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ESTIMATING AND PLANNING STEP STRESS ACCELERATED LIFE TEST FOR GENERALIZED LOGISTIC DISTRIBUTION UNDER TYPE-I CENSORING

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ABSTRACT

This paper presents estimation and derivation of optimum test plan for time step stress accelerated life test (SSALT). The maximum likelihood (ML) method is applied to estimate the unknown parameters of the generalized logistic distribution, to construct the asymptomatic confidence intervals, and to predict the value of the scale parameter and the reliability function under the usual conditions. The scale parameter of the lifetime distribution is assumed to be an inverse power law function of the stress level. Moreover, we consider minimizing the determinant of Fisher information matrix to obtain the optimum time of changing stress point, and also the optimum censoring time. Finally, numerical simulation is introduced.

KEYWORDS: Accelerated Life Test, Step Stress, Type-I Censoring, Maxi-Mum Likelihood Estimation, Fisher Information Matrix, Optimum Test Plan, Generalized Logistic Distribution

INTRODUCTION

With rapidly changing technologies, higher customer expectations for better reliability, the need for rapid device development, the necessity and the creativeness of more advanced technology in manufacturing field, it is difficult to produce enough amounts of failure units from ALT with only a constant stress level. Therefore, step stress ALT is required as an alternative to a constant stress ALT. The step-stress test has been widely used in many fields, for example electronic applications to reveal failure modes. The step-stress scheme applies stress to test units in such a way that the stress setting of test units is changed at certain specified times. Generally, a test unit starts at a low stress, if the unit does not fail at a specified time, stress on it is raised to a higher level and held a specified time. Stress is repeatedly increased and held, until the test unit fails. Several authors have studied the two popular and relate issues in CSALT and SSALT: estimation of the parameters and derivation of optimum test plans. For ex-ample, Singpurwalla (1971), Watkins (1991) and Abdel-Ghaly, et al. (1998) have studied statistical inference of CSALT. Additional to the statistical inference studies of CSALT, optimum CSALT plans were studied for different lifetime distributions based on different censoring scheme; for example, Nelson and Kielpinski (1976), Nel-son (1990), Attia, et al. (2011a), and Attia, et al. (2011b).

Concerning estimation of SSALT, Nelson (1980) was the first one who estimate the parameters of weibull distribution using ML method under SSALT. Dharmad-hikari and Rahman (2003) obtained ML estimators and their confidence intervals of weibull and log-normal distributions. Sang (2005) applied SSALT on the generalized exponential distribution and used ML method for estimating its parameters. Aly (2008) dealt with a k-level step-stress accelerated life test under progressive type-I censoring with grouped data to estimate the parameters of the log-logistic distribu-tion using ML approach. Abdel-Hamid and AL-Hussaini (2009) considered simple step stress ALT under type-I censoring using the exponentiated exponential distri-bution and obtained its ML estimators. Kateri, et al. (2009) developed inference for multi-sample simple step-stress model under exponentially distributed lifetime with Type-II censoring, while in (2010), they considered the exact and explicit inferential results for Type-I censored case. Bing (2010) derived the exact and approximate con-fidence intervals for the exponential distribution in case of SSALT under progressive Type-II censoring.

Many authors studied optimum SSALT plans for different lifetime distributions based on different censoring scheme. For example, Bai, et al. (2002) obtained op-timum simple time step and failure step stress when the lifetime was exponentially distributed. Nesar, et al. (2006) suggested optimal ALT plans for units whose lifetime follows the exponentiated weibull distribution under periodic inspection and type-I censoring. Shuo-Jye, et al. (2006) investigated the methods for obtaining test plan by using the variance optimality and the D-optimality criteria for k-stage step stress under progressive type-I censoring with grouped data. Al-Haj and Al-Masri (2007) obtained optimum test plan for simple step stress ALT using the log-logistic distribu-tion and considering the case of a pre-specified censoring time. Srivastava and Shukla (2008) presented estimation and optimum test plan for simple time-step-stress accel-erated life tests. Al-Haj and Al-Masri (2010) presented optimum times of changing stress level for simple step-stress and three-step stress plans, respectively. Nesar et al. (2010) discussed ALT design for the generalized exponential distribution with log linear model under periodic inspection and type-I censoring.

This paper is organized as follows: In Section 2, we describe the Cumulative Exposure (CE) model and its assumptions. In Section 3, we use the ML method to obtain point and interval estimation of the model parameters. Moreover, the asymptomatic variance-covariance matrix, and the predictive values of both the scale parameter and the reliability function under usual conditions are also introduced in Section 3. Optimum test plan for time of changing stress, and censoring time step stress ALT are addressed in Section 4. Section 5 presents the numerical results.

THE CUMULATIVE EXPOSURE MODEL

Since a test unit in a step-stress test is exposed to several different stress levels, its lifetime distribution combines lifetime distributions from all stress levels used in the test. The cumulative exposure model of the lifetime in a step-stress life testing continuously pieces these lifetime distributions in the order that the stress levels are applied. More specifically, the step-stress cumulative exposure model assumes that the remaining lifetime of a test unit depends on the current cumulative fraction failed and the current stress, regardless of how the fraction is accumulated. In addition, if held at the current stress, survivors will fail according to the cumulative distribution for that stress but starting at the previously accumulated fraction failed (for more details, see Nelson (1990)). We assume the following assumptions for the step stress ALT procedure

- V_0 denote the design stress that is the stress level under normal use conditions.
- $V_1 < \ldots < V_k$ denote the k-stress levels gradually applied in that order in a step stress testing.
- τ_j be the time that stress changed from V_j to V_{j+1}, $1 \le j \le k-1$; $\tau_1 < \ldots < \tau_{k-1}$.
- The failure times x_{ij}, $i = 1,2,\ldots,n_j$, and $j = 1,2,\ldots,k$ at stress levels V_j, $j = 1,2,\ldots,k$ are the three-parameter generalized logistic distribution with probability density function

$$f(x_{ij},\alpha_j,\gamma,\theta) = \alpha_j\,\gamma\, e^{\alpha_j x_{ij}}\left(1+\frac{\gamma}{\theta}e^{\alpha_j x_{ij}}\right)^{-(\theta+1)},\ -\infty < x_{ij} < \infty,\ \alpha_j,\ \gamma,\ \theta > 0, \tag{1}$$

- The scale parameter α_j, $j = 1, \ldots, k$ of the underlying lifetime distribution (1) is assumed to have an inverse power law function on the stress levels i.e.,

$$\alpha_j = CS_j^P, \quad j = 1, 2, ..., k, \; C, P > 0, \tag{2}$$

where $S_j = \frac{V^*}{V_j}$, $V^* = \prod_{j=1}^{k} V_j^{b_j}$, $b_j = \frac{n_j}{N}$, $N = \sum_{j=1}^{k} n_j$, C is the constant of proportionality, and is the power of the applied stress.

According to the cumulative exposure model in (Nelson, (1990)), the cumulative distribution function of X is given by

$$G(x) = \begin{cases} F_1(x) & 0 \le x < \tau_1 \\ F_2(x - \tau_1 + u_{11}) & \tau_1 \le x < \tau_2, \\ \vdots & \\ F_k(x - \tau_{k-1} + u_{k-1}) & \tau_{k-1} \le x < \infty \end{cases} \tag{3}$$

where $F_j(x - \tau_{j-1} + u_{j-1}) = 1 - \left(1 + \frac{\gamma}{\theta} e^{CS_k^P[x - \tau_{j-1} + u_{j-1}]}\right)^{-\theta}$ and u_{j-1} is determined by

$$F_j(u_{j-1}) = F_{j-1}(\tau_{j-1} - \tau_{j-2} + u_{j-2}), \quad j = 2, ..., k; u_0 = 0. \tag{4}$$

By solving equations (4) for gives

$$u_{j-1} = \left(\frac{S_{j-1}}{S_j}\right)^P (\tau_{j-1} - \tau_{j-2} + u_{j-2}) \tag{5}$$

Thus, the associated probability density

$$g(x) = \begin{cases} f_1(x) & 0 \le x < \tau_1 \\ f_j(x - \tau_{j-1} + u_{j-1}) & \tau_1 \le x < \tau_2, \\ \vdots & \\ f_k(x - \tau_{k-1} + u_{k-1}) & \tau_{k-1} \le x < \infty \end{cases} \tag{6}$$

MAXIMUM LIKELIHOOD (ML) ESTIMATION

As defined by the assumptions in section 2, and if we consider the case of time censored samples, the likelihood function of the experiment is considered to have the following form

$$L = \prod_{j=1}^{k} \prod_{i=1}^{n_j} \left[f_j(x_{ij} - \tau_{j-i} + u_{j-i})\right]\left[1 - F_k(T - \tau_{k-i} + u_{k-i})\right]^{B}, \tag{7}$$

Where,

$$f_j[x_{ij} - \tau_{j-1} + u_{j-1}] = \gamma\, CS_j^P\, e^{CS_j^P[x_{ij} - \tau_{j-1} + u_{j-1}]} \left(1 + \frac{\gamma}{\theta} e^{CS_j^P[x_{ij} - \tau_{j-1} + u_{j-1}]}\right)^{-(\theta+1)} \tag{8}$$

and

$$F_k\left[T-\tau_{k-1}+u_{k-1}\right]=1-\left(1+\frac{\gamma}{\theta}\,e^{CS_k^P\left[T-\tau_{k-1}+u_{k-1}\right]}\right)^{-\theta} \tag{9}$$

$\mathrm{B}=N-\sum_{j=1}^{k} r_j$ is the number of survival units, and is the censored time. Ac-cording to the equations (8), and (9), the likelihood function (7) will be written as the following form

$$L=\prod_{j=1}^{k}\prod_{i=1}^{n_j}\left[CS_j^P\gamma\;\; e^{CS_j^P\left[x_{ij}-\tau_{j-1}+u_{j-1}\right]}\left(1+\frac{\gamma}{\theta}\,e^{CS_j^P\left[x_{ij}-\tau_{j-1}+u_{j-1}\right]}\right)^{-(\theta+1)}\right]\left[\left(1+\frac{\gamma}{\theta}\,e^{CS_k^P\left[T-\tau_{k-1}+u_{k-1}\right]}\right)\right]^{-\mathrm{B}\,\theta} \tag{10}$$

The log-likelihood function

$$\begin{aligned} ln\,L = N\,ln\,C + N\,ln\,\gamma + C\sum_{j=1}^{k}\sum_{i=1}^{n_j} S_j^P\left(x_{ij}-\tau_{j-1}+u_{j-1}\right)-(\theta+1)\\ \sum_{j=1}^{k}\sum_{i=1}^{n_j} ln\left(1+\frac{\gamma}{\theta}e^{C\,S_j^P\left(x_{ij}-\tau_{j-1}+u_{j-1}\right)}\right)-\theta\mathrm{B}\,ln\left(1+\frac{\gamma}{\theta}e^{C\,S_k^P\left(T-\tau_{k-1}+u_{k-1}\right)}\right)\end{aligned} \tag{11}$$

where, $\sum_{j=1}^{k} n_j\,ln\,S_j=0.$

ML Estimation of the Parameters

The first derivatives of the log-likelihood function (11) with respect to the unknown parameters (C,P,γ,θ) are

$$\frac{\partial\,ln\,L}{\partial C}=\frac{N}{C}+\sum_{j=1}^{k}\sum_{i=1}^{n_j} S_j^P\left(x_{ij}-\tau_{j-1}+u_{j-1}\right)-(\theta+1)\sum_{j=1}^{k}\sum_{i=1}^{r_j}\;\xi_{ij}-\theta\;Z_{\boldsymbol{k}}\,. \tag{12}$$

$$\frac{\partial\,ln\,L}{\partial P}=C\left\{\sum_{j=1}^{k}\sum_{i=1}^{n_j}\sigma_{ij}-(\theta+1)\sum_{j=1}^{k}\sum_{i=1}^{n_j}\xi_{\boldsymbol{ij}}\;ln\,S_j-\theta Z_k\;ln\,S_j\right\}. \tag{13}$$

$$\frac{\partial\,ln\,L}{\partial\gamma}=\frac{1}{\gamma}\left\{N-(\theta+1)\sum_{j=1}^{k}\sum_{i=1}^{n_j}\nu_{ij}-\;\theta\mathrm{B}\mu_k\right\}. \tag{14}$$

$$\frac{\partial\,\ln L}{\partial\theta}=\left\{\sum_{j=1}^{k}\sum_{i=1}^{n_j}\left(\frac{(\theta+1)}{\theta}\nu_{ij}-\pi_{ij}\right)+\mathrm{B}\left(\mu_k-\Lambda_k\right)\right\}. \tag{15}$$

where

$$\begin{aligned}
\xi_{ij}&=S_j^P\left(x_{ij}-\tau_{j-1}+u_{j-1}\right)\,\nu_{\boldsymbol{ij}}\;,\\
\nu_{\boldsymbol{ij}}&=\left(1+\frac{\theta}{\gamma}\,e^{-C\;S_j^P\left(x_{ij}-\tau_{j-1}+u_{j-1}\right)}\right)^{-1},\\
\sigma_{ij}&=S_j^P\left(x_{ij}-\tau_{j-1}+u_{j-1}\right)ln\,S_j\;,\qquad ,\\
\pi_{ij}&=ln\left(1-\nu_{ij}\right),\\
Z_k&=\mathrm{B}S_k^P\left(T-\tau_{k-1}+u_{k-1}\right)\;\mu_{\boldsymbol{k}}\;,\\
\mu_{\boldsymbol{k}}&=\left(1+\frac{\theta}{\gamma}\,e^{-C\;S_k^P\left(T-\tau_{k-1}+u_{k-1}\right)}\right)^{-1},and\\
\Lambda_k&=ln\left(1-\mu_k\right).
\end{aligned}$$

Since the first derivative equations (12) to (15) are non-linear equations, their solutions will be obtained numerically as will be seen in section (5.1). The second partial derivatives of the log-likelihood function (11) with respect to the parameters are as follows

$$\frac{\partial^2 \ln L}{\partial C^2} = -\Big\{ \frac{N}{C^2} + (\theta+1)\sum_{j=1}^{k}\sum_{i=1}^{n_j} S_j^P \left(x_{ij} - \tau_{j-1} + u_{j-1}\right) \xi_{ij}\phi_{ij} + \theta S_k^P \left(T - \tau_{k-1} + u_{k-1}\right) Z_k \varphi_k \Big\}, \tag{16}$$

$$\frac{\partial^2 \ln L}{\partial P^2} = -C\Big\{ \sum\sum \left[(\theta+1)\xi_{ij}\left(C\sigma_{ij}\phi_{ij} + \ln S_j\right) - \sigma_{ij}\right]\ln S_j + \theta Z_k \left(C\,\Omega_k \varphi_k + \ln S_k\right)\ln S_k \Big\}, \tag{17}$$

$$\frac{\partial^2 \ln L}{\partial \gamma^2} = \frac{-1}{\gamma^2}\left\{ N - (\theta+1)\sum_{j=1}^{k}\sum_{i=1}^{n_j} \nu_{ij}^2 - \theta B \mu_k^2 \right\}, \tag{18}$$

$$\frac{\partial^2 \ln L}{\partial \theta^2} = \frac{-1}{\theta}\left\{ \sum_{j=1}^{k}\sum_{i=1}^{n_j} \nu_{ij}\left[(\theta+1)\,\phi_{ij} + (1-\theta)\right] - \theta B \mu_k^2 \right\}, \tag{19}$$

$$\frac{\partial^2 \ln L}{\partial C \partial P} = -\Big\{ \sum_{j=1}^{k}\sum_{i=1}^{n_j} \left[(\theta+1)\xi_{ij}\left(C\sigma_{ij}\phi_{ij} + \ln S_j\right) - \sigma_{ij}\right] + \theta Z_k \left(C\,\Omega_k \varphi_k + \ln S_k\right)\ln S_k \Big\}, \tag{20}$$

$$\frac{\partial^2 \ln L}{\partial C \partial \gamma} = \frac{-1}{\gamma}\left\{ (\theta+1)\sum_{j=1}^{k}\sum_{i=1}^{n_j} \xi_{ij}\phi_{ij} + \theta Z_k \phi_k \right\}, \tag{21}$$

$$\frac{\partial^2 \ln L}{\partial C \partial \theta} = -\left\{ \sum_{j=1}^{k}\sum_{i=1}^{n_j} \xi_{ij}\left(1 - \frac{(\theta+1)}{\theta}\phi_{ij}\right) + \mu_k Z_k \right\}, \tag{22}$$

$$\frac{\partial^2 \ln L}{\partial P \partial \gamma} = \frac{-C}{\gamma}\ \left\{ (\theta+1)\ \sum_{j=1}^{k}\sum_{i=1}^{n_j} \xi_{ij}\phi_{ij}\ \ln S_j + \theta Z_k \varphi_k\ \ln S_k \right\}, \tag{23}$$

$$\frac{\partial^2 \ln L}{\partial P\, \partial \theta} = -C\left\{ \sum_{j=1}^{k}\sum_{i=1}^{n_j} \xi_{ij}\left(1 - \frac{(\theta+1)}{\theta}\varphi_{ij}\right)\ln S_j + Z_k \mu_k \ln S_k \right\}, \tag{24}$$

$$\frac{\partial^2 \ln L}{\partial \gamma \partial \theta} = \frac{-1}{\gamma}\left\{ \sum_{j=1}^{k}\sum_{i=1}^{n_j} \nu_{ij}\left(1 - \frac{(\theta+1)}{\theta}\phi_{ij}\right) + B\mu_k^2 \right\}. \tag{25}$$

where

$$\phi_{ij} = 1 - \nu_{ij},$$
$$\varphi_k = 1 - \mu_k, and$$
$$\Omega_k = S_k^P \left(T - \tau_{k-1} + u_{k-1}\right) \ln S_k$$

Therefore, the elements of Fisher information matrix for the MLE can be obtained as the expectations of the negative of the second partial derivatives, i.e.,

$$F=\begin{bmatrix} f_{11} & f_{12} & f_{13} & f_{14} \\ & f_{22} & f_{23} & f_{24} \\ & & f_{33} & f_{34} \\ & & & f_{44} \end{bmatrix} = -E\begin{bmatrix} \frac{\partial^2 lnL}{\partial^2 C} & \frac{\partial^2 lnL}{\partial C\partial P} & \frac{\partial^2 lnL}{\partial C\partial \gamma} & \frac{\partial^2 lnL}{\partial C\partial \theta} \\ & \frac{\partial^2 lnL}{\partial^2 P} & \frac{\partial^2 lnL}{\partial P\partial \gamma} & \frac{\partial^2 lnL}{\partial P\partial \theta} \\ & & \frac{\partial^2 lnL}{\partial^2 \gamma} & \frac{\partial^2 lnL}{\partial \gamma\partial \theta} \\ & & & \frac{\partial^2 lnL}{\partial^2 \theta} \end{bmatrix} \tag{26}$$

The asymptotic variance-covariance matrix for the MLE is defined as the inverse of Fisher's information matrix (26), i.e.,

$$\sum = F^{-1} \tag{27}$$

where

$$\hat{F}=-\begin{bmatrix} \frac{\partial^2 lnL}{\partial^2 C} & \frac{\partial^2 lnL}{\partial C\partial P} & \frac{\partial^2 lnL}{\partial C\partial \gamma} & \frac{\partial^2 lnL}{\partial C\partial \theta} \\ & \frac{\partial^2 lnL}{\partial^2 P} & \frac{\partial^2 lnL}{\partial P\partial \gamma} & \frac{\partial^2 lnL}{\partial P\partial \theta} \\ & & \frac{\partial^2 lnL}{\partial^2 \gamma} & \frac{\partial^2 lnL}{\partial \gamma\partial \theta} \\ & & & \frac{\partial^2 lnL}{\partial^2 \theta} \end{bmatrix} \downarrow \left(\hat{C},\hat{P},\hat{\gamma},\hat{\theta}\right) \tag{28}$$

Prediction of the Scale Parameter and the Reliability Function

To predict the value of the scale parameter α_u under the usual condition stress Vu, the invariance property of MLE is used (for more details see, Meeker and Escobar (1998)), i.e.,

$$\hat{\alpha} = \hat{C}S_u^{\hat{P}}, \tag{29}$$

where,

$$S_u = \frac{V^*}{V_u}, \quad V^* = \prod_{j=1}^{k} V_j^{b_j} \text{, and } b_j = \frac{n_j}{N}.$$

The MLE of the reliability function at the lifetime x0 under the usual condition stress Vu, is given

$$\hat{R}_u(x_0) = \left(1+\frac{\gamma}{\theta}e^{\hat{\alpha}_u x_0}\right)^{-\theta} \tag{30}$$

OPTIMUM TEST PLAN

Before starting an ALT (which is sometimes an expensive and difficult endeavor), it is advisable to have a plan that helps in accurately estimating reliability at operating conditions while minimizing test time and cost. Poor planning means waste time, effort and money and may not even yield the desired information. But good planning does not only lead to shorter test time or fewer test specimens or both; but more importantly; a good test plan will result in a more precise estimate for the reliability measure.

Time of Changing Stress Test Plan

This section presents the commonly used test planning method for determining the optimum time of changing stress values $\tau_{j-1}, j = 2,\ldots,k$ According to the D-Optimality criterion which is based on minimizing the determinant of the Fisher information matrix of the MLE of the model parameters (Gouno(2007)), the optimal value of $\tau_{j-1}, j = 2,\ldots,k$ at each stress level can be obtained by solving the following equation

$$\frac{\partial |F|}{\partial \tau_{j-1}} = 0, \quad j = 2,\ldots,k. \tag{31}$$

The determinant of F and the derivation of the equation (31) are placed in Appendix A. To get the optimum time of changing stress value τ1 that minimized |F| of the MLE under simple step stress level V_j, $j = 1, 2$ numerical results will be given in section (5.2) to illustrate the application of the planning methodology.

The Censoring Time Test Plan

In this section, we determine the best choice value of the censoring time T by minimizing the determinant of Fisher information matrix of the MLE of the model parameters. Therefore, the optimal value of T can be obtained by solving the following equation

$$\frac{\partial |F|}{\partial T} = 0. \tag{32}$$

The determinant of F and the derivation of the equation (32) are placed in Appendix B. The numerical solution for determining the optimum value of the censoring time T , is obtained as will be shown in section (5.2).

SIMULATION STUDIES

In this section, numerical examples are given for ML estimation and optimum test plan under type-I censoring. The Math-Cade program is used to calculate the MLE of the unknown parameters (C,P,γ,θ), their properties, and optimum test plan of the SSALT.

MLE under Type-I Censoring

In this section, the numerical solution is performed as follows. For given values of C,P and under three step stress level $V_j, j = 1, 2, 3$ the values of $\alpha_j, j = 1, 2, 3$ are calculated according to the equation (2). Generate a random sample of size N from the three-parameter generalized logistic distribution and obtained the random variables x_{ij} ($i = 1,2,\ldots,n_j$, and $j = 1,2,\ldots,k$) for given values of n_j, τ_j , $j = 1, 2, 3$ and different initial guesses of the true parameters α, γ, θ say $\alpha_0, \gamma_0, \theta_0$. Based on the values of n_j, τ_j, V_j, x_{ij} ($i = 1,2,\ldots,n_j$, $j = 1,2,\ldots,k$), and V_u , maximum likelihood estimators (MLE), mean square error (MSE), relative bias (RAB), lower bound (LB), upper bound (UB), and estimated the scale parameter $\hat{\alpha}_u$ and the reliability function $\hat{R}_u(x_0)$ are obtained. The points are repeated more than 150 times until got the MLE as shown in table (1). The numerical results which are placed in tables (1) to (3) are based on $n_1 = 20$, $n_2 = 20$, $n_3 = 20$, $T = 10, \mathrm{B} = 9, \tau_0 = 0, \tau_1 = 1.5,$

$\tau_2 = 1.75, \quad V_1 = 0.75, \; V_2 = 1.0, \; V_3 = 2.0,$ and $V_u = 0.5$.

From the results of the tables (1) to (3), we observe the MSE of the scale parameter $\alpha_j, j = 1, 2, 3$ decreases as the stress value $V_j, j = 1, 2, 3$. In addition, we note the covariance between C and θ is the smallest one and it is converges to zero. On the other hand, the reliability decreases when the mission time x_0 increases. Moreover, there is an inverse proportional relationship between $\hat{\alpha}_u$ and $\hat{R}_u(x_0)$ at the same mission time.

Table 1: The MLE, RAB, and MSE

(C_0=0.5, P_0=1.0, γ_0=1.0, θ_0=0.5, α_{01} =0.7631, α_{02} =0.5724, α_{03} =0.2862)							
Parameter	**C**	**P**	**γ**	**θ**	**α_1**	**α_2**	**α_3**
MLE	0.5126	1.2431	1.1671	0.4327	0.8671	0.6064	0.2562
RAB	0.0252	0.2431	0.1671	0.1347	0.1362	0.0595	0.1048
MSE	0.0002	0.0591	0.0279	0.0045	0.0108	0.0012	0.0009
(C_0=0.5, P_0=1.2, γ_0=1.0, θ_0=0.45, α_{01} =0.8305, α_{02} =0.5880, α_{03} =0.256)							
Parameter	**C**	**P**	**γ**	**θ**	**α_1**	**α_2**	**α_3**
MLE	0.5092	1.3032	1.3233	0.4288	0.8834	0.6072	0.2461
RAB	0.0183	0.0872	0.3233	0.0472	0.0637	0.0326	0.0386
MSE	0.0001	0.0106	0.1046	0.0005	0.0028	0.0004	0.0001
(C_0=0.4, P_0=1.2, γ_0=1.0, θ_0=0.5, α_{01} =0.6644, α_{02} =0.4704, α_{03} =0.2048)							
Parameter	**C**	**P**	**γ**	**θ**	**α_1**	**α_2**	**α_3**
MLE	0.4905	1.2060	1.1988	0.3894	0.8168	0.5773	0.2502
RAB	0.2262	0.0050	0.1988	0.2213	0.2293	0.2272	0.2221
MSE	0.0082	0.0000	0.0395	0.0122	0.0232	0.0114	0.0114
(C_0=0.4, P_0=1.0, γ_0=0.95, θ_0=0.5, α_{01} =, α_{02} =, α_{03} =)							
Parameter	**C**	**P**	**γ**	**θ**	α_1	α_2	α_3
MLE	0.4754	1.1659	1.0323	0.4020	0.7784	0.5566	0.2480
RAB	0.1885	0.1659	0.0866	0.1960	0.2749	0.2155	0.0834
MSE	0.0057	0.0275	0.0096	0.0068	0.0282	0.0097	0.0004
(C_0=0.4, P_0=1.0, γ_0=0.95, θ_0=0.6, α_{01} =0.6105, α_{02} =0.4579, α_{03} =0.2289)							
Parameter	**C**	**P**	**γ**	**θ**	**α_1**	**α_2**	**α_3**
MLE	0.4380	1.2105	0.9770	0.4255	0.7308	0.5159	0.223
RAB	0.0950	0.2105	0.0284	0.2908	0.1970	0.1266	0.026
MSE	0.0014	0.0443	0.0007	0.0305	0.0145	0.0034	0.0000
(C_0=0.3, P_0=1.2, γ_0=0.9, θ_0=0.5, α_{01} =0.4983, α_{02} =0.3528, α_{03} =0.1536)							
Parameter	**C**	**P**	**γ**	**θ**	**α_1**	**α_2**	**α_3**
MLE	0.4208	1.1177	0.9765	0.3573	0.6751	0.4895	0.2256
RAB	0.4028	0.0686	0.0850	0.2854	0.3548	0.3873	0.4687
MSE	0.0146	0.0068	0.0068	0.0204	0.0313	0.0187	0.0052
(C_0=0.3, P_0=1.3, γ_0=1.0, θ_0=0.6, α_{01} =0.5198, α_{02} =0.3576, α_{03} =0.1452)							
Parameter	**C**	**P**	**γ**	**θ**	**α_1**	**α_2**	**α_3**
MLE	0.4404	1.1353	1.0573	0.3479	0.7117	0.5134	0.234
RAB	0.4679	0.1267	0.0573	0.4202	0.3668	0.4356	0.609
MSE	0.0197	0.0271	0.0033	0.0636	0.0368	0.0243	0.008
(C_0=0.3, P_0=1.3, γ_0=0.8, θ_0=0.45, α_{01} =0.5198, α_{02} =0.3576, α_{03} =0.1452)							
Parameter	**C**	**P**	**γ**	**θ**	**α_1**	**α_2**	**α_3**
MLE	0.3646	1.2136	0.9456	0.3864	0.6091	0.4296	0.1852
RAB	0.2154	0.0665	0.1821	0.1413	0.1718	0.2013	0.2754
MSE	0.0042	0.0080	0.0212	0.0040	0.0080	0.0052	0.0016

Table 2: Asymptotic Var-Cov Matrix and the Confidence Intervals

$(C_0=0.5, P_0=1.0, \gamma_0=1.0, \theta_0=0.5)$

Parameter	Var-Cov Matrix				L.B	U.B
	C	P	γ	θ		
C	0.0086	-0.0155	0.0199	-0.0084	0.3597	0.6655
P		0.1051	-0.0347	0.0175	0.7097	1.7764
γ			0.1612	-0.0292	0.5066	1.8276
θ				0.015	0.2312	0.6341

(C0=0.5, P0=1.2, γ0=1.0, θ0=0.45)

Parameter	Var-Cov Matrix				L.B	U.B
	C	P	γ	θ		
C	0.0083	-0.0168	0.0234	-0.0078	0.3589	0.6594
P		0.11010	-0.04500	0.0183	0.7574	1.8489
γ			0.2142	-0.0327	0.5619	2.0847
θ				0.0139	0.2348	0.6227

$(C_0=0.4, P_0=1.0, \gamma_0=0.95, \theta_0=0.5)$

Parameter	Var-Cov Matrix				L.B	U.B
	C	P	γ	θ		
C	0.0076	-0.0122	0.0163	-0.0075	0.3317	0.6192
P		0.0987	-0.0229	0.0129	0.6492	1.6827
γ			0.1313	-0.0241	0.4362	1.6284
θ				0.0129	0.2149	0.5891

(C0=0.4, P0=1.2, γ0=1.0, θ0=0.5)

Parameter	Var-Cov Matrix				L.B	U.B
	C	P	γ	θ		
C	0.0077	-0.0136	0.0204	-0.0068	0.3458	0.6352
P		0.1015	-0.032	0.0133	0.682	1.7301
γ			0.1863	-0.0268	0.4887	1.9089
θ				0.0111	0.2162	0.5626

$(C_0=0.4, P_0=1.0, \gamma_0=0.95, \theta_0=0.6)$

Parameter	Var-Cov Matrix				L.B	U.B
	C	P	γ	θ		
C	0.0067	-0.0116	0.0136	-0.0077	0.3036	0.5724
P		0.1002	-0.0197	0.0136	0.6898	0.6898
γ			0.1113	-0.024	0.4281	1.5259
θ				0.0154	0.2216	0.6294

$(C_0=0.3, P_0=1.2, \gamma_0=0.9, \theta_0=0.5)$

Parameter	Var-Cov Matrix				L.B	U.B
	C	P	γ	θ		
C	0.0061	-0.0093	0.0144	-0.0059	0.2922	0.5495
P		0.0961	-0.0153	0.0086	0.6078	1.6275
γ			0.1302	-0.0207	0.3829	1.5701
θ				0.0098	0.1946	0.5201

$(C_0=0.3, P_0=1.3, \gamma_0=1.0, \theta_0=0.6)$

Parameter		Var-Cov Matrix			L.B	U.B
	C	P	γ	θ		
C	0.0065	-0.0101	0.0167	-0.0057	0.3077	0.573
P		0.0966	-0.0193	0.0088	0.6241	1.6466
γ			0.1578	-0.0217	0.4039	1.7108
θ				0.0088	0.1936	0.5021

$(C_0=0.3, P_0=1.3, \gamma_0=0.8, \theta_0=0.45)$

Parameter		Var-Cov Matrix			L.B	U.B
	C	P	γ	θ		
C	0.0046	-0.0087	0.0112	-0.0056	0.2525	0.4768
P		0.0999	-0.0127	0.0091	0.6936	1.7336
γ			0.1131	-0.0211	0.3924	1.4988
θ				0.0119	0.2068	0.5660

Table 3: Estimates of α and R(x0) under Normal Conditions

(C_0=0.5, P_0=1.0, γ_0=1.0, θ_0=0.5)			(C_0=0.5, P_0=1.2, γ_0=1.0, θ_0=0.45)		
$\hat{\alpha}_u$	x_0	$R_u(x_0)$	$\hat{\alpha}_u$	x_0	$R_u(x_0)$
1.4353	0.01	0.5653	1.4984	0.01	0.5442
	0.50	0.4441		0.50	0.4208
	1.00	0.3372		1.00	0.3148
	1.50	0.2518		1.50	0.2319
(C_0=0.4, P_0=1.0, γ_0=0.95, θ_0=0.5)			(C_0=0.4, P_0=1.0, γ_0=095, θ_0=0.6)		
$\hat{\alpha}_u$	x_0	$R_u(x_0)$	$\hat{\alpha}_u$	x_0	$R_u(x_0)$
1.2488	0.01	0.5975	1.1938	0.01	0.5999
	0.50	0.4935		0.50	0.4970
	1.00	0.3970		1.00	0.4008
	1.50	0.3149		1.50	0.3181
(C_0=0.4, P_0=1.2, γ_0=1.0, θ_0=0.5)			(C_0=0.3, P_0=1.2, γ_0=0.9, θ_0=0.5)		
$\hat{\alpha}_u$	x_0	$R_u(x_0)$	$\hat{\alpha}_u$	x_0	$R_u(x_0)$
1.3319	0.01	0.5762	1.0621	0.01	0.6229
	0.50	0.4689		0.50	0.5387
	1.00	0.3721		1.00	0.4578
	1.50	0.2915		1.50	0.3851
(C_0=0.3, P_0=1.3, γ_0=1.0, θ_0=0.6)			(C_0=0.3, P_0=1.3, γ_0=0.8, θ_0=0.45)		
$\hat{\alpha}_u$	x_0	$R_u(x_0)$	$\hat{\alpha}_u$	x_0	$R_u(x_0)$
1.1277	0.01	0.6135	0.9963	0.01	0.6182
	0.50	0.5259		0.50	0.5358
	1.00	0.4430		1.00	0.4561
	1.50	0.3695		1.50	0.3840

To illustrate the procedure of optimum test design, numerical simulations are given as follows: Suppose that a simple step stress test to estimate the optimum value of time of changing stress τ_1^*. The stress levels to test units are $V_1 = 1.0,$ and $V_2 = 3.0.$ Based on $\tau_0 = 0,$ $\tau_1 = 2.0,$ $T = 1.25$ with different initial guesses of the true parameters α, γ, θ say $\alpha_0, \gamma_0, \theta_0$ and different values of N, the optimum time of the changing stress τ_1^* is determined by solving equation (31). Table (4) presents the optimal value of τ_1^* at different specified values of (C, P, γ, θ) and the Generalized Asymptotic Variance (GAV).

Table 4: Optimum Values of τ1∗ and GAV

(C_0=1.0, P_0=0.5, γ_0=0.5, θ_0=0.75)					
N	n_1	n_2	R	τ_1*	GAV
30	15	15	11	3.9	0.0000123
40	20	20	13	2.8	0.0000052
50	25	25	22	3.2	0.0000010
60	30	30	32	3.8	0.0000003
70	35	35	37	3.3	0.0000002
110	55	55	37	1.3	0.0000002
154	77	77	57	1.5	0.0000001
(C_0=1.0, P_0=0.6, γ_0=0.5, θ_0=0.75)					
N	n_1	n_2	R	τ_1*	GAV
30	15	15	11	4.1	0.0000134
40	20	20	13	3.0	0.0000057
50	25	25	22	3.5	0.0000010
60	30	30	30	4.5	0.0000002
70	35	35	37	3.7	0.0000002
110	55	55	40	1.6	0.0000001

Table 4 – Contd.,

154	77	77	57	1.9	00000000
(C_0=0.9, P_0=0.45, γ_0=0.6, θ_0=0.7)					
N	**n_1**	**n_2**	**R**	**τ_1***	**GAV**
30	15	15	14	3.4	0.0000074
40	20	20	17	2.4	0.0000031
50	25	25	27	2.6	0.0000007
60	30	30	34	3.3	0.0000002
70	35	35	37	3.1	0.0000002
110	55	55	40	0.9	0.0000001
154	77	77	57	1.0	0.0000000
(C_0=0.9, P_0=0.4, γ_0=0.6, θ_0=0.65)					
N	**n_1**	**n_2**	**R**	**τ_1***	**GAV**
30	15	15	14	3.6	0.0000059
40	20	20	17	2.5	0.0000025
50	25	25	27	2.8	0.0000006
60	30	30	34	3.3	0.0000002
70	35	35	37	3.0	0.0000002
110	55	55	40	0.9	0.0000001
154	77	77	57	0.8	0.0000001
(C_0=0.8, P_0=0.4, γ_0=0.7, θ_0=0.7)					
N	**n_1**	**n_2**	**R**	**τ_1***	**GAV**
30	15	15	11	3.3	0.0000074
40	20	20	13	2.1	0.0000031
50	25	25	22	2.2	0.0000007
60	30	30	30	2.5	0.0000002
70	35	35	37	2.6	0.0000002
110	55	55	40	0.4	0.0000001
154	77	77	57	0.2	0.0000000
(C_0=0.8, P_0=0.6, γ_0=0.7, θ_0=0.65)					
N	**$^n1$**	**$^n2$**	**R**	**τ_1***	**GAV**
30	15	15	11	4.6	0.0000066
40	20	20	13	3.2	0.0000029
50	25	25	22	3.5	0.0000005
60	30	30	30	3.8	0.0000002
70	35	35	37	3.3	0.0000001
110	55	55	40	1	0.0000000
154	77	77	55	0.8	0.0000000
(C_0=1.2, P_0=0.3, γ_0=0.5, θ_0=0.8)					
N	**n_1**	**n_2**	**R**	**τ_1***	**GAV**
30	15	15	11	3.6	0.0000128
40	20	20	13	2.8	0.000005
50	25	25	22	4.9	0.0000011
60	30	30	30	2.8	0.0000004
70	35	35	37	3.6	0.0000002
110	55	55	40	2.3	0.0000001
154	77	77	57	2.8	0.0000000
(C_0=1.2, P_0=0.3, γ_0=0.4, θ_0=0.8)					
N	**n_1**	**n_2**	**R**	**τ_1***	**GAV**
30	15	15	11	3.4	0.0000154
40	20	20	13	2.5	0.0000062
50	25	25	22	3.1	0.0000014
60	30	30	30	3.6	0.0000005
70	35	35	37	3.3	0.0000003
110	55	55	40	1.5	0.00000071
154	77	77	57	2.1	0.0000005

To estimate the optimum value of the censoring time T^*, The stress levels to test units are V1 = 1.0 and V2 = 3.5. Based on $\tau_0 = 0,\ \ \tau_1 = 5.0,\ \ \ T = 2.0$ with different initial guesses of the true parameters α, γ, θ say $\alpha_0, \gamma_0, \theta_0$ and different values of N, the optimum of the censoring time T^* are determined by solving equation (32). Table (5) present the optimal value of at different specified values of (C, P, γ, θ) and the Generalized Asymptotic Variance (GAV). From the numerical results, we observe as the sample size increases GAV is decreased.

Table 5: Optimum Values of T1∗ and GAV

(C_0=0.5, P_0=0.7, γ_0=0.5, θ_0=0.5)					
N	**n_1**	**n_2**	**R**	**T_1***	**GAV**
44	22	22	11	1.2	0.0000002
54	27	27	19	1.9	0.0000001
100	50	50	16	1.1	0.0000000
120	60	60	22	1.4	0.0000000
140	70	70	21	1.5	0.0000000
(C_0=0.5, P_0=0.6, γ_0=0.5, θ_0=0.4)					
N	**$^n1$**	**$^n2$**	**R**	**T_1***	**GAV**
44	22	22	11	1.6	0.0000001
54	27	27	23	1.6	0.0000000
100	50	50	16	1.4	0.0000000
120	60	60	22	1.7	0.0000000
140	70	70	21	1.7	0.0000000
(C_0=0.5, P_0=0.5, γ_0=0.5, θ_0=0.5)					
N	**$^n1$**	**$^n2$**	**R**	**T_1***	**GAV**
44	22	22	11	2.5	0.0000002
54	27	27	19	3.1	0.0000001
100	50	50	16	2.9	0.0000000
120	60	60	22	3.0	0.0000000
140	70	70	21	3.2	0.0000000
(C_0=0.5, P_0=0.4, γ_0=0.5, θ_0=0.4)					
N	**n_1**	**n_2**	**R**	**T_1***	**GAV**
44	22	22	11	2.8	0.0000002
54	27	27	19	3.2	0.0000001
100	50	50	16	3.2	0.0000000
120	60	60	22	3.3	0.0000000
140	70	70	21	3.6	0.0000000
(C_0=0.6, P_0=0.7, γ_0=0.5, θ_0=0.4)					
N	**n_1**	**n_2**	**R**	**T_1***	**GAV**
44	22	22	11	1.2	0.0000002
54	27	27	19	1.5	0.0000001
100	50	50	16	1.1	0.0000000
120	60	60	22	1.0	0.0000000
140	70	70	21	0.9	0.0000000
(C_0=0.6, P_0=0.6, γ_0=0.6, θ_0=0.5)					
N	**n_1**	**n_2**	**R**	**T_1***	**GAV**
44	22	22	11	1.6	0.0000003
54	27	27	23	1.8	0.0000001
100	50	50	16	1.4	0.0000000
120	60	60	22	1.7	0.0000000
140	70	70	21	1.6	0.0000000
(C_0=0.6, P_0=0.5, γ_0=0.6, θ_0=0.4)					
N	**n_1**	**n_2**	**R**	**T_1***	**GAV**
44	22	22	11	1.9	0.0000003
54	27	27	19	2.4	0.0000001
100	50	50	16	1.8	0.0000000

Table 5 – Contd.,

120	60	60	22	2.1	0.0000000
140	70	70	21	2.1	0.0000000
(C_0=0.6, P_0=0.5, γ_0=0.5, θ_0=0.3)					
N	**n_1**	**n_2**	**R**	**T_1^***	**GAV**
44	22	22	11	2.0	0.0000001
54	27	27	19	2.3	0.0000001
100	50	50	16	2.3	0.0000000
120	60	60	22	2.2	0.0000000
140	70	70	21	2.3	0.0000000

CONCLUSIONS

This paper presented the Maximum Likelihood (ML) method of the parameter estimation with type-I censoring. The data failure times at each stress level are assumed to follow the 3-parameter generalized logistic distribution with scale parameter that is an inverse power law function. The ML estimation, Fisher's information matrix, the asymptomatic variance-covariance matrix, the prediction of the value of the scale parameter and the reliability function under the usual conditions stress were obtained for various combinations of the model parameters.

In additional, the corresponding optimum value of the switching time change stress and of the censoring time are obtained numerically by the D-optimality criterion. Since, standard Logistic, four-parameters extended GL , four-parameters extended GL type-I, two parameter GL , type-I GL , Generalized Log-logistic,standard Log-logistic, Logistic Exponential, Exponentiated Exponential (for x>0), Generalized Burr, Burr III, Burr XII distributions are special cases from the GL distribution then their results of the MLE and optimum test plan become particular cases of the results obtained here.

APPENDICES

Appendix A

The determinant of can be written as

$$|F| = \left(f_{33} f_{44} - f_{34}^2\right)\left(f_{11} f_{22} - f_{12}^2\right) - \left(f_{23} f_{44} - f_{24} f_{34}\right)\left(f_{11} f_{23} - f_{12} f_{13}\right) + \left(f_{23} f_{34} - f_{24} f_{33}\right)\left(f_{11} f_{24} - f_{12} f_{14}\right) - \left(f_{13} f_{44} - f_{14} f_{34}\right)\left(f_{13} f_{22} - f_{12} f_{23}\right) + \left(f_{13} f_{34} - f_{33} f_{14}\right)\left(f_{14} f_{22} - f_{12} f_{24}\right) - \left(f_{13} f_{24} - f_{23} f_{14}\right)\left(f_{14} f_{23} - f_{13} f_{24}\right) \quad (33)$$

The derivative of $|F|$ with respect to $\tau_{j-1}, j = 2,\ldots,k$

$$\frac{\partial |F|}{\partial \tau_{j-1}} = \left(f_{33} f_{44} - f_{34}^2\right)\left(f_{11} f_{22}' + f_{11}' f_{22} - 2 f_{12} f_{12}'\right) + \left(f_{33} f_{44}' + f_{33}' f_{44} - 2 f_{34} f_{34}'\right)\left(f_{11} f_{22} - f_{12}^2\right) - \left(f_{23} f_{44} - f_{24} f_{34}\right)\left(f_{11} f_{23}' + f_{11}' f_{23} - f_{12} f_{13}' - f_{12}' f_{13}\right) - \left(f_{23} f_{44}' + f_{23}' f_{44} - f_{24} f_{34}' - f_{24}' f_{34}\right)\left(f_{11} f_{23} - f_{12} f_{13}\right) + \left(f_{23} f_{34} - f_{24} f_{33}\right)\left(f_{11} f_{24}' + f_{11}' f_{24} - f_{12} f_{14}' - f_{12}' f_{14}\right) + \left(f_{23} f_{34}' + f_{23}' f_{34} - f_{24} f_{33}' - f_{24}' f_{33}\right)\left(f_{11} f_{24} - f_{12} f_{14}\right) - \left(f_{13} f_{44} - f_{14} f_{34}\right)\left(f_{13} f_{22}' + f_{13}' f_{22} - f_{12} f_{23}' - f_{12}' f_{23}\right) - \left(+ f_{13}' f_{44} - f_{14} f_{34}' - f_{14}' f_{34}\right)\left(f_{13} f_{22} - f_{12} f_{23}\right) + \left(f_{13} f_{34} - f_{33} f_{14}\right)\left(f_{14} f_{22}' + f_{14}' f_{22} - f_{12} f_{24}' - f_{12}' f_{24}\right) + \left(f_{13} f_{34}' + f_{13}' f_{34} - f_{14} f_{33}' - f_{14}' f_{33}\right)\left(f_{14} f_{22} - f_{12} f_{24}\right) - \left(f_{14} f_{23} - f_{13} f_{24}\right)\left(f_{14} f_{23}' + f_{14}' f_{23} - f_{13} f_{24}' - f_{13}' f_{24}\right) - \left(f_{13} f_{24}' + f_{13}' f_{24} - f_{23} f_{14}' - f_{23}' f_{14}\right)\left(f_{23} f_{14} - f_{13} f_{24}\right), \quad (34)$$

where

$$f'_{11} = -(\theta+1)\sum_{i=1}^{n_j} S_j^P \xi_{ij}\phi_{ij}\left[CS_j^P\left(x_{ij}-\tau_{j-1}+u_{j-1}\right)\left(\phi_{ij}-\nu_{ij}\right)+2\right], \quad (35)$$

$$f'_{22} = C\sum_{j=1}^{n_j} S_j^P\left(\ln S_j\right)^2\left\{(\theta+1)\nu_{ij}\left[CS_j^P\left(x_{ij}-\tau_{j-1}+u_{j-1}\right)\phi_{ij}\left(CS_j^P\left(x_{ij}-\tau_{j-1}+u_{j-1}\right)\right.\right.\right.$$
$$\left.\left.\left.\nu_{ij}-1\right)-\left(1+CS_j^P\left(x_{ij}-\tau_{j-1}+u_{j-1}\right)\phi_{ij}\right)^2\right]+1\right\}, \quad (36)$$

$$f'_{33} = \frac{2C(\theta+1)}{\gamma^2}\sum_{i=1}^{n_j} S_j^p \nu_{ij}^2\phi_k^2, \quad (37)$$

$$f'_{44} = \frac{C}{\theta^2}\sum_{i=1}^{n_j} S_j^P \nu_{ij}\phi_{ij}\left[(\theta+1)\left(\nu_{ij}-\phi_{ij}\right)+(\theta-1)\right], \quad (38)$$

$$f'_{12} = \sum_{j=1}^{n_j} S_j^P\left(\ln S_j\right)\left\{(\theta+1)\nu_{ij}\left[CS_j^P\left(x_{ij}-\tau_{j-1}+u_{j-1}\right)\phi_{ij}\left(CS_j^P\left(x_{ij}-\tau_{j-1}+u_{j-1}\right)\right.\right.\right.$$
$$\left.\left.\left.\nu_{ij}-1\right)-\left(1+CS_j^P\left(x_{ij}-\tau_{j-1}+u_{j-1}\right)\phi_{ij}\right)^2\right]+1\right\}, \quad (39)$$

$$f'_{13} = \frac{(\theta+1)}{\gamma}\sum_{i=1}^{n_j} S_j^P \nu_{ij}\phi_{ij}\left[1+CS_j^P\left(x_{ij}-\tau_{j-1}+u_{j-1}\right)\left(\phi_{ij}-\nu_{ij}\right)\right], \quad (40)$$

$$f'_{14} = -\sum_{i=1}^{n_j} S_j^P \nu_{ij}\left\{C\xi_{ij}\phi_{ij}\left(\frac{\theta+1}{\theta}\right)+\left(1+CS_j^P\left(x_{ij}-\tau_{j-1}+u_{j-1}\right)\phi_{ij}\right)\left(1-\left(\frac{\theta+1}{\theta}\right)\phi_{ij}\right)\right\}, \quad (41)$$

$$f'_{23} = \frac{C(\theta+1)}{\gamma}\sum_{i=1}^{n_j} S_j^P \nu_{ij}\phi_{ij}\ \ln S_j\left[1+CS_j^P\left(x_{ij}-\tau_{j-1}+u_{j-1}\right)\left(\phi_{ij}-\nu_{ij}\right)\right], \quad (42)$$

$$f'_{24} = -C\sum_{i=1}^{n_j} S_j^P \nu_{ij}\ \ln S_j\left\{C\xi_{ij}\phi_{ij}\left(\frac{\theta+1}{\theta}\right)+\left(1+CS_j^P\left(x_{ij}-\tau_{j-1}+u_{j-1}\right)\phi_{ij}\right)\left(1-\left(\frac{\theta+1}{\theta}\right)\phi_{ij}\right)\right\}, \quad (43)$$

And

$$f'_{34} = \frac{-C}{\gamma}\sum_{i=1}^{n_j}\nu_{ij}\phi_{ij}\left\{\left(\nu_{ij}-\phi_{ij}\right)\left(\frac{\theta+1}{\theta}\right)+1\right\}, \quad (44)$$

Appendix B

The determinant of the Fisher information matrix and its derivative with respect to have the same form of the equations (33) and (34), where the derivatives

$f'_{11},\ f'_{22},\ \ldots,\ f'_{34}$ with respect to is given as follows

$$f'_{11} = -\theta S_k^P\ Z_k\varphi_k\left\{CS_k^P\left(T-\tau_{k-1}+u_{k-1}\right)\left(\varphi_k-\mu_k\right)+2\right\} \quad (45)$$

$$f'_{22} = -C\theta B S_k^P \mu_k \ln S_k \{C\varphi_k [\Omega_k + (T - \tau_{k-1} + u_{k-1}) S_k^P (C \Omega_k (\varphi_k - \mu_k)) + 2 \ln S_k] + \ln S_k\}, \quad (46$$

$$f'_{33} = \frac{2C\theta}{\gamma^2} \{B S_k^P \varphi_k \mu_k^2\}, \quad (47)$$

$$f'_{44} = \frac{2C}{\theta} \{B S_k^P \varphi_k \mu_k^2\}, \quad (48)$$

$$f'_{12} = -\theta B S_k^P \mu_k \{C\varphi_k [\Omega_k + (T - \tau_{k-1} + u_{k-1}) S_k^P (C \Omega_k (\varphi_k - \mu_k)) + 2 \ln S_k] + \ln S_k\}, \quad (49)$$

$$f'_{13} = \frac{-\theta}{\gamma} S_k^P \varphi_k \{C Z_k (\varphi_k - \mu_k) + B\mu_k\}, \quad (50)$$

$$f'_{14} = -B S_k^P \mu_k^2 \{1 + 2C S_k^P (T - \tau_{k-1} + u_{k-1}) \varphi_{\boldsymbol{k}}\}, \quad (51)$$

$$f'_{23} = \frac{-C\theta}{\gamma} S_k^P \varphi_k \{C Z_k (\varphi_k - \mu_k) + B\mu_k\} \ln S_k, \quad (52)$$

$$f'_{24} = -C B S_k^P \mu_k^2 \{1 + 2C S_k^P (T - \tau_{k-1} + u_{k-1}) \varphi_{\boldsymbol{k}}\} \ln S_k, \quad (53)$$

And $f'_{34} = \frac{2C}{\gamma^2} \{B S_k^P \varphi_k \mu_k^2\},$ (54)

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