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# NEUTROSOPHIC FILTERS

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## ABSTRACT

In this paper we introduce the notion of filters on neutrosophic set which is considered as a generalization of fuzzy filters studies in [6], the important neutrosophic filters has been given. Several relations between different neutrosophic filters and neutrosophic topologies are also studied here. Possible applications to computer sciences are touched upon.

**KEYWORDS:** Fuzzy Filters, Neutrosophic Sets, Neutrosophic Filters, Neutrosophic Topology, Neutrosophic Ultrafilters

## INTRODUCTION

The fuzzy set was introduced by Zadeh [11] in 1965, where each element had a degree of membership. The intuitionstic fuzzy set (Ifs for short) on a universe X was introduced by K. Atanassov [1, 2, 3] as a generalization of fuzzy set, where besides the degree of membership and the degree of non- membership of each element. After the introduction of the neutrosophic set concept [8, 9, 10].

The fundamental concepts of neutrosophic set, introduced by Smarandache in 2002 [7, 8] and Salama in 2012[10], provides a natural foundation for treating mathematically the neutrosophic phenomena which exist pervasively in our real world and for building new branches of neutrosophic mathematics. Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their classical and fuzzy counterparts[1, 2, 3, 4, 5, 7, 11], such as a neutrosophic set theory

## PRELIMINARIES

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [8, 9], Atanassov in [1, 2, 3], Salama [10] and Kul Hur at el [6]. Smarandache introduced the neutrosophic components T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where $]^-0, 1^+[$ is nonstandard unit interval.

**Definition 2.1.** [10]

Let T, I, F be real standard or nonstandard subsets of $[^-0, 1^+]$, with

Sup_T=t_sup, inf_T=t_inf

Sup_I=i_sup, inf_I=i_inf

Sup_F=f_sup, inf_F=f_inf

n-sup=t_sup+i_sup+f_sup

n-inf=t_inf+i_inf+f_inf,

T, I, F are called neutrosophic components

**Definition 2.2**. [10]

Let X be a non-empty fixed set. $A$ neutrosophic set ( $NS$ for short or ( $(A \in N^X)$ ) $A$ is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ Where $\mu_A(x), \sigma_A(x)$ and $\gamma_A(x)$ which represent the degree of member ship function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$), and the degree of non-member ship (namely $\gamma_A(x)$) respectively of each element $x \in X$ to the set $A$ .

**Definition 2.3** [10] The NSs $0_N$ and $1_N$ in $X$ as follows:

$0_N$ may be defined as:

$(0_1)$ $0_N = \{\langle x, 0, 0, 1\rangle : x \in X\}$

$(0_2)$ $0_N = \{\langle x, 0, 1, 1\rangle : x \in X\}$

$(0_3)$ $0_N = \{\langle x, 0, 1, 0\rangle : x \in X\}$

$(0_4)$ $0_N = \{\langle x, 0, 0, 0\rangle : x \in X\}$

$1_N$ may be defined as:

$(1_1)$ $1_N = \{\langle x, 1, 0, 0\rangle : x \in X\}$

$(1_2)$ $1_N = \{\langle x, 1, 0, 1\rangle : x \in X\}$

$(1_3)$ $1_N = \{\langle x, 1, 1, 0\rangle : x \in X\}$

$(1_4)$ $1_N = \{\langle x, 1, 1, 1\rangle : x \in X\}$

## BASIC PROPERTIES OF NEUTROSOPHIC FILTERS

**Definition 3.1.** Let $N$ be a neutrosophic subsets in a set X. Then $N$ is called a neutrosophic filter on X, if it satisfies the following conditions:

$(N_1)$ Every neutrosophic set in X containing a member of $N$ belongs to $N$ .

$(N_2)$ Every finite intersection of members of $N$ belongs to $N$ .

$(N_3)$ $O_N$ not in $N$ .

In this case, the pair $(X, N)$ is called neutrosophic filtered by $N$ .

It follows from $(N_2)$ and $(N_3)$ that every finite intersection of members of $N$ is not $O_N$ . Furthermore, there is no neutrosophic set. We obtain the following results.

**Proposition 3.1.** The condition $(N_2)$ is equivalent to the following two conditions

$(N_{2a})$ The intersection of two members of $N$ belongs to $N$ .

$(N_{2b})$ $1_N$ belongs to $N$

**Proposition 3.2.** Le $N$ be a non-empty neutrosophic subsets in X satisfying $(N_1)$.Then,

- $1_N \in N$ iff $N \neq O_N$
- $O_N \notin N$ iff $N \neq$ all neutrosophic subsets of X.

From the above Propositions (3.1) and (3.2), we can characterize the concept of neutrosophic filter:

**Theorem 3.1.** Let $N$ be a neutrosophic subsets in a set X. Then $N$ is neutrosophic filter on X, if and only if it is satisfies the following conditions

- Every neutrosophic set in X containing a member of $N$ belongs to $N$.
- If $A, B \in N$, then $A \cap B \in N$.
- $N^X \neq N \neq O_N$.

**Proof:** It's clear.

**Theorem 3.2.** Let $X \neq \phi$. Then the set $\{1_N\}$ is a neutrosophic filter on X. Moreover if A is a non-empty neutrosophic set in X, then $\{B \in N^X : A \subseteq B\}$ is a neutrosophic filter on X

**Proof:** Let. $N = \{B \in N^X : A \subseteq B\}$. Since $1_N \in N$ and $O_N \notin N$, $O_N \neq N \neq N^X$. Suppose $U, V \in N$, then $A \subseteq U, A \subseteq V$. Thus $\mu_A(x) \leq \min(\mu_U(x), \mu_V(x))$, $\sigma_A(x) \leq \min(\sigma_U(x), \sigma_V(x))$ or $\sigma_A(x) \leq \max(\sigma_U(x), \sigma_V(x))$ and $\gamma_A(x) \leq \max(\gamma_U(x), \gamma_V(x))$ for all $x \in X$. So $A \subseteq U \cap V$ and hence $U \cap V \in N$.

## COMPARISON OF NEUTROSOPHIC FILTERS

**Definition 4.1.** Let $N_1$ and $N_2$ be two neutrosophic filters on a set X. Then $N_2$ is said to be finer than $N_1$ or $N_1$ coarser than $N_2$ if $N_1 \subset N_2$

If also $N_1 \neq N_2$, then $N_2$ is said to be strictly finer than $N_1$ or $N_1$ is strictly coarser than $N_2$.

Two neutrosophic filters are said to be comparable, if one is finer than the other. The set of all neutrosophic filters on X is ordered by the relation $N_1$ is coarser than $N_2$, this relation is induced the inclusion relation in $N^X$.

**Proposition 4.1**. Let $(N_j)_{j \in J}$ be any non-empty family of neutrosophic filters on X. Then $N = \cap_{j \in J} N_j$ is a neutrosophic filter on X. In fact $N$ is the greatest lower bound of the neutrosophic set $(N_j)_{j \in J}$ in the ordered set of all neutrosophic filters on X.

**Remark 4.1**.The neutrosophic filter by the single neutrosophic set $1_N$ is the smallest element of the ordered set of all neutrosophic filters on X.

**Theorem 4.1.**Let $A$ be a neutrosophic sets in X. Then there exists a nutrosophic filter $N(A)$ on X containing $A$ iff for any finite subset $\{S_1, S_2, ..., S_n\}$ of $A$, $\cap_{i=1} S_i \neq O_N$. In fact $N(A)$ is the coarest neutrosophic filter containing $A$.

**Proof** ($\Rightarrow$) Suppose there exists a nutrosophic filter $N(A)$ on X containing $A$. Let B be the set of all the finite intersections of members of $A$. Then by $(N_2)$, $B \subset N(A)$. By $(N_3)$, $O_N \notin N(A)$. Thus for each member $B$ of B, Hence the necessary condition holds

($\Leftarrow$) Suppose the necessary condition holds. Let $N(A) = \{A \in N^X : A \text{ contains a member of } B\}$. Where B is the family of all the finite intersections of members of A. Then we can easily check that $N(A)$ satisfies the conditions in Definition3.1

The neutrosophic filter $N(A)$ defined above is said to be generated by A and A is called a sub - base of $N(A)$.

**Corollary 4.1.** Let $N$ be a neutrosophic filter in a set X and A neutrosophic set. Then there is a neutrosophic filter $N^{/}$ which is finer than $N$ and such that $A \in N^{/}$ iff and A neutrosophic set. Then there is a neutrosophic filter $N^{/}$ which is finer than $N$ and such that $A \in N^{/}$ iff $A \cap U \neq O_N$ for each $U \in N$.

**Corollary 4.2** A set $\varphi$ of a neutrosophic filter on a non-empty set X, has a least upper bound in the set of all neutrosophic filters on X iff for all finite sequence $(N_j)_{j \in J}, 0 \leq j \leq n$ of elements of $\varphi$ and all $A_j \in N_j (1 \leq j \leq n), \cap_{j=1} A_j \neq O_N$

**Corollary 4.3**.The ordered set of all neutrosophic filters on a non-empty set X inductive.

If $\Lambda$ is a sub base of a neutrosophic filter $N$ on X, then $N$ is not in general the set of neutrosophic sets in X containing an element of $\Lambda$; for $\Lambda$ to have this property it is necessary and sufficient that every finite intersection of members of $\Lambda$ should contain an element of $\Lambda$. Hence we have the following result:

**Theorem 4.2.** Let $\beta$ is a set of neutrosophic sets on a set X. Then the set of neutrosophic sets in X containing an element of $\beta$ is a neutrosophic filter on X iff $\beta$ has the following two conditions

$(\beta_1)$ The intersection of two members of $\beta$ contain a member of $\beta$.

$(\beta_2)$ $\beta \neq O_N$ and $O_N \notin \beta$.

**Definition 4.2.** Let $\Lambda$ and $\beta$ are neutrosophic sets on X satisfying conditions $(\beta_1)$ and $(\beta_2)$ is called a base of neutrosophic filter it generates. Two neutrosophic bases are said to be equivalent, if they generate the same neutrosophic filter.

**Remark 4.2.**Let $\Lambda$ be a subbase of neutrosophic filter $N$. Then the set $\beta$ of finite intersections of members of $\Lambda$ is a base of filter $N$.

**Proposition 4.2.** A subset $\beta$ of a neutrosophic filter $N$ on X is a base of $N$ iff every member of $N$ contains a member of $\beta$.

**Proof** ($\Rightarrow$) Suppose $\beta$ is a base of $N$. Then clearly, every member of $N$ contains an element of $\beta$. ($\Leftarrow$) Suppose the necessary condition holds. Then the set of neutrosophic sets in X containing a member of $\beta$ coincides with $N$ by reason of $(N_j)_{j \in J}$.

**Proposition 4.3.** On a set X, a neutrosophic filter $N^{/}$ with base $\beta^{/}$ is finer than a neutrosophic filter $N$ with base $\beta$ iff every member of $\beta$ contains a member of $\beta^{/}$.

**Proof** This is an immediate consequence of Definitions 4.2 and 4.4.

**Proposition 4.4.** Two neutrosophic filters bases $\beta$ and $\beta^{/}$ on a set X are equivalent iff every member of $\beta$ contains a member of $\beta^{/}$ and every member of $\beta^{/}$ and every member of $\beta^{/}$ contains a member of $\beta$.

## NEUTROSOPHIC ULTRAFILTERS

**Definition 5.1.** A neutrosophic ultrafilter on a set X is a neutrosophic filter $N$ such that there is no neutrosophic filter on X which is strictly finer than $N$ ( in other words, a maximal element in the ordered set of all neutrosophic filters on X).

Since the ordered set of all the neutrosophic filters on X inductive, Zorn's lemma shows that

**Theorem 5.1.**If $N$ be any neutrosophic ultrafilter on a set X, then there is a neutrosophic ultrafilter than $N$.

**Propostition 5.1.** Let $N$ be a neutrosophic ultrafilter on a set X. If $A$ and $B$ are two neutrosophic subsets such that $A \cup B \in N$, then $A \in N$ or $B \in N$.

**Proof**: Suppose not. Then there exist neutrosophic sets $A$ and $B$ in X such that $A \notin N, B \notin N$ and $A \cup B \in N$. Let $\Lambda = \{M \in N^{X} : A \cup M \in N\}$. It is straightforward to check that $\Lambda$ is a neutrosophic filter on X, and $\Lambda$ is strictly finer than $N$, since $B \in \Lambda$.This contradiction the hypothesis that $N$ is a neutrosophic ultrafilter.

**Corollary 5.1**. Let $N$ be a neutrosophic ultrafilter on a set X and let $(N_j)_{1 \le j \le n}$ be a finite sequence of neutrosophic sets in X. If $\underset{j=1}{\cup} N_j \in N$, then at least one of the $N_j$ belongs to $N$.

**Definition 5.2**. Let $A$ be a neutrosophic set in a set X. If $U$ is any neutrosophic set in X, then the neutrosophic set $A \cap U$ is called trace of $U$ an A and denoted by $U_A$. For all neutrosophic sets $U$ and $V$ in X, we have $(U \cap V)_A = U_A \cap V_A$.

**Definition 5.3**. Let $A$ be a neutrosophic set in a set X. Then the set $\Lambda_A$ of traces an $A \in N^{X}$ of member of $\Lambda$ is called the trace of $\Lambda$ an $A$.

**Propsition 5.2**. Let $N$ be a neutrosophic filter on a set X and $A \in N^{X}$. Then the trace of $N_A$ of $N$ an $A$ is a neutrosophic filter iff each member of $N$ meets $A$.

**Proof**. From the result in Definition5.3, we see that $N_A$ satisfies $(N_2)$.If $M \cap A \subset P \subset A$, then $P = (M \cup P) \cap A$. Thus $N_A$ satisfies $(N_1)$. Hence $N_A$ is a neutrosophic filter iff it satisfies $(N_3)$. i.e. iff each member of $N$ meets $A$.

**Definition 5**.4. Let $N$ be a neutrosophic filter on a set X and $A \in N^{X}$. If the trace $N_A$ of $N$ an $A$, then $N_A$ is said to be induced by $N$ an $A$.

**Propsition 5.3.** Let $N$ be a neutrosophic filter on a set X induced a neutrosophic filter $N_A$ on $A \in N^{X}$. Then trace $\beta_A$ on $A$ of a base $\beta$ of $N$ is a base of $N_A$.

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