Theoretical Analysis of Different Fractal-Type Pipelines

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Abstract. For decades, Fractal-type pipelines have been utilized in industries and domestical applications for different practical applications. The present work is a comparative study of three types of fractals: T-shaped, Y-shaped, and irregular. The aim is to maintain a uniformflow distributionthroughout the system while maximizing the efficiency of the systems, and how different fractal systems can affect the properties of the fluid in motion. Finally, the theoretical analysis was utilized to assess systems after the optimizing process. Under same operational conditions no changes in flow characteristics were found when using different fractal types pipeline. Further numerical and experimental investigation is necessary to understand the influence of such design on the flow fields within the system.

INTRODUCTION

The fractal systems have been utilized by engineering industry intentionally or unintentionally in their structures implementation. The discovery of fractal systems by Benoit Mandelbrot revolutionized how we look practically at different geometry and structures. The word fractal means “to break into fragments.” These fractals have been proven to enhance the efficiency of structures and optimize their designs. The number of fractals present in nature is infinite. Some of the many applications of fractals in engineering are beams, trusses, slabs, etc. [1][2].

For example, the researcher investigates the different applications of fractals using simulation approaches are tackled, one of which is underground seepage location. In this application, the pipes are located at an optimal distance so that the seeped fluid does not flow in a backward direction. This helps overcome the flooding problem and reduces the overall maintenance and installation cost. Furthermore, this application proves how fractals help simulate an optimized design. Another application is the formation and growth of a coral reef, which will help understand how this fractal material can be utilized in fluid mechanics simulations. In the future, more engineering application frontiers will be opened by fractal geometry [3].

Some theories related to the turbulent flow prove that they can be used to explain how fractals behave and interact with the flow. For example, Hiter, in 1983, obtained a fractal behavior for Couette-Taylor flow, and in 1985 Tabeling, in his theoretical work, found that fractal dimensions increase systematically [4].

In this paper, an analysis of fractal systems is carried out to understand how to keep the distributed flow uniform and improve the efficiency of such systems. The working fluid is water at 373K.

FLOW UNIFORMITY AND EFFICIENCY OF FRACTAL SYSTEMS

Two main challenges present themselves when working with fractal systems. One minimizes global thermal resistance, and the other minimizes pumping power which helps in the fluid motion. For example, the suitable design for cooling a circular disc would be to make radial ducts to minimize the thermal resistance. Now in order to minimize the power required to pump the coolant, the most suitable design would be a tree-shaped structure. Results show that the optimization approaches (thermal and fluid mechanics) generally perform similarly. However, when concerned with smaller-scale applications, the tree-shaped structures perform better and can be more robust. Obviously, the ideal scenario is simultaneously lowering the thermal and flow resistance, but such structures become complex. Nevertheless, optimization of architectures concerning most degrees of freedom leads to robustness. For example, the case of a radial disc distributes the imperfections related to constructal theory and not uniform distribution over an area [5].

Deborah Pence observed the pressure drop and maximum wall temperature for a straight channel array and a fractal-like channel network while keeping the flow rate conditions the same. For the fractal-like design, the total length is 17.5 mm, and there are twelve branches at zero-level, four bifurcation levels, a fixed channel depth of 0.25 mm, and a terminal branch channel width of 0.1 mm. To achieve an essentially identical total convective heat transfer area between the two heat sinks, 77 straight channels 17.5 mm in length were required. The channel height and width were set to 0.143mm, equal to the terminal branch's hydraulic diameter. The flow was assumed to be fully developed hydrodynamically and thermally. Water was used as a working fluid; at the inlet, the water temperature was 293K. The observed pressure drops through, and the maximum wall temperature of the fractal-like channel network are 117 kPa and 16 K lower than in the straight channel array, respectively [6].

In 2001 Yongping Chen and Ping Cheng compared the new design with the traditional parallel net, which showed that the new fractal branching channel net has a stronger heat transfer capability and requires a lower pumping power. Fractal branching channel nets with different dimensions and total branching levels for designing a micro heat exchanger of rectangular shape are investigated. A comparison of heat transfer and pressure drop is made between fractal branching channel nets with the traditional parallel channel net. The comparison is based on the following two assumptions:

- The flow is laminar and fully developed.
- The effect of bifurcation on pressure drop is negligible.

It is found that the fractal net can increase the total heat transfer rate while it reduces the total pressure drop in the fluid. Furthermore, a larger fractal dimension or a more significant total number of branching levels is found to have a stronger heat transfer capability with a minor pumping power required. Thus, the fractal branching channel net enhances the efficiency of a micro heat exchanger [7].

The paper “Thermal-hydraulic performances and synergy effect between heat and flow distribution in a truncated doubled-layered heat sink with a Y-shaped fractal network” proposes a novel truncated double-layered (TDL) heat transfer sink with a Y-shaped fractal network. A comparative study was conducted between single-layered, double-layered, and truncated double-layered heat sinks. The double-layered heat sink is changed to a truncated one. It improves temperature uniformity around the inlet and also requires less power. But as pressure drop increases, the double-layered heat sink performs better than the truncated double-layered heat sink. The relationship between heat and flow distribution is discussed, and it’s revealed that only when heat distribution is accordant with flow distribution can the TDL heat sink achieve the best cooling performance [8]. These results prove that Fractal systems perform better than traditional channels, require less pumping power, and uniformly distribute the flow. Optimizing the design of any fractal channel is hard to achieve since many variables affect its geometry. Variables like branch angle and aspect ratio play an important role during the design phase. Aspect ratio affects the formation of vortices in the fluid, causing an improvement in heat transfer efficiency. Mainly fractal systems are studied for creating heat sinks for different electronics as they provide better temperature uniformity [9].

ANALYTICAL SOLUTIONS FOR DIFFERENT FRACTALS

All the analytical calculations in the study were carried out by considering specified standard values. As a result, the pressure drops and heat loss to the environment by water at 373K were observed while keeping the flow conditions the same for every fractal [8].

Regular T-shaped Fractal

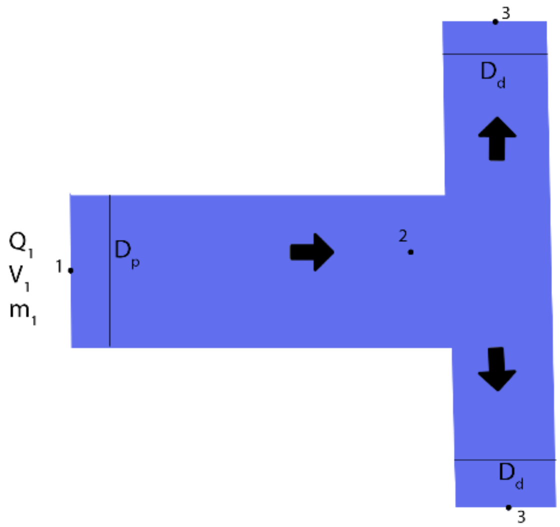

FIGURE 1. Regular T-shaped Fractal

Considering Water at:

- Initial Temperature = 100 ℃ (or 373 K)
- Density = ρ = 958.4 kg/m^3
- Dynamic Viscosity = μ = 0.282x10^{-3} Ns/m^2

Considering basic dimensions of pipe:

- Diameter of larger pipe = D_p = 0.01905 m
- Diameter of smaller pipe = D_d = 0.009525 m
- Length of larger pipe = L_p = 0.1 m
- Length of smaller pipe = L_d = 0.05 m
- Thickness of larger pipe = t_p = 0.002 m
- Thickness of smaller pipe = t_d = 0.0015 m

According to the diameter of the pipe, the flow rate is:

$$Q_1 = 1.565 \text{ x} 10^{-4} \text{ m}^3/\text{s}$$

$$m_1 = 0.15 \text{ kg/s}$$

Now, apply the formula for volume flow rate:

$$Q_1 = AV_1 \tag{1}$$

$$1.565 \text{ x}10^{-4} = \frac{\pi}{4}(0.01905)^2 V_1$$

$$V_1 = 0.549 \text{ m/s}$$

Considering,

$$Q_2 = Q_3 = \frac{Q1}{2} = \frac{1.565 \text{ x}10^{-4}}{2}$$

$$Q_2 = Q_3 = 7.825 \text{ x}10^{-5} \text{m}^3/\text{s}$$

Similarly.

$$Q_2 = AV_2$$

$$7.825 \text{ x}10^{-5} = \frac{\pi}{4} (0.009525)^2 \, V_2$$

$$V_2 = 1.098 \text{ m/s}$$

Finding Reynolds Number.

$$Re_1 = \frac{\rho VD}{\mu} \tag{2}$$

$$= \frac{(958.4)(0.549)(0.01905)}{0.282\text{x}10^{-3}}$$

$$Re_1 = 35544 > 2500 \text{ (Turbulent Flow)}$$

Similarly.

$$Re_2 = \frac{\rho VD}{\mu}$$

$$= \frac{(958.4)(1.098)(0.009525)}{0.282\text{x}10^{-3}}$$

$$Re_1 = 35544 > 2500 \text{ (Turbulent Flow)}$$

Finding Hydrodynamic entry length;

$$Le = 4.4 \, D_p Re^{\frac{1}{6}} \tag{3}$$

$$= 4.4 \, (0.01905) \, (35544)^{\frac{1}{6}}$$

$$Le = 0.4806 \text{ m}$$

Hence, after this length, the flow becomes fully developed.

Similarly:

$$Le = 4.4 \, D_p Re^{\frac{1}{6}}$$

$$= 4.4 \, (0.009525) \, (35544)^{\frac{1}{6}}$$

$$Le = 0.2403 \text{ m}$$

Hence, after this length, the flow becomes fully developed.

Finding pressure variation:

$$Q = \frac{\pi D^4 \Delta P}{128 \mu L} \tag{4}$$

$$1.56 \text{ x}10^{-4} = \frac{\pi (0.01905)^4 \Delta P}{128(0.282\text{x}10^{-3})(0.1)}$$

$$\Delta P = P_1 - P_2 = 1.3653 \text{ Pa}$$

Similarly;

$$Q = \frac{\pi D^4 \Delta P}{128 \mu L}$$

$$7.825 \text{ x}10^{-5} = \frac{\pi (0.009525)^4 \Delta P}{128(0.282\text{x}10^{-3})(0.05)}$$

$$\Delta P = P_2 - P_3 = 5.4614 \text{ Pa}$$

After solving these equations, we get:

P_1 = 101332 Pa
P_2 = 101330.5 Pa
P_3 = 101325 Pa

For Thermal calculations:
The area is provided by:

$$\text{Area} = \text{Total surface area of pipes}$$

$$= 2\,\pi rL + 2(2\,\pi rL)$$

$$= 2\,\pi\,(\frac{0.01905}{2})(0.1) + 4\,\pi\,(\frac{0.009525}{2})(0.05)$$

$$A = 8.977\ \text{x}10^{-3}\ \text{m}^2$$

Total heat transfer from pipe surface area to the environment is:

$$q = K\ A\ \Delta T \quad \textbf{(5)}$$

$$= (17)\ (8.977\ \text{x}10^{-3})\ (100 - 22)$$

$$q = 11.903\ \text{Watts}$$

Regular Y-shaped Fractal

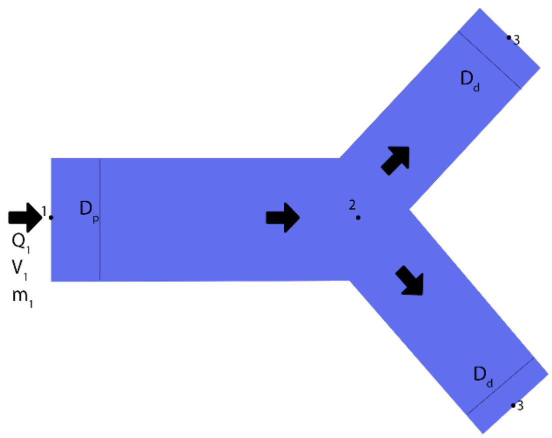

FIGURE 2. Regular Y-shaped Fractal

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$$Q_1 = AV_1$$

$$1.565 \text{ x}10^{-4} = \frac{\pi}{4} (0.01905)^2 V_1$$

$$V_1 = 0.549 \text{ m/s}$$

Considering,

$$Q_2 = Q_3 = \frac{Q1}{2} = \frac{1.565 \text{ x}10^{-4}}{2}$$

$$Q_2 = Q_3 = 7.825 \text{ x}10^{-5} m^3/s$$

Similarly;

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i.e., after this length, the flow becomes fully developed.
Similarly;

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After solving these equations, we get:
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Total heat transfer from pipe surface area to the environment is:

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$$q = 11.903 \text{ Watts}$$

Irregular Fractal

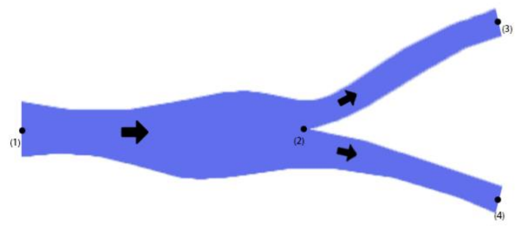

FIGURE 3. Irregular Fractal

Considering Water at:

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- Thickness of larger pipe = t_p = 0.002 m
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According to the diameter of the pipe, the flow rate is:

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Similarly;

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Hence, after this length, the flow becomes fully developed.
Similarly;

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Total heat transfer from pipe surface area to the environment is:

$$q = K\ A\ \Delta T$$

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$$q = 11.903 \text{ Watts}$$

RESULTS AND DISCUSSION

- The results show that for smaller dimension fractals, regardless of the fractal design, the pressure drops at outlet 2 and 3 remains almost the same. The pressure drop in the fractals is negligible under the same flow conditions. The inlet condition for the Y-shaped fractal was only 1 Pa greater than the other two fractal inlet conditions, but the outlet pressure remains the same.
- The cooling effect of the fractal is also identical in all three cases, which leads us to believe that analytically the thermal resistance remains the same regardless of the design.
- The optimization of these fractals may cause a change in the results of pressure drop and thermal resistance.

CONCLUSION

The influence of different types of fractals inspired pipes on the flow characteristics are investigated. The presented results prove that, under the same flow conditions, the design of fractal systems does not bring a significant change in the flow pressure and show that the same pumping power can be used for fluid motion. It also shows that the thermal resistance of fractals remains unchanged and unaffected for different fractal-type pipelines while keeping the material and operating conditions the same. A three-dimensional analysis and Computational Fluid Dynamics may give us a deeper insight into what type of fractal works best in different situations, as the 2D analysis does not consider the varying fluid interactions in three dimensional systems.

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