REGIONAL GRADIENT DETECTABILITY FOR INFINITE DIMENSIONAL SYSTEMS

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Abstract:

In the present paper, we deal with the linear infinite dimensional systems in a Hilbert space, where the dynamics of the system is governed by strongly continuous semi-group. Then, we examine the concept of regional gradient observability in connection with the structures of sensors on time finite horizon. Thus, the aim of this paper is to extend these results to the case of infinite time horizon. We explore the concept of regional gradient detectability in connection with the strategic gradient sensors. Furthermore, we consider two dimensional systems and the specific results related to some applications of sensors structures and G-observer.

Key words: *G*-Strategic sensors, *G*-observability, *G*-stability, *G*-detectability, *G*-observer, Infinite dimensional systems.

1. Introduction

In many practical problems of different classes of distributed parameter systems, one is interested in the reconstruction of the system state in a restricted given subregion, from the knowledge of the system dynamics and the output function [6, 9]. This is the so called regional observation problem and has been developed very recently by El Jai and Zerrik [7, 13]. An extension of the observability concept and that is very important in practical application is that of regional gradient observability [12]. This problem has been introduced and developed by Zerrik, Bourray and Budraoui [11]. The results in regional gradient analysis have been achieved on time finite horizon. It remains that to extend these results to the case of infinite time horizon (regional gradient detectability).

In our work, we explore the notion of regional gradient detectability by using the choice of sensors. That is to say, one may be concerned with the gradient state asymptotic detection in a given part ω of the domain Ω (see Figure 1),

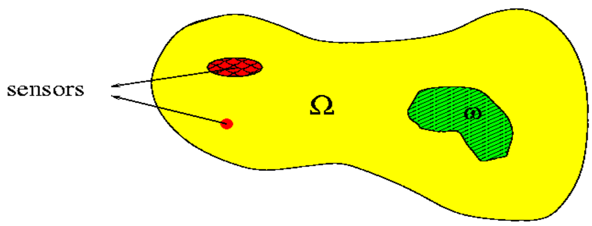

Figure 1: The domain Ω, the region ω, the location of sensors

which is an extension of region detectability concept, was introduced recently by Al-Saphory and El Jai [1-4], and was focused on the state asymptotic detection only in a critical subregion. This paper is organized as follows:

Section 2 concerns to the problem statement and considered system. First we give some definitions and characterizations related to regional gradient observability and regional strategic sensors. Section 3 develop a characterization result that links the regional asymptotic gradient detectability and regional gradient strategic sensors and we give the sufficient condition of this concept. Section 4 gives an application to the location gradient sensors and examines the possibility to construct an estimation of regional gradient state of diffusion system.

2. Regional Gradient Detectability

An extension of the detectability notion as in ref. [4] and that is very important in practical application [11] is that of regional gradient detectability, this notion is related to the problem of detection the state gradient not in the whole domain Ω, where the system is defined but only in a given subregion $\omega \subset \Omega$.

2 .1 Problem statement:

Let Ω be an open bounded set of R^n with boundary $\partial\Omega$ and ω be a non-empty subregion of Ω. We denote $\Theta = \Omega \times (0,\infty)$ and $\Pi = \partial\Omega \times (0,\infty)$. Consider the following parabolic system:

$$\begin{cases} \dfrac{\partial x}{\partial t}(\xi,t) = Ax(\xi,t) + Bu(t) & \Theta \\ x(\xi,0) \quad = x_0(\xi) & \Omega \\ x(\eta,t) \quad = 0 & \Pi \end{cases} \qquad (2.1)$$

augmented with the output function:

$$y(.,t) = Cx(.,t) \qquad (2.2)$$

where A is a second order linear differential operator, which generates a strongly continuous semi-group $(S_A(t))_{t\geq 0}$ on the state space X and is self-adjoint with compact resolvent. The spaces X, U and O be separable Hilbert spaces represent respectively the state space, control space and the observation space. We consider $X = H^1(\Omega)$, $U = L^2(0,\infty,R^p)$ and $O = L^2(0,\infty,R^q)$, where p and q are the numbers of controls and sensors. The operators $B \in L(R^p, X)$ and $C \in L(R^q, X)$ depend on the structure of controls and sensors [7, 10]. The measurements can be obtained by the use of zone or pointwise sensors which may be located in Ω or $\partial\Omega$ [7].

- Sensors are any couple $(D_i, f_i)_{1\leq i\leq q}$ where D_i denote closed subsets of $\overline{\Omega}$, which is spatial supports of sensors and $f_i \in L^2(D_i)$ define the spatial distributions of measurements on D_i.

- Depending on the nature of D_i and f_i, one could have different types of sensors**:**

  ◦ It may be zone if $D_i \subset \Omega$ and $f \in L^2(D)$. The output function (2.2) may be written in the form:

$$y(.,t) = \int_{D_i} x(\xi,t)\ f_i(\xi)\, d\xi \qquad (2.3)$$

  ◦ It may be pointwise if $D_i = \{b_i\}$ with $b_i \in \Omega$ and $f = \delta(. - b_i)$, where δ is the Dirac mass concentrated in b. The output function (2.2) can be given by the form:

$$y(.,t) = \int_{\Omega} x(\xi,t)\delta_{b_i}(\xi - b_i)\, d\xi \qquad (2.4)$$

- The system (2.1) has a unique solution given by:

$$x(\xi,\ t) = S_A(t)\ x_0(\xi) + \int_0^t S_A(t-s)Bu(s)\ ds,$$

$\forall s \in [0,t]$ (2.5)

- In the case when (2.1) is autonomous system the equation (2.5) allows to given the following equation:

$$x(\xi,\ t) = S_A(t)x_0(\xi)$$

- The initial state x_0 and its gradient ∇x_0 are supposed to be unknown, the problem concerns the reconstruction of the initial gradient ∇x_0 on the subregion ω of the system domain Ω.
- Define the operator $K : X \longrightarrow O$, as follows:

$$x \longrightarrow CS_A(.)x,$$

which is, in the zone case, linear and bounded (see, e.g., [9]). And the adjoint operator K^* of K is defined by:

$$K^* y = \int_0^t S_A^*(s)\, C^* y(s)\, ds$$

- The operator ∇ denotes the gradient is given by:

$\nabla : H^1(\Omega) \longrightarrow (L^2(\Omega))^n$

$$x \longrightarrow \nabla x = \left(\frac{\partial x}{\partial \xi_1}, \frac{\partial x}{\partial \xi_2}, \dots, \frac{\partial x}{\partial \xi_n} \right)$$

While ∇^* denotes its adjoint.

- For a non-empty subset ω of Ω, the operators $\tilde{\chi}_\omega$ is defined by:

$$\tilde{\chi}_\omega : (L^2(\Omega))^n \longrightarrow (L^2(\omega))^n$$

$$x = (x_1,\ x_2, \dots, x_n) \longrightarrow \tilde{\chi}_\omega x = (x_1\big|_\omega, x_2\big|_\omega, \dots, x_n\big|_\omega$$

with adjoint denoted by $\tilde{\chi}^*_\omega$.

- Finally introduce the operator $H = \tilde{\chi}_\omega \nabla K^*$ from O into $(L^2(\omega))^n$ with adjoint $H^* = K\nabla^* \tilde{\chi}^*_\omega$.

2.2 Definitions and characterizations:

In this subsection, we present some definitions and characterizations related to the regional gradient observability and regional gradient sensors as in ref.s [11, 12].

Definition 2.1: The system (2.1) augmented with the output function (2.2) is said to be regionally exactly gradient observable on ω (or exactly *G*-observable on ω), if:

$$\operatorname{Im} H = (L^2(\omega))^n$$

Definition 2.2: The system (2.1) augmented with the output function (2.2) is said to be regionally weakly gradient observable on ω (or weakly *G*-observable on ω), if:

$$\overline{\operatorname{Im} H} = (L^2(\omega))^n$$

Remark 2.3: The definition (2.2) is equivalent to say that the system (2.1) augmented with the output function (2.2) is weakly *G*-observable on ω, if:

$$\operatorname{Ker} H^* = \{0\}$$

Now, we can give the characterization of the exactly and weakly *G*-observable on ω by the following two propositions:

Proposition 2.4: The system (2.1) augmented with the output function (2.2) is said to be regionally exactly *G*-observable on ω, if and only if:

$$\operatorname{Ker} \tilde{\chi}_\omega + \operatorname{Im} \tilde{\chi}^*_\omega \tilde{\chi}_\omega \nabla K^* = (L^2(\Omega))^n$$

Proposition 2.5: The system (2.1) augmented with the output function (2.2) is regionally weakly *G*-observable on ω if and only if:

$$\operatorname{Ker} \tilde{\chi}_\omega + \overline{\operatorname{Im} \tilde{\chi}_\omega{}^* \tilde{\chi}_\omega \nabla K^*} = (L^2(\Omega))^n.$$

For the proof of the propositions of 2.4 and 2.5 is similar to ref. [1] with technical changes.

In this subsection, we recall results that link regional gradient observability to sensor structures. Let us consider the system (2.1) observed by q sensors $(D_i, f_i)_{1 \le i \le q}$, which may be pointwise or zone (for more details see ref. [7]).

Definition 2.6: A sensor (D, f) is gradient strategic on ω if the corresponding system (2.1) augmented with the output function (2.2) is *G*-observable on ω, a such sensor will be said *G*-strategic on ω.

Definition 2.7: A suite of sensors $(D_i, f_i)_{1 \le i \le q}$ is said to be *G*-strategic on ω if there exist at least one sensor (D_i, f_i) which is *G*-strategic on ω.

3. Sensors and Regional Gradient Detectability:

we give some definitions concern the concept of regional asymptotical gradient stability**.**

Definition 3.1: A semi-group $(S_A(t))_{t \ge 0}$ is regionally asymptotically gradient stable (or *G*-stable) on $(L^2(\omega))^n$, if there exist positive constants M_ω and α_ω, such that:

$$\| \tilde{\chi}_\omega \nabla S_A(.) \|_{L((L^2(\omega))^n, H^1(\Omega))} \le M_\omega e^{-\alpha_\omega t},\ t \ge 0.$$

(3.1)

Definition 3.2: The system (2.1) is said to be regionally asymptotically gradient stable (or *G*-stable) on ω, if the operator A generates a semi-group which is regionally asymptotically gradient stable in $(L^2(\omega))^n$.

If $(S_A(t))_{t \ge 0}$ is *G*-stable semi-group on $(L^2(\omega))^n$, then for all $x_0(.) \in H^1(\Omega)$ the solution of the associated autonomous system of (2.1) satisfies:

$$\| \tilde{\chi}_\omega \nabla x(.,\ t) \|_{L((L^2(\omega))^n, H^1(\Omega))} = \| \tilde{\chi}_\omega \nabla S_A(.)\ x_0(.) \|_{L((L^2(\omega))^n, H^1(\Omega))}$$

$$\le M_\omega e^{-\alpha_\omega t} \| x_0(.) \|_{H^1(\Omega)}$$

and therefore

$$\lim_{t\to\infty} \| \tilde{\chi}_\omega \nabla x(.,t) \|_{L((L^2(\omega))^n, H^1(\Omega))} = 0.$$

Definition 3.3: The system (2.1) augmented with the output (2.2) is regionally asymptotically gradient detectable (or *G*-detectable) on ω if there exists an operator $H_\omega : R^q \longrightarrow (L^2(\omega))^n$, such that $(A - H_\omega C)$ generates a strongly continuous semi-group $S_{H_\omega}(t)_{t\geq 0}$, which is *G*-stable on $(L^2(\omega))^n$. We have the following important result:

Corollary 3.4: If the system (2.1) augmented with the output function (2.1) is exactly *G*-observable on ω, then it is *G*-detectable on ω. This gives the following relation: $\exists\, r > 0$, such that

$$\| \tilde{\chi}_\omega \nabla x \|_{L((L^2(\omega))^n, H^1(\Omega))} \leq r \, \| C S_A(.) x \|_{L(L^2(0,\infty,R^n), H^1(\Omega))},$$

$$\forall\, x \in (L^2(\omega))^n \quad (3.2)$$

Thus, the notion of regional *G*-detectable is far less restrictive than that of exact regional *G*-observable in ω. In this subsection, we shall develop a characterization result that links the regional asymptotic gradient detectability and sensors structures. For that purpose, we assume that the operator *A* has a complete set of eigenfunctions in $H^1(\Omega)$ denoted by φ_{m_j} orthonormal in $L^2(\omega)$ and the associated eigenvalues λ_m are of multiplicity r_m and assume also that the system (2.1) has *J* unstable modes. Thus, the sufficient condition of *G*-detectability on ω is formulated in the following theorem:

Theorem 3.5: Suppose there are *q* sensors $(D_i, f_i)_{1\leq i\leq q}$, the system (2.1) together with the output function (2.2) is *G*-detectable on ω if the sensors are *G*-strategic for the unstable subsystem of (2.1).

Proof: Under the assumptions of section 2, the system (2.1) can be decomposed by the projections P and $I - P$ on two parts, unstable and stable. The sensors $(D_i, f_i)_{1\leq i\leq q}$ is *G*-strategic on ω for the unstable part of the system (2.1), may be written in the form:

$$\begin{cases} \dfrac{\partial x_1}{\partial t}(\xi,t) = A_1 x_1(\xi,t) + PBu(t) & \Theta \\ x_1(\xi,0) = x_{0_1}(\xi) & \Omega \\ x_1(\eta,t) = 0 & \Pi \end{cases} \qquad (3.3)$$

where the operator A_1 is represented by a matrix of order $\left(\sum_{m=1}^{J} r_m, \sum_{m=1}^{J} r_m\right)$ given by

$$A_1 = \text{diag}\,[\lambda_1, \ldots, \lambda_1, \lambda_2, \ldots, \lambda_2, \ldots, \lambda_J, \ldots, \lambda_J]$$

and $PB = [G_1^{tr}\, G_2^{tr} \ldots G_J^{tr}]$,

then that sensors satisfies the conditions:

(1) $q \geq m$,

(2) rank $G_m = r_m$, $\forall\, m$, $m = 1, 2, \ldots, J$, with

$$(G_m)_{ij} = \begin{cases} \displaystyle\sum_{k=1}^{n} \left\langle \frac{\varphi_{m_j}}{\partial \xi_k}, f_i(.) \right\rangle_{L^2(D_i)} & \text{zone sensor case,} \\ \displaystyle\sum_{k=1}^{n} \frac{\varphi_{m_j}}{\partial \xi_k}(b_i) & \text{pointwise sensor case,} \end{cases}$$

where $\sup r_m = m < \infty$ and $1 \leq j \leq r_m$.

The subsystem (3.3) is weakly *G*-observable on ω, and since it is finite dimensional, then it is exactly *G*-observable on ω [10]. Therefore, it is *G*-detectable on ω, and hence there exists and operator H_ω^1 such that $A_1 - H_\omega^1 C$ generates a *G*-stable semi-group on $(L^2(\omega))^n$, which satisfies the following: $\exists$ $M_\omega^1, \alpha_\omega^1 > 0$, such that

$$\| \tilde{\chi}_\omega \nabla e^{(A_1 - H_\omega^1 C)t} \|_{L((L^2(\omega))^n, H^1(\Omega))} \leq M_\omega^1 e^{-\alpha_\omega^1 t}$$

The solution of (3.3) is given by:

$$x_1(\xi, t) = e^{(A_1 - H_\omega^1 C)t} x_{0_1}(\xi, t)$$

and then we have:

$$\lim_{t\to\infty} \| \tilde{\chi}_\omega \nabla x_1(\xi, t) \|_{L((L^2(\omega))^n, H^1(\Omega))} = 0$$

The stable part of the system (2.1) may be written in the form:

$$\begin{cases} \dfrac{\partial x_2}{\partial t}(\xi,t) = A_2 x_2(\xi,t) + (I-P)Bu(t) & \Theta \\ x_2(\xi,0) = x_{0_2}(\xi) & \Omega \\ x_2(\eta,t) = 0 & \Pi \end{cases} \qquad (3.4)$$

where the solution of the subsystem (3.4) is given by:

$$x_2(\xi,t) = S_{A_2}(t-s)x_0(\xi) + \int_0^t S_{A_2}(t-s)(I-P)Bu(s)\, ds$$

and since this subsystem is stable, then it is *G*-stable. Therefore there exists $M_\omega^2, \alpha_\omega^2 > 0$, such that

$$\| \tilde{\chi}_\omega \nabla\, S_{A_2}(.) \|_{L((L^2(\omega))^n, H^1(\Omega))} \leq M_\omega^2 e^{-\alpha_\omega^2 t}$$

,

and then we have:

$$\lim_{t\to\infty} \| \tilde{\chi}_\omega \nabla x_2(\xi, t) \|_{L((L^2(\omega))^n, H^1(\Omega))} = 0$$

Consequently,

$$\lim_{t\to\infty} \| \tilde{\chi}_\omega \nabla x(\xi, t) \|_{L((L^2(\omega))^n, H^1(\Omega))} = 0$$

Finally, the system (2.1)-(2.2) is *G*-stable on ω.

4. Application to Sensors Structures:

In this section, we give specific results related to some examples of sensors structures and apply these results to different figures of domain, which is usually follow from symmetry consideration. We consider the two-dimensional system defined on $\Omega = (0,1)\times(0,1)$ by the form

$$\begin{cases} \dfrac{\partial x}{\partial t}(\xi_1,\xi_2,t) = \dfrac{\partial x^2}{\partial \xi_1^2}(\xi_1,\xi_2,t) + \dfrac{\partial x^2}{\partial \xi_2^2}(\xi_1,\xi_2,t) + x(\xi_1,\xi_2,t) & \Theta \\ x(\xi_1,\xi_2,0) = x_0(\xi_1,\xi_2) & \Omega \\ x(\eta_1,\eta_2,t) = 0 & \Pi \end{cases} \qquad (4.1)$$

Let $\omega = (\alpha_1, \beta_1) \times (\alpha_2, \beta_2)$ be the considered region is subset of $(0, 1) \times (0, 1)$. In this case the eigenfunctions of the system (4.1) for Dirichlet boundary conditions are given by:

$$\varphi_{m_n}(\xi_1,\xi_2) = \frac{2}{\sqrt{(\beta_1-\alpha_1)(\beta_2-\alpha_2)}} \sin m\pi(\frac{\xi_1-\alpha_1}{\beta_1-\alpha_1}) \sin n\pi(\frac{\xi_2-\alpha_2}{\beta_2-\alpha_2}) \quad (4.2)$$

associated with eigenvalues:

$$\lambda_{m_n} = -\left(\frac{m^2}{(\beta_1-\alpha_1)^2} + \frac{n^2}{(\beta_2-\alpha_2)^2}\right)\pi^2, n \geq 1, m \geq 1 \quad (4.3)$$

We suppose that $(\beta_1 - \alpha_1)^2/(\beta_2 - \alpha_2)^2 \notin Q$, since if this not the case the multiplicity of the eigenvalues may be infinite. With this hypothesis $r_m = 1$, for all m, *where* $m = 1, 2, \ldots, J$ and one sensor may be sufficient for *G*-detectable on ω. We illustrate some practical examples of the linear system (4.1).

4.1 Internal zone sensor:

Consider the system (4.1) together with the output function (2.3) where the sensor supports D are located in Ω. The output function (2.3) can be written by the form:

$$y(t) = \int_D x(\xi_1,\xi_2,t) f(\xi_1,\xi_2) d\xi_1 d\xi_2$$

(4.4)

where $D \subset \Omega$ is the location of the zone sensor and $f \in L^2(D)$

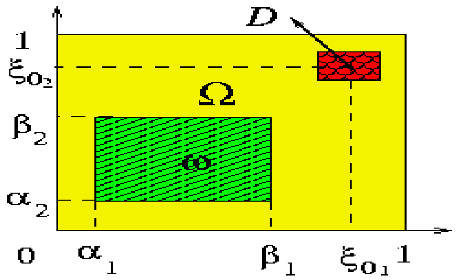

Figure 2: *Rectangular domain* Ω*, region* ω *and location* D *of zone sensor*

In this case of figure 2, the eigenfunctions and the eigenvalues are given in equation (4.2)-(4.3). Let the measurements support is rectangular with

$$D = [\xi_1 - l_1, \xi_1 + l_2] \times [\xi_2 - l_2, \xi_2 + l_2] \in \Omega$$

then we have the following results:

Corollary 4.1: Suppose that f is symmetric about $\xi_1 = \xi_{0_1}$ and f is symmetric with respect to $\xi_2 = \xi_{0_2}$, then the system (4.1) together with the output function (4.4) is not *G*-detectable on ω if $m(\xi_{0_1} - \alpha_1)/(\beta_1 - \alpha_1)$ and $n(\xi_{0_2} - \alpha_2) / (\beta_2 - \alpha_2) \in N$, for some $m, n = 1, 2, \ldots, J$.

4.2 Internal pointwise sensor:

We can discuss the case where the system (4.1) augmented with the following output:

$$y(t) = \int x(\xi_1,\xi_2,t)\delta(\xi_1 - b_1, \xi_2 - b_2) d\xi_1 d\xi_2 \quad (4.5)$$

where $b = (b_1, b_2)$ is the location of pointwise sensor in Ω (see figure 3). If $(\beta_1 - \alpha_1)/(\beta_2 - \alpha_2) \notin Q$, then $m = 1$ and one sensor (b, δ_b) may be sufficient for *G*-detectable on ω. Then we obtain:

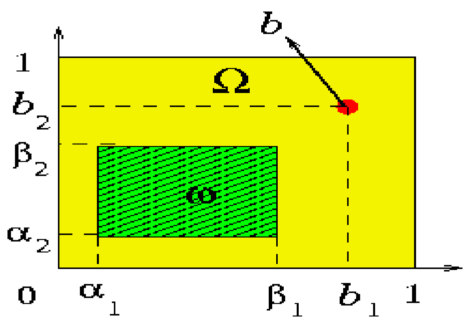

Figure 3: *Rectangular domain* Ω*, region* ω *and location* b *of pointwise sensor*

Corollary 4.3: The system (4.1)-(4.5) is *G*-detectable on ω if $m(b_1 - \alpha_1)/(\beta_1 - \alpha_1)$ $m \in N$ and $n(b_2 - \alpha_2)/(\beta_2 - \alpha_2) \notin N$, for all $m, n = 1, 2, \ldots, J$.

4.3 Application to a regional G-observer

Consider the system described by the parabolic equation

$$\begin{cases} \frac{\partial x}{\partial t}(\xi,t) = \frac{\partial x^2}{\partial \xi^2}(\xi,t) + x(\xi,t) + Bu(t) & \Theta \\ x(\xi,0) = x_0(\xi) & \Omega \\ x(0,t) = x(1,t) = 0 & \Pi \end{cases}$$

(4.Let $\omega = (0, \beta)$ be a subregion of $\Omega = (0, 1)$ and suppose that there exists a single sensor (D, f) with $D = [\xi_0, -l, \xi_0 + l] \subset (0, 1)$. Then, the augmented output function is given by:

$$y(., t) = \int_D x(\xi,t) f(\xi) d\xi \quad (4.7)$$

The eigenfunctions of he operator $(\frac{\partial^2}{\partial \xi^2} + 1)$ for the Dirichlet boundary conditions are defined by:

$$\varphi_m(\xi) = \left(\frac{2}{\beta}\right)^{1/2} \sin m\pi\left(\frac{\xi_0}{\beta}\right)$$

associated with the eigenvalues:

$$\lambda_m = 1 - \left(\frac{m\pi}{\beta}\right)^2$$

Now, consider the dynamical system:

$$\begin{cases} \frac{\partial z}{\partial t}(\xi,t) = \frac{\partial z^2}{\partial \xi^2}(\xi,t) + z(\xi,t) + Bu(t) - H_\omega C - (y(t) - z(\xi,t)) & \Theta \\ z(\xi,0) = z_0(\xi) & \Omega \\ z(0,t) = z(1,t) = 0 & \Pi \end{cases} \quad (4.8)$$

and suppose that the sensor (D, f) is *G*-strategic on ω for the unstable part of the system (4.6), then we have the following result:

Corollary 4.9: The system (4.8) is an identity regional gradient observer for the system (4.6)-(4.7) if:

$$\int_D \frac{\partial \varphi_m}{\partial \xi} f(\xi) \neq 0$$

(1) **Zone case:** If f is symmetric about $\xi = \xi_0$, then the system (4.8) is an identity regional gradient observer if $m\xi_0/\beta \in N$, $\forall$ m, $m = 1, 2, \ldots, J$.

(2) **Pointwise case:** If $f = \delta(\xi - b)$, where $D = \{b\} \in (0, 1)$ then the system (4.8) is an identity

regional gradient observer if $mb/\beta \in N, \forall m$, $m = 1, 2, \ldots, J$.

5. Conclusion

The original concept developed in this paper is related to the state gradient asymptotic detection possibility in a desired region in connection with the sensors structures. In this work, we have presented the existence of the sufficient condition of regional gradient detectability. Moreover, we have shown that it is possible to construct a regional gradient observer for regional gradient state of parabolic distributed systems. An extension of these results to the problem of the region ω is a part of the boundary of the system domain is under consideration.

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