A Modified Theodolite Instrument: Conceptual Work

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TECHNICAL NOTE

Mohammed Taleb Obaidat[1] *and Zaid A. Al-Smadi*[2]

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ABSTRACT: A proposed modified theodolite instrument is presented. The instrument concept, sequence of operation of measurements, and design plans are reported. The proposed instrument has a potential similar to metric photogrammetry in computing 3-D coordinates without physically touching the target point. A sliding graduated rod (arm) has been introduced to connect the upper and lower parts, each of which has a separate vertical vernier. The rod supports the theodolite head that can be extended upward an arbitrary distance, thus effectively providing a second position for the head. Appropriate angular measurements, together with the length of the extension (the rod) can be used to calculate the inaccessible distances. The instrument relies on simple trigonometric functions to measure the height of objects and the horizontal and slope distances, and to perform trigonometric leveling. Accuracy potential of the instrument has been presented based on the law of propagation of random error. The flow of operations for different measurement cases has been mathematically demonstrated. The instrument is anticipated to bypass the cost-effectiveness and technological bottlenecks between plane surveying and photogrammetric systems.

KEYWORDS: theodolite, plane surveying, precision, 3-D, photogrammetry

The theodolite is an instrument which is suited for angular measurements, including both horizontal and vertical angles, as well as performing trigonometric leveling. The instrument is currently available in various forms starting from the old models of vernier transit of an angular accuracy of about 30 s and reaching the automatic digital or electronic theodolites which have accuracies of better than 0.5 and 1.0 s, respectively [*1,2*].

In conventional surveying practice, the location of any visible point at any height can be computed by measuring the elevation (or depression) angle and the horizontal distance to the point considered [*3*]. Unfortunately, measuring distances can be difficult because of obstacles such as rivers, trees, or buildings. These obstacles sometimes cannot be physically removed, passed through or over. Other instruments, such as Electronic Distance Measurement (EDM) or Total Stations, are used for this purpose. A self-reducing tacheometer (using a constant stadia interval factor), with the aid of a rod (leveling staff), might also be used to measure angles and distances with a relative accuracy of distance measurement better than 1/500. The measurement process involves observing the rod intercept from the image of the rod [*4*]. Use of the previous three instruments precludes the need to physically pass between the two points of concern. Instead, the instrument is set up at a convenient point while a reflector unit or a rod is placed at the target point for use with the EDM/Total Station or tacheometer. The problems of obscured vision and the prevention of direct measurement can be overcome by taking parallel lines at right-angle offsets. However, measuring distance and physically reaching the target point are still essential in all these possibilities in order to compute the 3-D point location. This makes the 3-D point location procedures inconvenient, costly, and time-consuming.

Using two sighting positions to avoid the difficulty of inaccessibility is another alternative which can be used to find the location of a visible point. For example, the field artillery range finder might be used in order to find linear distances, but its measurement accuracy diminishes as distance increases [*5*]. Also, it is common surveying practice to move the transit to a second position at a right angle to the original sighting line, perhaps 5 to 10 metres away, to get the necessary triangle for distance calculation. An arbitrary surveying baseline of two control points might also be constructed and the horizontal and vertical angles between the control points and the target could be measured. Then, point location of the target point could be found using intersection geometry [*6*].

Photogrammetry also has been shown to be a viable tool for remote 3-D measurements. However, this process involves cameras, a calibration field for the cameras, images, data reduction and processing, and specialists in the area [*7–9*]. Thus, for a limited number of object points, it is neither worthwhile nor cost-effective to use this technology rather than plane surveying methods to find heights, distances, and 3-D coordinates.

This paper describes a theoretical framework for a modified theodolite instrument. The proposed instrument is anticipated to have a potential similar to metric photogrammetry in computing 3-D coordinates without physically touching the target point. The instrument does not use any measurement aid such as a reflector or rod. Instead, a single human operator could operate the instrument by using the mechanism described in the body of the paper for measuring heights of objects, measuring vertical and horizontal

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[1]Assistant professor of civil engineering, Civil Engineering Department, Jordan University of Science and Technology, Irbid, P. O. Box 3030, Jordan.

[2]Research assistant, Mechanical Engineering Department, Jordan University of Science and Technology, Irbid, P. O. Box 3030, Jordan.

angles, measuring horizontal and slope distances, and performing trigonometric leveling. The development of such an instrument is anticipated to bypass the cost-effectiveness and technological bottlenecks found in plane surveying and photogrammetric systems. The paper also shows the design plans of the proposed theodolite instrument and the flow of operations for different measurements cases.

Instrument Concept

The basic components of the proposed instrument are similar to any other theodolite with the addition of a lower part, which is the contribution of this paper. The proposed instrument has two parts: the upper part and lower part. (The numbers enclosed within parentheses refer to the numbers that appear in Figs. 1 through 3. Missing numbers appear in the lower part components).

The upper part is exactly like that of any available theodolite with an extra sliding side mirror (3) and a separate graduation of the vertical angle vernier. The sliding mirror is used to help in measuring angles in case of critical target locations which require a short instrument height. The graduation of the vertical vernier is used in order to measure vertical angles with respect to a rod axis connecting the upper and lower parts. The vernier of the vertical angle is attached to the telescope with an angle graduation of 90° always coincident with the direction of the rod, whereas the 0° is coincident with the horizontal plane only when the instrument is leveled. The conventional theodolite measures vertical angles with respect to the plumb-line. The rod will be described later in the section on the lower part of the instrument. This means that the 90° angle mark is not always coincident with the plumb-line. Instead, it coincides with a rod which might be tilted toward the target point direction. The upper part consists of optical rough sighting (1), foot-screws for leveling (2), base-plate for leveling (4), vertical vernier (5), air bubble (6), leveling unit (7), telescope (8), supporting arm (9), horizontal vernier (10), objective piece (12), eyepiece for reading the graduating circles (14), scaling vernier (15), telescope eye piece, telescope focusing ring, eyepiece for optical plummet, slow motion for telescope tilt in horizontal and vertical planes, vertical and horizontal circle display for angles, micrometer knob for seconds, switching knob between horizontal and vertical circle displays, clamps for fastening the instrument on a leveling base, clamping levers for horizontal and vertical setting clamps, and all other accessories.

The basic function of the upper part is to measure vertical angles with respect to the rod as a reference line as well as horizontal angles. Figure 1 shows a 3-D perspective of the upper portion.

The lower part has a sliding rod (11), another vertical vernier scale (16) and scaling vernier (17), and two holding plates which serve as supports for the components of this part (18) and have grooves that permit a rod to pass through. This part is connected with the tripod. Two vertical scale verniers are attached to the outer sides of the two plates with angle graduations always at 90° to the direction of the plumb-line. The function of these scale verniers is to measure complementary vertical angles with respect to the plumb-line that passes through the telescope vertical axis when the rod slides to a convenient height for the human operator and the telescope is tilted to sight to the target point. A groove exists between the two supporting plates. A sliding scale (rod) is another component of this part. The rod connects the upper and lower parts together and passes through the groove opening. It could also be used in order to measure the height of the instrument. Rod length can vary, depending on convenience and the height of the human operator. It has the capability to slide up and down, varying the distance between the upper and lower parts. When the sliding rod goes down, it passes through the tripod in order to adjust the height of the instrument.

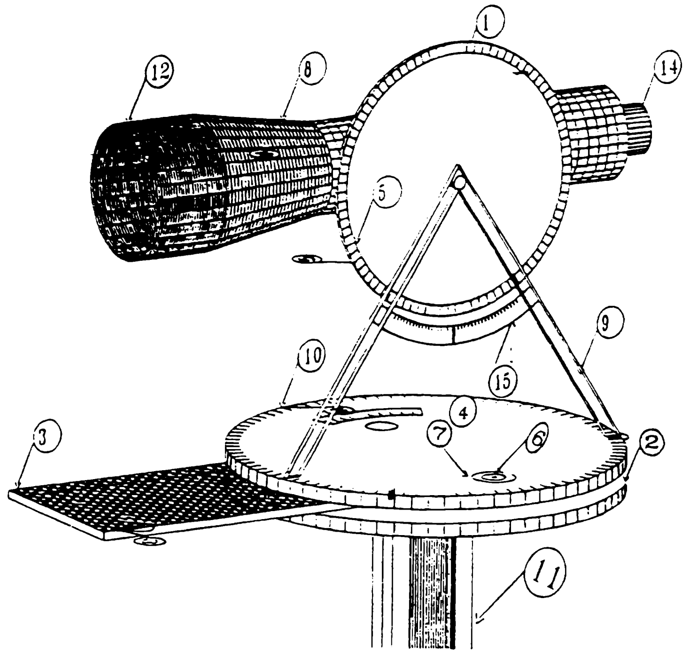


FIG. 1—*3-D perspective of the upper portion.*

Figure 2 shows the components of the lower part. A combination of 3-D representations of the upper and lower parts, as well as different views and the main components of the instrument, are shown in Fig. 3.

Sequence of Operations and Orientations

In order to determine the height of an object or distance to the object of concern, it is essential to make the following measurements and computational procedures.

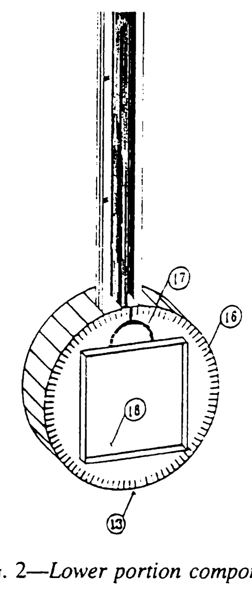
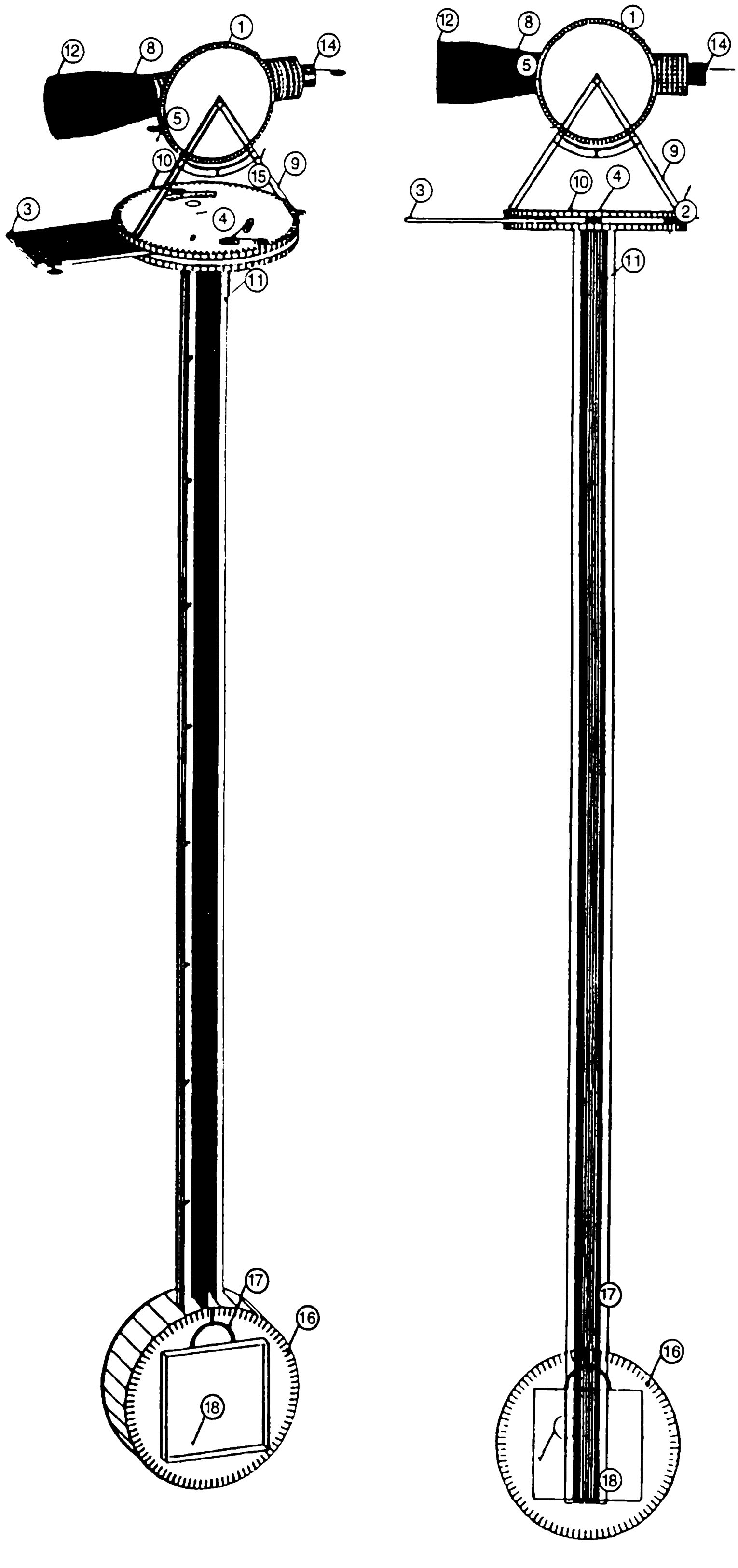
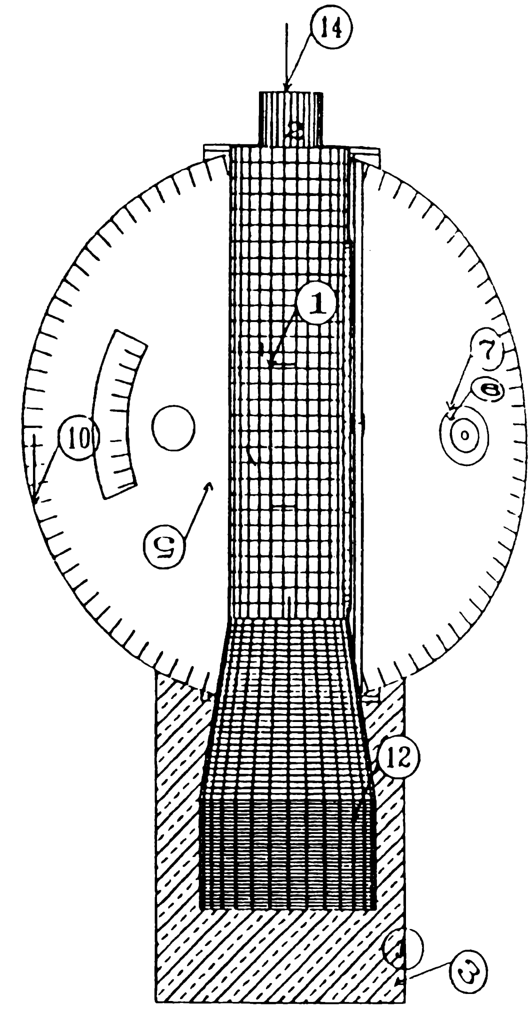


FIG. 2—*Lower portion components.*





NO	NAME
1	ROUGH SIGHT
2	FOOT SCREW
3	MIRROR
4	CIRCULAR PLATE
5	VERNIER
6	AIR BUBBLE
7	LEVELING UNIT
8	TELESCOPE
9	SUPPORTING ARM
10	VERNIER ON HORIZONTAL PLANE
11	SCALE ROD
12	OBJECT_PIECE
13	TRIPOD CONNECTION
14	EYE_PIECE
15	SCALING VERNIER
16	VERNIER
17	SCALING VERNIER
18	SUPPORTING PLATE

FIG. 3—*Different views and main components of the instrument.*

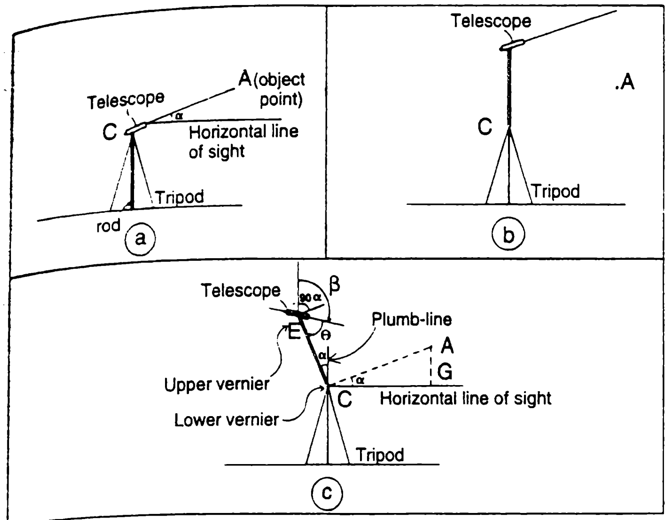


FIG. 4—*Sequence of operations and orientations of measurement procedures.*

1. Set up the instrument above a convenient selected point, i.e., center and level the instrument. It is worthwhile mentioning here that the leveling process is important for the lower-part vertical vernier while it is not for the upper-part vertical vernier, unlike a conventional theodolite.

2. Sight and focus the telescope on the target point and measure the vertical angle (α) of the horizontal line of sight of the telescope to the target point using the upper vertical vernier, the vertical angle with respect to the plumb-line (Fig. 4*a*).

3. Slide the rod that connects the upper and lower parts vertically to a convenient distance of separation between the upper and lower parts. This distance would not be greater than operator height. The rod is still coincident with the plumb-line and the telescope vertical angle is still α (Fig. 4*b*).

4. Tilt the rod in a clockwise or counter-clockwise direction, keeping the telescope vertical angle with respect to the rod α, with the vertical tilt angle α measured on the vernier of the lower part of the instrument. The tilt will be clockwise if the height of the point in concern is lower than the height of instrument, while it is counter-clockwise if it is higher. Notice that there will be a right angle between the rod and the imaginary line that connects the target point and the rotation center of the lower vertical vernier of the instrument (Fig. 4*c*). The rod tilt will give a vertical angle measurement without changing the horizontal angle in either the upper or lower verniers such that the tilt will be in a plane.

5. Tilt the telescope without tilting the rod in order to target the point in concern. Thus, angle β shown in Fig. 4*c* could be measured. It is worthwhile mentioning here that the vertical angle graduation of 90° of the upper vertical vernier is always coincident with the direction of the rod (the rod, not the plumb-line, is the vertical angle reference in the upper part). Consequently, angle θ will be $180 - \alpha - \beta$. Computation of the angle θ using this relation could be the simplified if a nomograph is prepared with inputs of α and β.

The outcome of this procedure is a right triangle formed by the three points representing the center of rotation of the lower vertical vernier, the center of rotation of the upper vertical vernier, and the target point. The rod length between the two verniers can be measured accurately since the rod is graduated linearly. Consequently, trigonometric functions could be simply applied in order to find the missing angles as well as lengths in the triangle. Horizontal and slope distances, trigonometric leveling, and height measurements can then be determined, as seen in the next section.

Computation Concept

The measured rod length and vertical angle θ give an indication of scale for the horizontal and slope distance values between the setup point of the theodolite and the target point. The smaller the angle θ and the longer the rod length, the stronger will be the intersection geometry at the target point of the two optical rays generated from the two telescope positions, the upper and lower positions because of the effect of rod length. Consequently, the accuracy of measurements is expected to be highly correlated to the values of both rod length and angle θ.

As shown in Fig. 4, for target point A, rod length EC, and a theodolite at point C, the slope distance AC could be found as $CE \tan \theta$. Consequently, knowing the vertical angle α, read by the lower vernier (Fig. 4*a*), the horizontal projection of AC (CG) is found to be $AC \cos \alpha$. Furthermore, the difference in elevation between the center of the lower vernier and the target point A will be $(AC) \sin \alpha$. Nomograph charts could be developed in order to quantify AC, CG, and the difference in elevation. The input parameters for these nomographs are rod length, angle α, and angle θ.

Propagation of Random Error

If the estimated standard errors or precision measures, of angle θ and rod length are σ_θ and σ_{CE} respectively, then the precision of the computed length AC or σ_{AC}, can be estimated from the law of propagation of random error [*10*] and [*6*]:

$$\sigma_{AC}^2 = (\partial F/\partial CE)^2\sigma_{CE}^2 + (\partial F/\partial \theta)^2\sigma_\theta^2 \quad (1)$$

where the function $F = CE \tan \theta$.

Two similar equations could be derived using the same concept in order to estimate precision of the horizontal distance and of the elevation difference.

Using the previous equation for a typical situation, a rod length of 1.00 metre, for example, and a measured slope distance of 100 metres, θ is 89.43 degrees, and using estimated standard errors for rod length and angle measurements as 1 mm and 5 s (2.42×10^{-5} radians), respectively, the computed standard error of the slope distance will be about 10 cm. If the rod length is increased to 1.20 m and the measured slope distance decreased to 50.0 metres, θ is 88.63°, and the computed standard error of the slope distance will be decreased to about 4.2 cm. Further, increasing the precision of measurement by enhancing the measurement capabilities of the instrument; i.e., decreasing the estimated standard errors for rod length and angle measurements, will decrease the computed standard error of the slope distance. For example, for θ of 88.63°, rod length of 1.20 m, and estimated standard errors for rod length and angle measurements of 0.5 mm and 2 s (9.69×10^{-6} radians) respectively, the computed standard error of the slope distance will be about 2.1 cm.

Measurement Cases

This section shows the measurement procedures and computations of height, elevation difference, and distances using the developed instrument for different target configurations. The configurations include:

1. Object having its base at the same level as the theodolite set-up point.

2. Object having its base at the same level as the theodolite set-up point with an obstacle that prevents distance measurement using conventional methods.
3. Object having both its base and elevation of highest point below the theodolite horizontal line of sight.
4. Object having both its base and elevation of highest point above the theodolite horizontal line of sight.
5. Object having its base below the theodolite horizontal line of sight, with the elevation of the highest point above the theodolite horizontal line of sight.

Case 1 is the only case in which the object height can be found using a conventional theodolite if instrument height is known. In the other cases object height cannot be found using a conventional theodolite unless the horizontal distance between the theodolite and object is known. The following sections discuss finding object height, elevation difference, and distance for each of the previously mentioned cases using the proposed instrument. In each of the cases, the top of the object is A, the bottom B.

Case 1

This case could be done using any conventional method, that is, a regular theodolite, of known height of instrument (*CD*). Figure 5 shows the flow of operations of measurements done in order to find the three parameters: distance, height, and difference in elevation.

The rod need not be used in this case. Instead, vertical angles to the highest and lowest target points are measured (α and β in Fig. 5). Since $BG = CD$ and $BD = CG$, trigonometry is used to show that the horizontal distance BD is $CD \cot \beta$, whereas AG is $CG \tan \alpha$; i.e., $AG = CD \cot \beta \tan \alpha$, and the height of AB is $CD + CD \cot \beta \tan \alpha$. Consequently, knowing the elevation of point D (h_D), the elevation of point A, h_A, will be ($h_D + AB$).

Case 2

The existence of an obstacle prevents measuring horizontal distance using conventional methods. Figure 6 shows a tree in the way of measuring the horizontal distance between the instrument and the object; a wall in this case. The conventional theodolite does not have the capability to compute the height of the wall unless this horizontal distance is known.

The proposed instrument solves this problem by measuring the vertical angle α to the highest object point when the rod length is zero. A rod length (*CE*) is selected. The telescope and rod are tilted backward at an angle α with respect to the plumb-line. Then

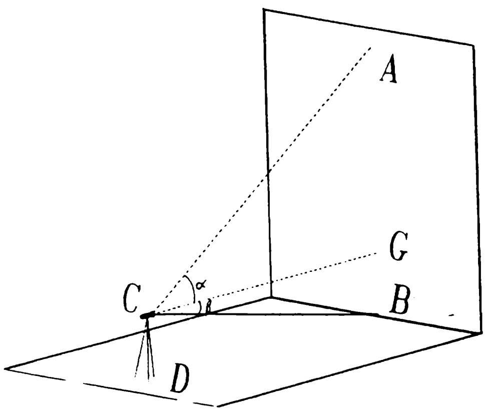


FIG. 5—*Flow of operations of measurement for Case 1.*

the telescope is tilted, without moving the rod, to sight toward the highest point. Consequently, angle γ can be measured and *CE* is known. These procedures are mentioned previously in steps 3, 4, and 5 in the *Sequence of Operations and Orientations* section. These procedures will be repeated in cases 3, 4, and 5 and are not mentioned again.

In Fig. 6, triangle ACE is a right triangle at C. Consequently, the slope distance AC will be $CE \tan \gamma$, and the object height AB will be $AC \sin \alpha + CD$; i.e., $AB = CE \tan \gamma \sin \alpha + CD$. The horizontal distance from the instrument to the object ($BD = CG$) is $(AC) \cos \alpha$; i.e., $BD = (CE) \tan \gamma \cos \alpha$. The elevation of point A is $h_A = h_D + CD + AG$; i.e. $h_A = h_D + CD + AC \sin \alpha$ or $h_A = h_D + CD + CE \tan \gamma \sin \alpha$. The 3-D coordinates of point A or any object point can be computed from the previous relations.

Case 3

This case is useful when the object points are not reachable due to physical obstacles or steep natural slopes. Figure 7 shows a column of height AB with its base (B) and highest elevation point (A) lower than the horizontal line of sight of the instrument or the level of point G, due to the change of slope of natural terrain. The figure also shows the flow of operations of measurements of this case. Vertical depression angles α and β are measured to points A and B, respectively, using the upper vertical vernier based upon the concept of a conventional theodolite in which the rod reading is zero (Fig. 7*a*). Then the telescope and the rod are tilted clockwise at an angle (α) measured on the lower vernier in order to sight to the highest point (A) (Fig. 7*b*). After that, angle γ can be measured. The same orientation procedures are repeated to sight at the base point (B) at a tilt angle β (Fig. 7*c*). Angle δ can be measured, too. Triangles ACE and BCF are right triangles at C, and triangle ACB is an oblique triangle of measured angle (β-α).

Using trigonometric relations and measuring rod lengths *CE* and *CF*, as well as the two vertical angles γ and δ, the following relationships can be proved:

From triangle ACE: $AC = CE \tan \gamma$; whereas triangle BCF gives: $BC = CF \tan \delta$.

Then, applying the cosine law into triangle ACB gives the height of the object (AB):

$$AB = [AC^2 + BC^2 - 2(AC)(BC)\cos(\beta - \alpha)]^{1/2} \tag{2}$$

The horizontal distance (CG) can also be found using this equation:

$$\begin{aligned} CG &= AC \cos \alpha \\ &= BC \cos \beta \\ &= CE \tan \gamma \cos \alpha \\ &= CF \tan \delta \cos \beta \end{aligned} \tag{3}$$

Elevation of point A (h_A) can be found if the height of the instrument CD is measured and the elevation of point D is given any arbitrary value (h_D):

$$\begin{aligned} h_A &= h_D + CD - AG \\ &= h_D + CD - AC \sin \alpha \\ &= h_D + CD - CE \tan \gamma \sin \alpha \end{aligned} \tag{4}$$

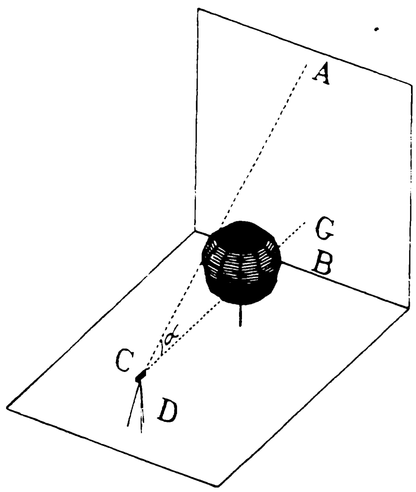


Figure 6-a

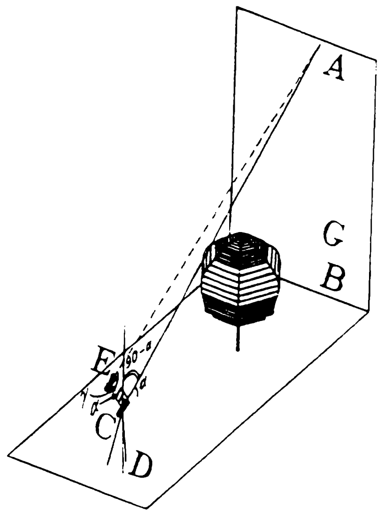


Figure 6-b

FIG. 6—*Flow of operations of measurement for Case 2.*

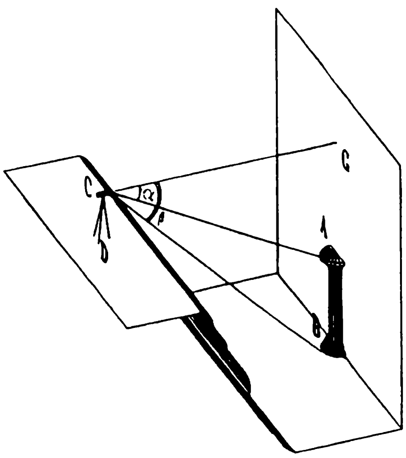


Figure 7-a

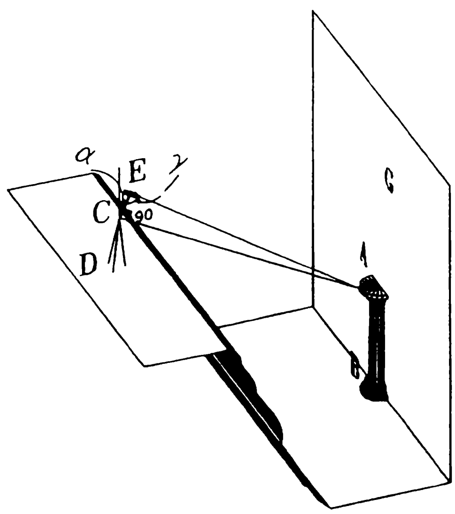


Figure 7-b

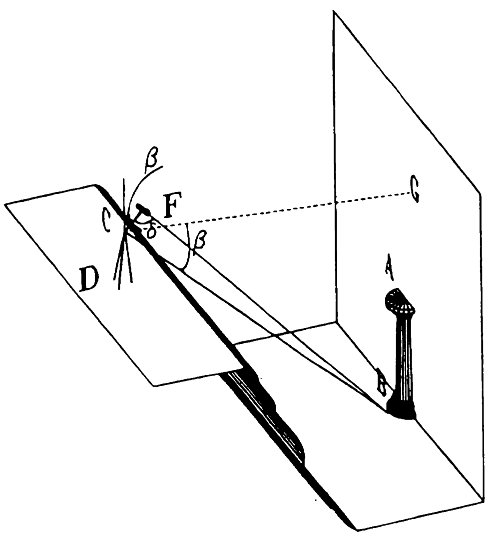


Figure 7-c

FIG. 7—*Flow of operations of measurement for Case 3.*

A similar equation can be written to find the elevation of point B:

$$\begin{aligned} h_B &= h_D + CD - BG \\ &= h_D + CD - BC \sin \beta \\ &= h_D + CD - CF \tan \delta \sin \beta \end{aligned} \tag{5}$$

Case 4

This case is similar to Case 3, but the object is higher than the horizontal line of sight of the instrument. This is normally the case for objects or constructions located in high, hilly areas. Figure 8 shows the flow of operations used to implement measurements in this case. In Fig. 8*a*, vertical elevation angles of α and β are measured to points A and B, respectively, using the upper vertical vernier when the rod length is zero. After the selection of a suitable rod length, tilt angles will be α and β to points A and B, respectively, (Figs. 8*b* and 8*c*), and the measured interior angles of triangles ACE and BCF are γ and δ, respectively. It is worthwhile mentioning that the tilt angles here are exchangeable with Case 3; i.e., the tilt angle for the base of the column here equals the tilt angle for the highest point in Case 3 and vice versa.

As in Case 3, the following equations can be defined easily using trigonometric formulas for the imaginary triangles (if the two rod lengths *CE* and *CF*, are measured as well as the two vertical angles γ and δ).

From triangle ACE: $AC = CE \tan \gamma$; and from triangle BCF, length *BC* is known: $BC = CF \tan \delta$.

Then, applying the cosine law into triangle ABC will produce a formula similar to Eq 2 to find the height of the object (*AB*):

$$AB = [AC^2 + BC^2 - 2(AC)(BC)\cos(\alpha - \beta)]^{1/2} \tag{6}$$

The horizontal distance (*CG*) can also be found using Eq 3.

Elevation of point A (h_A) can be found if the height of the instrument *CD* is measured and the elevation of point D is given any arbitrary value (h_D). Then Eq 4 is applicable with a different sign:

$$\begin{aligned} h_A &= h_D + CD + AG = h_D + CD + AC \sin \alpha \\ &= h_D + CD + CE \tan \gamma \sin \alpha \end{aligned} \tag{7}$$

A similar equation can also be written to find the elevation of point B:

$$\begin{aligned} h_B &= h_D + CD + BG = h_D + CD + BC \sin \beta \\ &= h_D + CD + CF \tan \delta \sin \beta \end{aligned} \tag{8}$$

Case 5

This case has object heights with elevations both higher and lower than the horizontal line of sight of the instrument. This case is most common in rolling terrain. Figure 9 shows a building with its base lower than the line of sight of the instrument, while its highest point has a higher elevation than the line of sight. With the rod length zero, the vertical elevation and depression angles α and β are measured to points A and B, respectively, using the upper vertical vernier. As in previous cases, two rod lengths are selected, like *CE* and *CF*, with tilt angles α and β to points A and B, respectively. The angles γ and δ can be measured. The result is right triangles ACE and BCF. It is worthwhile mentioning here that the rod tilt is required only to either point A or B, but tilting the rod to the two points is useful for computational check purpose.

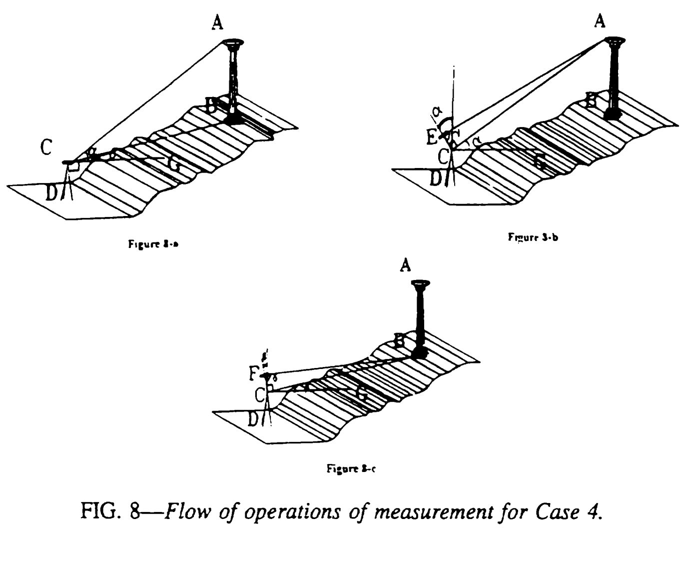


FIG. 8—*Flow of operations of measurement for Case 4.*

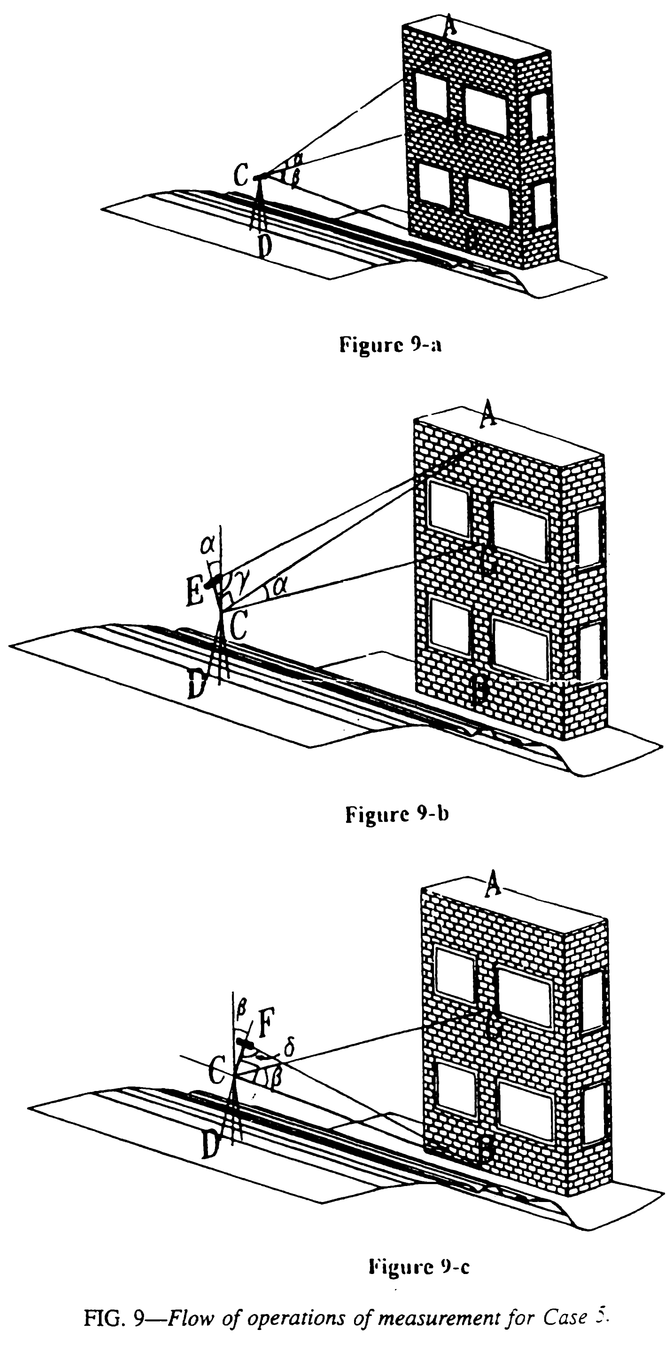


FIG. 9—*Flow of operations of measurement for Case 5.*

Using trigonometric relations and by measuring the rod lengths CE and CF as well as the vertical angles γ and δ, the following relationships can be proved:

From triangle ACE, $AC = CE \tan \gamma$. Triangle BCF is a right triangle at C. Consequently, $BC = CF \tan \delta$. From triangle ACG, the horizontal distance $CG = AC \cos \alpha$; i.e., $CG = CE \tan \gamma \cos \alpha$. Using triangle BCG will also give the following formula for CG: $CG = BC \cos \beta$; i.e., $CG = CF \tan \delta \cos \beta$.

The elevation of point A could also be found using the following equation:

$$\begin{aligned} h_A &= h_D + CD + AG \\ &= h_D + CD + AC \sin \alpha \\ &= h_D + CD + CE \tan \gamma \sin \alpha \end{aligned} \tag{9}$$

A similar equation can also be written to find the elevation of point B:

$$\begin{aligned} h_B &= h_D + CD - BG \\ &= h_D + CD - BC \sin \beta \\ &= h_D + CD - CF \tan \delta \sin \beta \end{aligned} \tag{10}$$

All the previous computational procedures could also be done by tilting the telescope to either point A or B. For example, if the telescope is tilted to point A, using trigonometric functions of triangles ACG, BCG, and ACE, the slope distance (AC), the horizontal distance (CG), and the elevations of points A and B could be computed. Then, repetition of measurement procedures to the other point, point B, gives a potential check in order to strengthen the precision of the measurements by taking the average values of all computed parameters.

General Case

Figure 10 shows a geometrical demonstration of a case that represents the last four cases, which require rod tilt in order to compute the height of an object, the elevation difference between points A and B. The following procedure is used to find the vertical distance AB as a function of measured angles α, β, and δ, and the rod length CF:

From triangle BCF, the slope distance $BC = CF \tan \delta$. Triangles ABH and BCH also give the length of line BH to be:

$$BH = BC \cos \varphi = AB \cos \omega \tag{11}$$

Consequently,

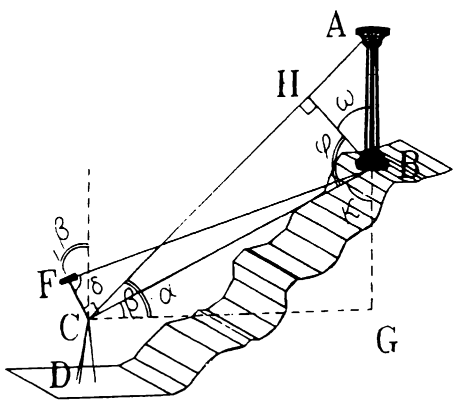


FIG. 10—*Flow of operations of measurement for general case.*

$$AB = BC \cos \varphi \sec \omega \tag{12}$$

The following angle relations could also be derived:

$$(\alpha - \beta) + \varphi = \kappa + \beta = 90° \tag{13}$$

$$180° - \kappa = \varphi + \omega \tag{14}$$

From Eqs 13 and 14: $\omega = \alpha$, and $\cos \varphi = \sin(\alpha - \beta)$. Applying these relations into Eq 12 will give:

$$AB = BC \sin(\alpha - \beta) \sec \alpha \tag{15}$$

or

$$AB = CF \tan \delta \sin(\alpha - \beta) \sec \alpha \tag{16}$$

Equation 16 indicates that the elevation difference between points A and B can be computed if a rod length (CF), and angles α, β, and δ are measured. This means that tilting the telescope to the other point is not necessary.

Equation 16 can also be used to compute the elevation difference between points A and B for the cases required to tilt the rod; i.e., the last four cases. Each case could be represented as a special case of the general case by applying the following procedure:

Case 2: Find (AG) by measuring CE (instead of CF), α, and γ (instead of δ), while targeting point A with and without tilting the rod. The height of the instrument CD is known and β is not required.

Case 3: Mirror image of the general case. Measure CE (instead of CF), α, β, and γ (instead of δ). This means Figs. 7*a* and 7*b* only are required and there is no need to tilt the rod again as in Fig. 7*c*.

Case 4: Represents the general case.

Case 5: Divide AB into two parts (AG and BG), and apply operations of Case 2 to every part.

A similar approach could also be used to show that computations for slope and horizontal distances for all cases could be represented using this general case.

Conclusions

The development of such an innovative instrument will facilitate the measurement procedures to locate 3-D point positioning without the need of physically reaching the object being measured. Other by-product measurements such as areas and volumes might also be computed once the 3-D point locations are known. Thus, the instrument herein presented is expected to bridge the technological gap between photogrammetric stereo-vision techniques and conventional plane surveying instruments.

One of the essential requirements of using this instrument is the development of tables or nomographs to solve the measurement equations. This will reduce the time required for measurement. Accuracy assessment factors such as rod length and intersection angle at the target point also play major roles in convincing users to employ the presented instrument. Equipping the instrument with electronic angle measurement potential rather than transit or mechanical procedures will give the instrument suitability and convenience.

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