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ORIGINAL ARTICLE

A new modification of variational iteration method for solving reaction–diffusion system with fast reversible reaction

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Abstract This paper presents a new modification of He's variational iteration method using Adomian's polynomials (VIMAP) to solve reaction–diffusion system with fast reversible reaction. An auxiliary parameter is introduced into the VIMAP and optimally identified to adjust the convergence region of the approximate solution. The results reveal that the VIMAP is very accurate comparing with those obtained by the VIM but is not valid for large solution domain, while the new modification have a remarkable accuracy for large domains.

2010 MATHEMATICS SUBJECT CLASSIFICATION: 35K57; 49J40; 49M27; 65M99; 65D99

1. Introduction

Nonlinear phenomena are of fundamental importance in various fields of science and engineering. The nonlinear models of real-life problems are still difficult to solve either numerically or theoretically. There has recently been much attention devoted to the search for better and more efficient solution methods for determining a solution, approximate or exact, analytical or numerical, to nonlinear models [1].

Many promising numeric–analytic methods have been proposed recently such as the variational iteration method (VIM) by He [2–5] and the Adomian's decomposition method (ADM) [6–9]. In recent years, many authors have successfully applied the VIM [10–13] to solve a wide variety of linear and nonlinear problems with approximations converging rapidly to accurate solutions. With the passage of time some modifications in He's variational iteration method (VIM) has been introduced by various authors [14–28].

The reaction diffusion equations (RDEs) have recently attracted considerable attention, partly due to their occurrence in many fields of science, in physics as well as in chemistry or biology, partly due to their interesting features and rich variety of properties of their solutions [29].

Recently, Eymard et al. [30] studied the numerical solution of the reaction–diffusion system with fast reversible chemical reaction of type $mA \rightleftharpoons nB$ by using the finite volume method.

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Also, Al-Sawoor and Al-Amr [31] applied the VIM and the ADM to solve this system and compared the obtained results.

The motivation of this paper is to present a new modification of the variational iteration method using Adomian's polynomials (VIMAP) by introducing an optimal auxiliary parameter into the VIMAP. The VIMAP and its modification are successfully applied to solve reaction–diffusion system which describes a reversible chemical reaction. Comparisons are made between the standard VIM, the VIMAP and the proposed method.

In this work, we consider a reversible chemical reaction between mobile species A and B, that takes place inside a bounded region $\Omega \subset \mathbb{R}$, we have the reaction–diffusion system of partial differential equations [30,32]:

$$\begin{aligned} u_t &= a\Delta u - \alpha k(r_A(u) - r_B(v)), \quad \text{in } \Omega \times (0,T), \\ v_t &= b\Delta v + \beta k(r_A(u) - r_B(v)), \quad \text{in } \Omega \times (0,T), \end{aligned} \tag{1}$$

where $T > 0$ and Ω is a bounded set of $\mathbb{R}$, with the boundary conditions

$$\nabla u \cdot n = \nabla v \cdot n = 0, \quad \text{on } \partial\Omega \times (0,T), \tag{2}$$

and the initial conditions

$$u(x,0) = u_0(x), \quad v(x,0) = v_0(x), \quad \text{in } \Omega. \tag{3}$$

For a reversible reaction $\alpha A \underset{k_2}{\overset{k_1}{\rightleftharpoons}} \beta B$, the rate functions are of the form $r_A(u) = k_1 u^\alpha$ and $r_B(u) = k_2\, v^\beta$, where k_1 and k_2 are rate constants, a and b are diffusion coefficients and k is the chemical kinetics factor (for further details see [33,34]).

2. Variational iteration method using Adomian's polynomials (VIMAP)

To illustrate the methodology of the VIMAP, we first consider the system of partial differential equations written in an operator form

$$\begin{aligned} L_t u + R_1(u,v) + N_1(u,v) &= g_1, \\ L_t v + R_2(u,v) + N_2(u,v) &= g_2, \end{aligned} \tag{4}$$

with initial data

$$\begin{aligned} u(x,0) &= f_1(x), \\ v(x,0) &= f_2(x), \end{aligned} \tag{5}$$

where L_t is considered, without loss of generality, a first order partial differential operator, R_1 and R_2 are linear operators, N_1 and N_2 are nonlinear operators, and g_1 and g_2 are inhomogeneous terms.

According to the VIM, we can construct a correctional functional as follows [11,13]:

$$\begin{aligned} u_{n+1}(x) &= u_n(x) + \int_0^x \lambda_1 [L_t u_n(\tau) + R_1(\tilde{u}_n, \tilde{v}_n) + N_1(\tilde{u}_n, \tilde{v}_n) - g_1(\tau)] d\tau, \\ v_{n+1}(x) &= v_n(x) + \int_0^x \lambda_2 [L_t v_n(\tau) + R_2(\tilde{u}_n, \tilde{v}_n) + N_2(\tilde{u}_n, \tilde{v}_n) - g_2(\tau)] d\tau, \end{aligned} \tag{6}$$

where λ_1 and λ_2 are general Lagrange multipliers, which can be identified optimally via the variational theory [5], the subscript n denotes the nth order approximation, $\tilde{u}_n$ and $\tilde{v}_n$ are considered as restricted variations, i.e., $\delta\tilde{u}_n = 0$ and $\delta\tilde{v}_n = 0$.

The ADM assumes a series that the unknown functions $u(x,t)$ and $v(x,t)$ can be expressed by an infinite series of the form [16,17]

$$\begin{aligned} u(x,t) &= \sum_{k=0}^{\infty} u_k(x,t), \\ v(x,t) &= \sum_{k=0}^{\infty} v_k(x,t). \end{aligned} \tag{7}$$

And the nonlinear operators $N_1(u,v)$ and $N_2(u,v)$ can be decomposed by the infinite series of the so-called Adomian polynomials

$$\begin{aligned} N_1(u,v) &= \sum_{k=0}^{\infty} A_k, \\ N_2(u,v) &= \sum_{k=0}^{\infty} B_k. \end{aligned} \tag{8}$$

The Adomian polynomials A_k and B_k are generated according to the following algorithms [16]

$$\begin{aligned} A_k &= \frac{1}{k!}\left[\frac{\partial^k}{\partial\lambda^k} N_1\left(\sum_{i=0}^{\infty}\lambda^i u_i, \sum_{i=0}^{\infty}\lambda^i v_i\right)\right]_{\lambda=0}, \\ B_k &= \frac{1}{k!}\left[\frac{\partial^k}{\partial\lambda^k} N_2\left(\sum_{i=0}^{\infty}\lambda^i u_i, \sum_{i=0}^{\infty}\lambda^i v_i\right)\right]_{\lambda=0}, \quad k > 0. \end{aligned} \tag{9}$$

Substituting Eqs. (7) and (8) into the variational iteration formula (6), we obtain

$$\begin{aligned} u_{n+1}(x) &= \int_0^x \lambda_1 \left[\sum_{k=0}^{n} L_t u_k(\tau) + R_1\left(\sum_{k=0}^{n} u_k, \sum_{k=0}^{n} v_k\right) + \sum_{k=0}^{n} A_k - g_1(\tau)\right] d\tau, \\ v_{n+1}(x) &= \int_0^x \lambda_2 \left[\sum_{k=0}^{n} L_t v_k(\tau) + R_2\left(\sum_{k=0}^{n} u_k, \sum_{k=0}^{n} v_k\right) + \sum_{k=0}^{n} B_k - g_2(\tau)\right] d\tau. \end{aligned} \tag{10}$$

The successive approximations $u_{n+1}(x,t)$, $v_{n+1}(x,t)$, $n \geqslant 0$, of the solutions $u(x,t)$ and $v(x,t)$ will be readily obtained by using selected functions f_1 and f_2. Consequently, the solutions are given by Eqs. (7).

3. Variational iteration method using Adomian's polynomials with an optimal auxiliary parameter (VIMAPOAP)

We assume that an unknown auxiliary parameter h can be inserted into the correction functional (10) of VIMAP, so that we obtain

$$\begin{aligned} u_{n+1}(x,h) &= h\int_0^x \lambda_1 \left[\sum_{k=0}^{n} L_t u_k(\tau) + R_1\left(\sum_{k=0}^{n} \tilde{u}_k, \sum_{k=0}^{n} \tilde{v}_k\right) + \sum_{k=0}^{n} A_k - g_1(\tau)\right] d\tau, \\ v_{n+1}(x,h) &= h\int_0^x \lambda_2 \left[\sum_{k=0}^{n} L_t v_k(\tau) + R_2\left(\sum_{k=0}^{n} \tilde{u}_k, \sum_{k=0}^{n} \tilde{v}_k\right) + \sum_{k=0}^{n} B_k - g_2(\tau)\right] d\tau. \end{aligned} \tag{11}$$

The auxiliary parameter h can be determined by means of the so-called h-curve and the error of norm 2 of the residual functions to ensures that the approximations $u_n(x,h)$, $v_n(x,h)$, $n \geqslant 1$, that contain the auxiliary parameter h, converge to the exact solutions. In fact, the proposed method gives a simple and a powerful mathematical tool for nonlinear problems and is cable to approximate the solution more accurately in a large solution domain.

4. Numerical experiment

We consider the reaction of the reversible dimerization of *o*-phenylenedioxy-dimethylsilane (for further details see [35]). Since the reaction is of the type $2A \underset{k_2}{\overset{k_1}{\rightleftharpoons}} B$, the reaction terms take the form $r_A(u) = k_1u^2$ and $r_B(v) = k_2v$ [30]. Moreover, $\alpha = 2$ and $\beta = 1$. So that, we have the following reaction–diffusion system:

$$\begin{aligned} u_t &= au_{xx} - 2k(k_1u^2 - k_2v), \quad \text{in } [0,0.1]\times(0,T),\\ v_t &= bv_{xx} + k(k_1u^2 - k_2v), \quad \text{in } [0,0.1]\times(0,T), \end{aligned} \tag{12}$$

and the initial conditions are defined as follows:

$$u_0(x) = \begin{cases} 0 & \text{for } x\in[0,0.03],\\ \frac{1}{2}\sin\left(\frac{50\pi}{7}(x-0.03)\right) & \text{for } x\in[0.03,0.1], \end{cases} \tag{13}$$

$$v_0(x) = \begin{cases} \frac{1}{4}\cos\left(\frac{50\pi}{7}x\right) & \text{for } x\in[0,0.07],\\ 0 & \text{for } x\in[0.07,0.1]. \end{cases} \tag{14}$$

The rate constants for both reactions can be estimated at the temperature 298 K,

$$k_1 \approx 1.072\cdot 10^{-4}\ \mathrm{L^2\ mol^{-2}} \quad \text{and} \quad k_2 \approx 2.363\cdot 10^{-6}\ \mathrm{L^2\ mol^{-2}},$$

and diffusion coefficients

$$a \approx 1.579\cdot 10^{-9}\ \mathrm{m^2\ s^{-1}} \quad \text{and} \quad b \approx 1.042\cdot 10^{-9}\ \mathrm{m^2\ s^{-1}}.$$

To solve the system (12) by means of the VIMAP, we construct correctional functionals which read

$$\begin{aligned} u_{n+1}(x,t) &= u_n(x,t) + \int_0^t \lambda_1\left[u_{n\tau} - au_{nxx} + 2kk_1\tilde{u}_n^2 - 2kk_2\tilde{v}_n\right]d\tau,\\ v_{n+1}(x,t) &= v_n(x,t) + \int_0^t \lambda_2\left[v_{n\tau} - bv_{nxx} - kk_1\tilde{u}_n^2 + kk_2\tilde{v}_n\right]d\tau. \end{aligned} \tag{15}$$

Its stationary conditions can be obtained as follows:

$$\begin{aligned} \lambda_1'(\tau) &= 0, \quad 1 + \lambda_1(\tau)|_{\tau=t} = 0,\\ \lambda_2'(\tau) &= 0, \quad 1 + \lambda_2(\tau)|_{\tau=t} = 0. \end{aligned}$$

The Lagrange multipliers, therefore, can be identified as $\lambda_1 = \lambda_2 = -1$, and the iteration formulas are given by

$$\begin{aligned} u_{n+1}(x,t) &= u_n(x,t) - \int_0^t \left[u_{n\tau} - au_{nxx} + 2kk_1u_n^2 - 2kk_2v_n\right]d\tau,\\ v_{n+1}(x,t) &= v_n(x,t) - \int_0^t \left[v_{n\tau} - bv_{nxx} - kk_1u_n^2 + kk_2v_n\right]d\tau. \end{aligned} \tag{16}$$

We assume that the unknown functions $u(x,t)$ and $v(x,t)$ can be expressed by

$$\begin{aligned} u(x,t) &= \sum_{k=0}^{\infty} u_k(x,t),\\ v(x,t) &= \sum_{k=0}^{\infty} v_k(x,t), \end{aligned} \tag{17}$$

and the nonlinear terms can be decomposed by the Adomian polynomials

$$N(u) = u^2 = \sum_{k=0}^{\infty} A_k, \tag{18}$$

where the Adomian polynomials A_k can be generated by (9).

Substituting Eqs. (17) and (18) into the iteration formulas (16), we obtain

$$\begin{aligned} u_{n+1}(x,t) &= -\int_0^t \left[\sum_{k=0}^{n} u_{k\tau} - a\sum_{k=0}^{n} u_{kxx} + 2kk_1\sum_{k=0}^{n} A_k - 2kk_2\sum_{k=0}^{n} v_k\right]d\tau,\\ v_{n+1}(x,t) &= -\int_0^t \left[\sum_{k=0}^{n} v_{k\tau} - b\sum_{k=0}^{n} v_{kxx} - kk_1\sum_{k=0}^{n} A_k + kk_2\sum_{k=0}^{n} v_k\right]d\tau. \end{aligned} \tag{19}$$

The first three components of Adomian polynomials read

$$\begin{aligned} A_0 &= u_0^2,\\ A_1 &= 2u_0u_1,\\ A_2 &= 2u_0u_2 + u_1^2. \end{aligned}$$

The solutions are obtained using the initial conditions only. Consequently, the pair of zeroth components is given by

$$u_0(x,t) = \begin{cases} u_{01}(x,t) & \text{for } x\in[0,0.03],\\ u_{02}(x,t) & \text{for } x\in[0.03,0.07],\\ u_{03}(x,t) & \text{for } x\in[0.07,0.1], \end{cases} \tag{20}$$

$$\begin{aligned} u_{01}(x,t) &= 0,\\ u_{02}(x,t) &= u_{03}(x,t) = \frac{1}{2}\sin\left(\frac{50\pi}{7}(x-0.03)\right), \end{aligned}$$

$$v_0(x,t) = \begin{cases} v_{01}(x,t) & \text{for } x\in[0,0.03],\\ v_{02}(x,t) & \text{for } x\in[0.03,0.07],\\ v_{03}(x,t) & \text{for } x\in[0.07,0.1], \end{cases} \tag{21}$$

$$\begin{aligned} v_{01}(x,t) &= v_{02}(x,t) = \frac{1}{4}\cos\left(\frac{50\pi}{7}x\right),\\ v_{03}(x,t) &= 0, \end{aligned}$$

and the pair of first components read

$$u_1(x,t) = \begin{cases} u_{11}(x,t) & \text{for } x\in[0,0.03],\\ u_{12}(x,t) & \text{for } x\in[0.03,0.07],\\ u_{13}(x,t) & \text{for } x\in[0.07,0.1], \end{cases}$$

$$\begin{aligned} u_{11}(x,t) &= \tfrac{kk_2}{2}\cos\left(\tfrac{50\pi}{7}x\right)t,\\ u_{12}(x,t) &= -\tfrac{1250}{49}\pi^2a\sin\left(\tfrac{50\pi}{7}(x-0.03)\right)t\\ &\quad -\tfrac{kk_1}{2}\sin^2\left(\tfrac{50\pi}{7}(x-0.03)\right)t + \tfrac{kk_2}{2}\cos\left(\tfrac{50\pi}{7}x\right)t,\\ u_{13}(x,t) &= -\tfrac{1250}{49}\pi^2a\sin\left(\tfrac{50\pi}{7}(x-0.03)\right)t - \tfrac{kk_1}{2}\sin^2\left(\tfrac{50\pi}{7}(x-0.03)\right)t, \end{aligned}$$

$$v_1(x,t) = \begin{cases} v_{11}(x,t) & \text{for } x\in[0,0.03],\\ v_{12}(x,t) & \text{for } x\in[0.03,0.07],\\ v_{13}(x,t) & \text{for } x\in[0.07,0.1], \end{cases}$$

$$\begin{aligned} v_{11}(x,t) &= -\tfrac{625}{49}\pi^2b\cos\left(\tfrac{50\pi}{7}x\right)t - \tfrac{kk_2}{4}\cos\left(\tfrac{50\pi}{7}x\right)t,\\ v_{12}(x,t) &= -\tfrac{625}{49}\pi^2b\cos\left(\tfrac{50\pi}{7}x\right)t + \tfrac{kk_1}{4}\sin^2\left(\tfrac{50\pi}{7}(x-0.03)\right)t\\ &\quad -\tfrac{kk_2}{4}\cos\left(\tfrac{50\pi}{7}x\right)t,\\ v_{13}(x,t) &= \tfrac{kk_1}{4}\sin^2\left(\tfrac{50\pi}{7}(x-0.03)\right)t. \end{aligned}$$

And so on, the rest of components can be easily obtained.

A comparison between the numerical results of the proposed method with those obtained by VIM [31] is given by Table 1. The results show that the two solutions obtained

Table 1 Comparison of the approximate values of $u(x,t)$ and $v(x,t)$ obtained by using three iterations of VIMAP and VIM for $k = 1$ at time $t = 1$.

x	u_{VIMAP}	u_{VIM}[31]	v_{VIMAP}	v_{VIM}[31]
0	1.18149782428374e−06	1.18149782428374e−06	0.249999278075943	0.249999278075943
0.02	1.06449275718299e−06	1.06449275718299e−06	0.225241566544505	0.225241566544505
0.04	0.111258461144262	0.111258461144262	0.155873327356743	0.155873327356743
0.06	0.311724080934518	0.311724080934518	0.0556404903603831	0.0556404903603832
0.08	0.450440570555619	0.450440570555619	2.17526106277004e−05	2.17526106276869e−05
0.1	0.499946008340295	0.499946008340295	2.67970603236263e−05	2.67970603236036e−05

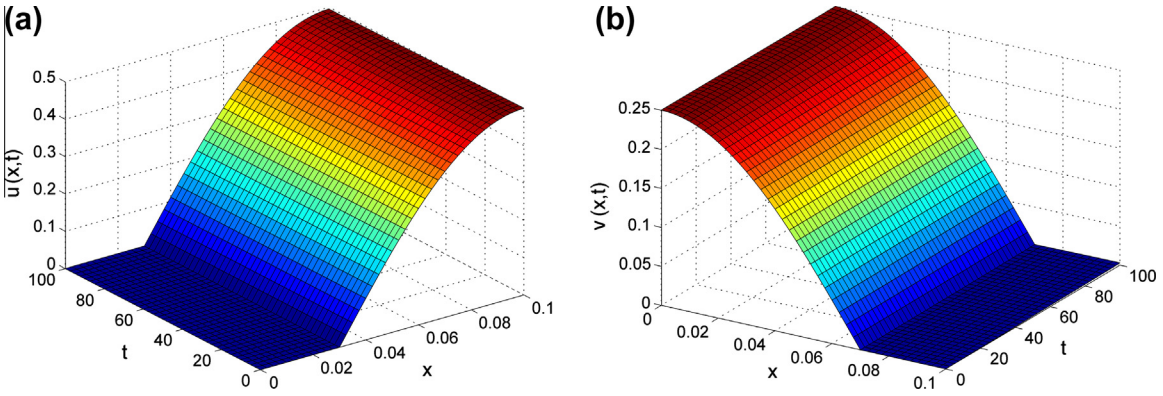

Fig. 1 The numerical results for: (a) $u(x,t)$ and (b) $v(x,t)$ by VIMAP for different values of t with $k = 1$.

are in excellent agreement (see Fig. 1). So that the VIMAP is very accurate for small solution domains.

To solve the system (12) by means of the VIMAPOAP, we propose that an unknown auxiliary parameter h can be inserted into the correction functionals (19) as follows:

$$u_{n+1}(x,t) = -h\int_0^t \left[\sum_{k=0}^{n} u_{k\tau} - a\sum_{k=0}^{n} u_{kxx} + 2kk_1\sum_{k=0}^{n} A_k - 2kk_2\sum_{k=0}^{n} v_k\right] d\tau,$$
$$v_{n+1}(x,t) = -h\int_0^t \left[\sum_{k=0}^{n} v_{k\tau} - b\sum_{k=0}^{n} v_{kxx} - kk_1\sum_{k=0}^{n} A_k + kk_2\sum_{k=0}^{n} v_k\right] d\tau. \tag{22}$$

We will find the approximations $\sum_{n=0}^{2} u_n(x,t)$ and $\sum_{n=0}^{2} v_n(x,t)$, denoted by $u^*(x,t)$ and $v^*(x,t)$ respectively, with $(x,t) \in [0,0.1] \times [0,10{,}000]$ by means of the proposed method.

We start with the initial conditions given by (20) and (21), with the above iteration formulas, we found the pair of first components

$$u_1(x,t) = \begin{cases} u_{11}(x,t) & \text{for } x \in [0,0.03], \\ u_{12}(x,t) & \text{for } x \in [0.03,0.07], \\ u_{13}(x,t) & \text{for } x \in [0.07,0.1], \end{cases}$$

$$u_{11}(x,t,h_1) = \tfrac{h_1 kk_2}{2}\cos\left(\tfrac{50\pi}{7}x\right)t,$$
$$u_{12}(x,t,h_2) = -h_2\left(\tfrac{1250}{49}\pi^2 a\sin\left(\tfrac{50\pi}{7}(x-0.03)\right)t + \tfrac{kk_1}{2}\sin^2\left(\tfrac{50\pi}{7}(x-0.03)\right)t - \tfrac{kk_2}{2}\cos\left(\tfrac{50\pi}{7}x\right)t\right),$$
$$u_{13}(x,t,h_3) = -h_3\left(\tfrac{1250}{49}\pi^2 a\sin\left(\tfrac{50\pi}{7}(x-0.03)\right)t + \tfrac{kk_1}{2}\sin^2\left(\tfrac{50\pi}{7}(x-0.03)\right)t\right),$$

$$v_1(x,t) = \begin{cases} v_{11}(x,t) & \text{for } x \in [0,0.03], \\ v_{12}(x,t) & \text{for } x \in [0.03,0.07], \\ v_{13}(x,t) & \text{for } x \in [0.07,0.1], \end{cases}$$

$$v_{11}(x,t,h_1) = -h_1\left(\tfrac{625}{49}\pi^2 b\cos\left(\tfrac{50\pi}{7}x\right)t + \tfrac{kk_2}{4}\cos\left(\tfrac{50\pi}{7}x\right)t\right),$$
$$v_{12}(x,t,h_2) = -h_2\left(\tfrac{625}{49}\pi^2 b\cos\left(\tfrac{50\pi}{7}x\right)t - \tfrac{kk_1}{4}\sin^2\left(\tfrac{50\pi}{7}(x-0.03)\right)t + \tfrac{kk_2}{4}\cos\left(\tfrac{50\pi}{7}x\right)t\right),$$
$$v_{13}(x,t,h_3) = \tfrac{h_3 kk_1}{4}\sin^2\left(\tfrac{50\pi}{7}(x-0.03)\right)t.$$

And so on. In order to find a proper value of h_1 for the approximations $\sum_{n=0}^{2} u_{n1}(x,t,h_1)$ and $\sum_{n=0}^{2} u_{n1}(x,t,h_1)$, denoted by $u_1^*(x,t,h_1)$ and $v_1^*(x,t,h_1)$ respectively, we plot the h_1-curve of $\partial u_1^*(x,t,h_1)/\partial t$ and $\partial v_1^*(x,t,h_1)/\partial t$ when $x = 0.02$ and $t = 5000$ (see Fig. 2a) to discover the valid region of h, which corresponds to the line segments nearly parallel to the horizontal axis.

According to the system (12), we can define the following residual functions:

$$r_1^*(x,t,h_1) = \tfrac{\partial u_1^*(x,t,h_1)}{\partial t} - a\tfrac{\partial^2 u_1^*(x,t,h_1)}{\partial x^2} + \alpha kk_1 u_1^{*2}(x,t,h_1) - \alpha kk_2 v_1^*(x,t,h_1),$$
$$s_1^*(x,t,h_1) = \tfrac{\partial v_1^*(x,t,h_1)}{\partial t} - b\tfrac{\partial^2 v_1^*(x,t,h_1)}{\partial x^2} - \beta kk_1 u_1^{*2}(x,t,h_1) + \beta kk_2 v_1^*(x,t,h_1).$$

To determine an optimal value of h_1, we plot the error of norm 2 of the above residual functions with respect to h_1 (see Fig. 2b), i.e.,

$$\left(\tfrac{1}{(51)^2}\sum_{i=0}^{50}\sum_{j=0}^{50}\left(r_1^*\left(\tfrac{i}{500},200j,h_1\right)\right)^2\right)^{\frac{1}{2}},$$
$$\left(\tfrac{1}{(51)^2}\sum_{i=0}^{50}\sum_{j=0}^{50}\left(s_1^*\left(\tfrac{i}{500},200j,h_1\right)\right)^2\right)^{\frac{1}{2}}.$$

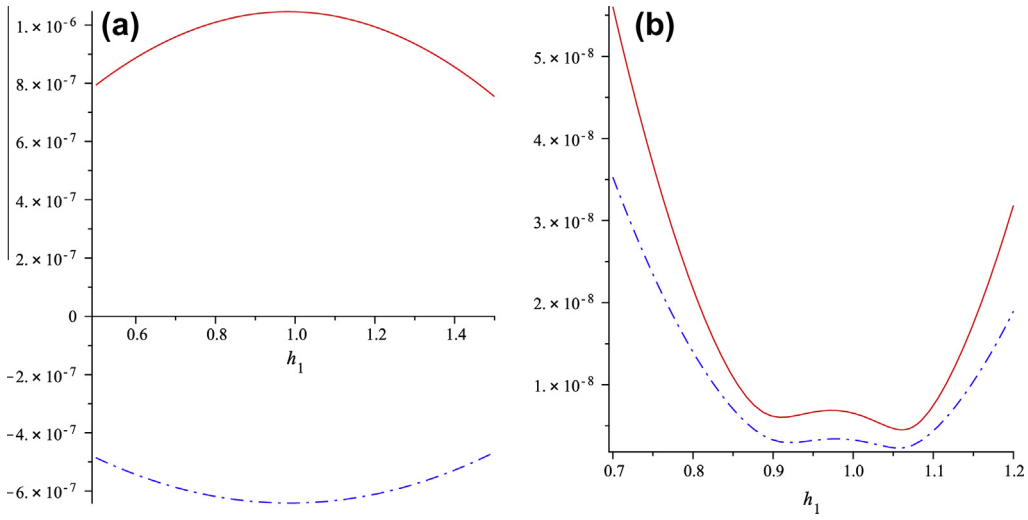

Fig. 2 (a) The h_1-curve of $\partial u_1^*(x,t,h_1)/\partial t$ (solid line) and $\partial v_1^*(x,t,h_1)/\partial t$ (dash-dot) when $x = 0.02$ and $t = 5000$. (b) The error of norm 2 of $r_1^*(x,t,h_1)$ (solid line) and $s_1^*(x,t,h_1)$ (dash-dot) with respect to h_1.

Table 2 Comparison of the approximate values of $u(x,t)$ and $v(x,t)$ obtained by using two iterations of VIMAP and VIMAPOAP for $k = 1$ at time $t = 10{,}000$.

x	u_{VIMAP}	$u_{VIMAPOAP}$	v_{VIMAP}	$v_{VIMAPOAP}$
0	0.0115974381284457	0.0115456005366113	0.242884984176699	0.242913716409086
0.025	0.00981983151240772	0.00977593936897188	0.205656593731590	0.205680922008050
0.05	0.163237100088741	0.152050418931326	0.133441330722535	0.139620972048625
0.075	0.393980535536189	0.223115586936561	0.0133288130659265	0.0988667152668504
0.1	0.549488659313337	0.249419698888176	−0.0259995049389023	0.123916955251160

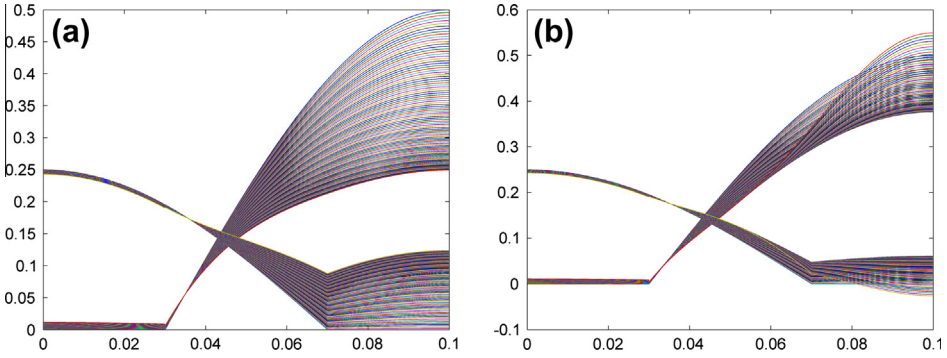

Fig. 3 The numerical results for $u(x,t)$ and $v(x,t)$ obtained by using two iterations of (a) VIMAPOAP and (b) VIMAP with $k = 1$.

According to Fig. 2, we can select $h_1 = 1.06$.

In the same manner, one can select $h_2 = 0.56$ and $h_3 = 0.56$. These calculations can be easily performed by using MAPLE software.

A comparison between the numerical results of the VIMAPOAP with those obtained by VIMAP with $(x,t) \in [0,0.1] \times [0,10{,}000]$ shows that the second one is not valid for large solution domain while the first has an improved accuracy (see Table 2).

We can compare the obtained numerical results (see Fig. 3) with those obtained [30] by using finite volume method to insure that the new technique is more accurate than VIMAP. The same mesh and time step sizes are used in Fig. 3, so that we take $x = 0.000125, 0.000375, \ldots, 0.099875$ and $t = 0, 100, \ldots, 10{,}000$.

5. Conclusions

In this work, we present a new modification of the variational iteration method using Adomian's polynomials (VIMAP). The proposed technique was implemented to find the approximate solution of reaction–diffusion system describes a fast reversible chemical reaction. The numerical results reveal that the proposed method is reliable, effective and accurate tool for solving

nonlinear problems by introducing an auxiliary parameter into the VIMAP to accelerate the convergence for large solution domains. However, the accuracy of the VIMAP was proved for small domains. In our work, we use MATLAB software to obtain the approximate solutions.

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