/6 | 110/6 |
| $G'_2$ | 104/6 | 87/6 | 137/6 | 64/6 | 85/6 |

**Step 3:** The non decreasing sequence in which jobs are processed is **3-2-4-1-5**

**Step 4:** Processing time for each job is as follows

| Job | $M_1$ | | $M_2$ | |
|---|---|---|---|---|
| | Time In | Time Out | Time In | Time Out |
| 3 | 0 | 74/6 | 74/6 | 211/6 |
| 2 | 74/6 | 149/6 | 211/6 | 298/6 |
| 4 | 149/6 | 228/6 | 298/6 | 362/6 |
| 1 | 228/6 | 319/6 | 362/6 | 466/6 |
| 5 | 319/6 | 429/6 | 466/6 | 551/6 |

Hence $P_1$ = 466/6, $P_2$ = 298/6, $P_3$ = 211/6, $P_4$ = 362/6, $P_5$ = 551/6.

**Step 5:** The slack time for each job is as follows

| Job | $P_i$ | $d_i$ | $sl_i$ | $e_i$ | $l_i$ |
|---|---|---|---|---|---|
| 1 | 466/6 | 18 | 358/6 | 2 | 3 |
| 2 | 298/6 | 17 | 196/6 | 2 | 3 |
| 3 | 211/6 | 15 | 121/6 | 2 | 3 |
| 4 | 362/6 | 18 | 254/6 | 2 | 3 |
| 5 | 551/6 | 19 | 437/6 | 2 | 3 |

**Step 6:** The optimal sequence is **3-2-4-1-5**.

**Step 7:** The total flow time of the system and the total optimized penalty cost due to earliness/tardiness of the jobs.

| Job | $P_i$ | $d_i$ | $sl_i$ | Cost |
|---|---|---|---|---|
| 3 | 0 – 211/6 | 18 | 103/6 | (103/6)(2) = 103/3 |
| 2 | 211/6 – 509/6 | 17 | 407/6 | (407/6)(3) = 407/2 |
| 4 | 509/6 – 871/6 | 15 | 781/6 | (781/6)(3) = 781/2 |
| 1 | 871/6 – 1337/6 | 18 | 1229/6 | (1229/6)(3) = 1229/2 |
| 5 | 1337/6 – 1888/6 | 19 | 1774/6 | (1774/6)(3) = 1774/2 |

Total optimized penalty cost = 20963.16

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