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Numerical Treatment Of Laser Interaction With Solid In One Dimension

Hameed H Hameed
Assist lecturer
Foundation Of Technical Education
Technical Institute Of Alsuwayra

Hayder M Abaas
Assist lecturer
Baghdad University
College Of Science for women
Department Of mathematics

Abstract:

The laser radiation has great important in many applications. In this paper we have studied the influence of laser radiation on solid material. We use a (800 µs) pulsed laser ND: YAG with (1.06 µ m) wave length, (3 J) energy and power density of (7.6 X 10^6 W/cm^2) . We have deduced a mathematical function for laser energy with time, then deducing a function for laser power density with time because practically assumed to be constant.

A lead material used in this paper for study, we have deduced mathematical functions for thermal properties (conductivity, specific heat, density, diffusivity) with temperature, and solve the partial differential equation (P.D.E) which represent the heat transfer of laser radiation to the material in one dimension, presumes variable and constant laser density ($I=I_0$ and I=I(t)) , also the thermal properties conductivity , specific heat ,density and diffusivity ($K=K_0$ and K=K(T),C=C0 and C=C(T), $\rho = \rho_0 and \rho = \rho(T)$,du=du_0 and du=du(T)) respectively .

The results reveal that the time of evaporation increase when the laser density is taken as variable with time , also it increase when the thermal properties were vary with temperature .
We used (Matlab 7.0) to perform all programs which related with this paper

Introduction

The development of laser has been an exciting chapter in the history of science and engineering. It has produced a new type of advice with potential for application in an extremely wide variety of fields. Mach basic development in lasers were occurred during last 35 years .The laser's interaction with metal and vaporize of metals made due it's ability for welding , cutting and drilling applicable.

The status of laser development and application were still rather rudimentary. The light emitted by laser is electromagnetic radiation, this radiation has a wave nature , the waves consists of vibrating. electric and magnetic fields , many studies have tried to find and solve models of laser interactions Remi Senti [1] proposed the mathematical model related to the laser- plasma interaction, in [2] the authors have developed an analytical model to study the temperature distribution in IR optical materials heated by laser pulses. in [3] a mathematical model simulating the surface hardening of steel laser beam was described, in [4] the authors have studied two separate non linear effects which influence pulsed laser propagation and target interaction, in [5] the authors have studied the interaction of nanosecond pulsed lasers with material from thermal point of view using experimental technique and theoretical approach of dimensional analysis.

In this paper we have evaluated the solution of (P.D.E) that represent the laser interaction with solid situation in one dimension assuming that the power density of laser and thermal properties are functions with time and temperature respectively .

One dimension laser heating equation

In general the one dimension laser heating processes of opaque solid slab is represented as[6,7]

$$\rho C T_t = \frac{\partial}{\partial x}(K T_x) \text{.........(1)}$$

With boundary conditions and initial condition which represent the prevaporization stage:-

$$-KT_x = 0 \quad for \quad x = l, 0 \le t \le tv$$
$$-KT_x = \alpha I(t) \quad for \quad x = 0, 0 \le t \le tv \qquad \text{..........(2)}$$
$$T(x,0) = T\infty \quad for \quad t = 0, 0 \le x \le l$$

Where:
K: represents the thermal conductivity.
ρ : represents the density.
C: represents the specific heat.
T: represents the temperature.
T_∞ : represents the ambient temperature.
tv: represents the front surface vaporization.
αI(t): represents the surface heat flux density absorbed by the slab.

Now , if we assume that ρ, C, K are constant , the equation (1) becomes

$$T_t = duT_{xx} \quad \text{........(3)}$$

With the same boundary conditions as in equation (2)

Where : $du = \dfrac{K}{\rho C}$ which represents the thermal diffusion .

but in general $K = K(T), \rho = \rho(T), C = C(T)$, therefore the derivation equation (1) with this assuming implies [6]

$$T_t = \frac{1}{\rho(T)C(T)}[K(T)T_{xx} + \overline{K}T_x^2] \text{ ....(4)}$$

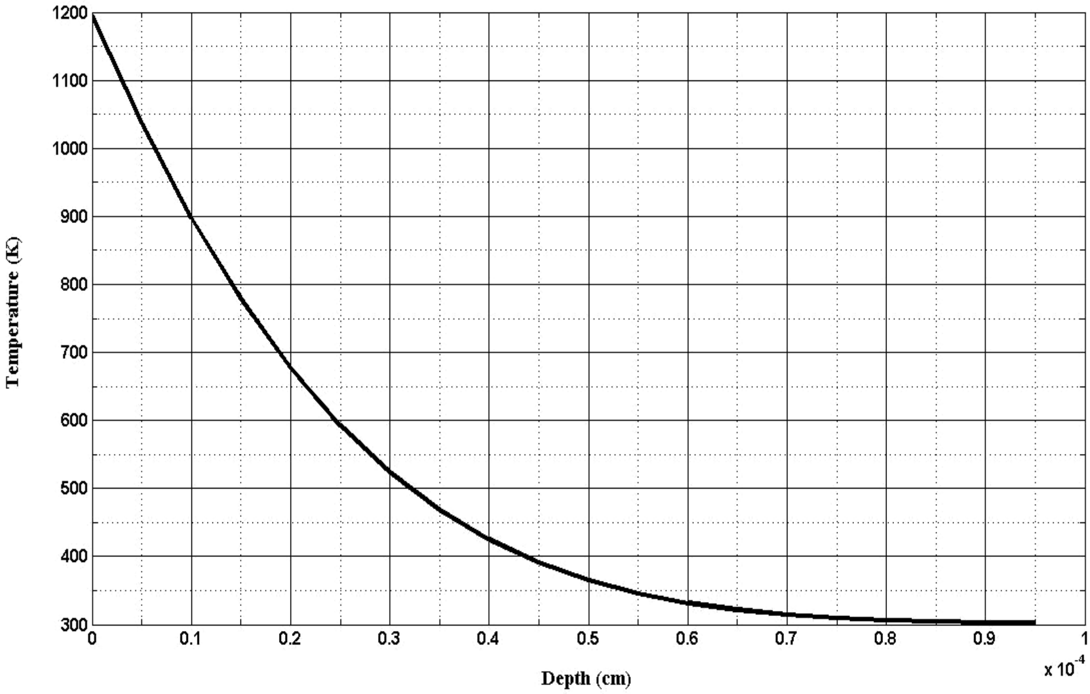
with the same boundary and initial conditions in equation(2). Where $\overline{K}$ represents the derivative of K with respect the temperature

Numerical Solution with constant laser power density and constant thermal properties

First we have taken the lead metal (pb) with thermal properties [8].
K=22.506 X 10^{-5} J/msec .cm .K^0 .
C=0.14016 J/g . K^0 .
ρ =10.751 g/cm^3 .
T_m=melting point =600 K^0 .
T_v= vapor point = 1200 K^0 .

And we have taken the laser energy E=3J , A=1.34 X 10^{-3} cm^2 , where A: represent the area under laser influence .

The numerical solution of equations (3) with boundary and initial conditions in equation (2) assuming ($I=I_0$ =7.6 X10^6W/cm^2) with thermal properties of lead metal by explicit method [9,10,11] using matlab program give us the results as shown in Fig (1) :-



Fig(1) : Depth dependence of the temperature with the laser power density I_0 =7.6 $X10^6$ (W/cm^2), tv=3.84 E-8 Millie second

Evaluation of function I(t) of laser flux density .

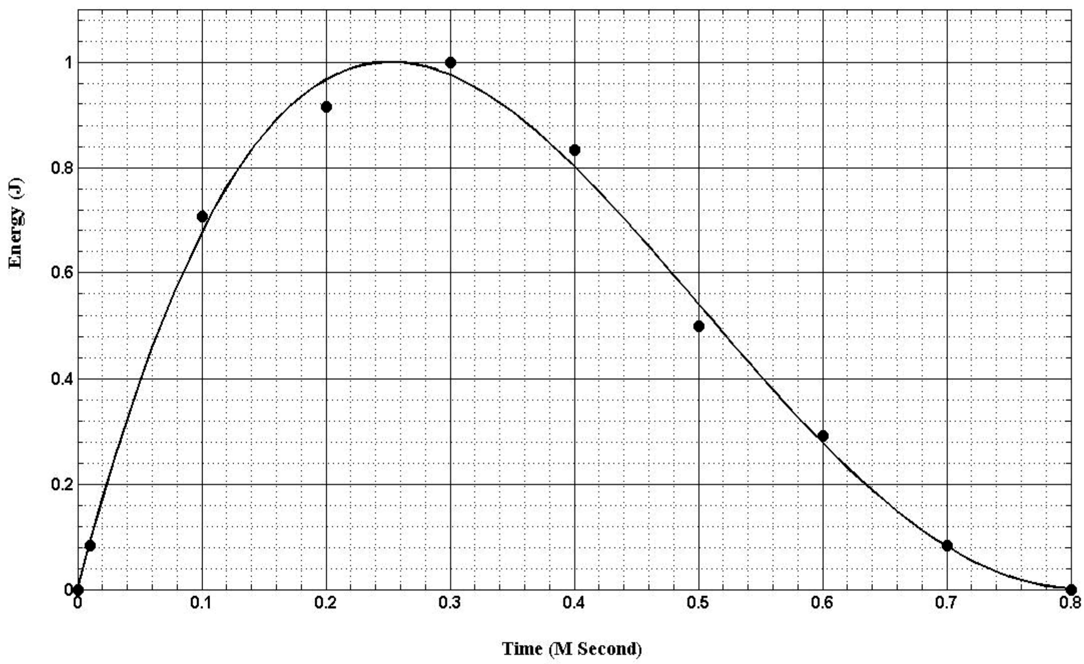
From following data [6,7] that represent the energy (J) with time (Millie second) :-

Time:-	0	0.01	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
Energy:-	0	0.02	0.17	0.22	0.24	0.2	0.12	0.07	0.02	0

by using matlab program , the pest polynomial which deduced from above data was:-

$E(t)= 3.7110\ E\text{-}4 +2.1582\ t -5.7582\ t^2 +3.6746\ t^3+9.9414\ E\text{-}1\ t^4-1.0069\ t^5$ ……(5)

As shown in Fig (2) :-

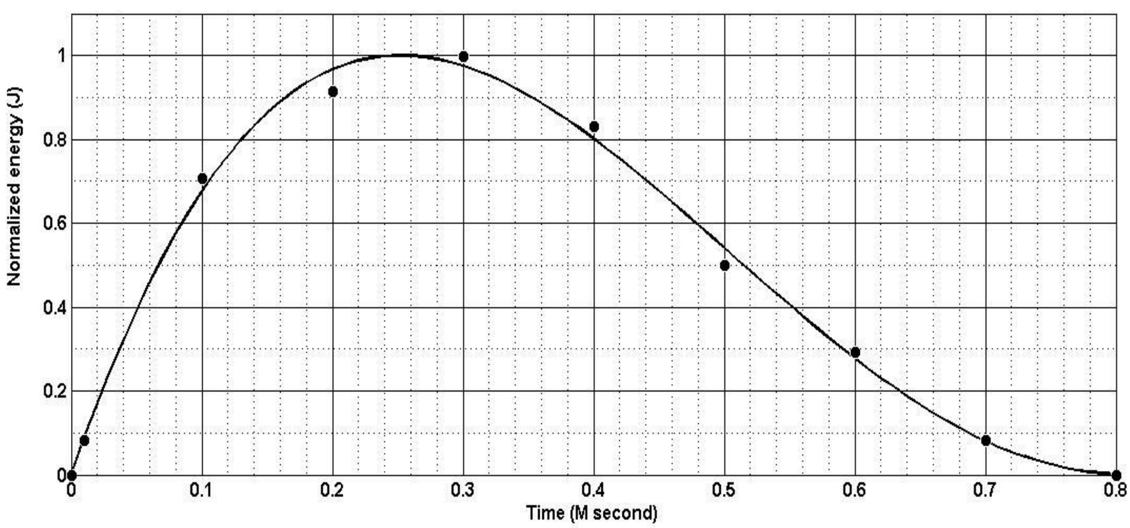


Fig(2): The time dependence of energy

With maximum value E_{max}=0.2403
The normalized function [$E_{normalized}$) is deduced by dividing E(t) by the maximum value (E_{max})

$$E_{normalized} = \frac{E(t)}{E_{\max}} \qquad \ldots\ldots (6)$$

As shown in Fig(3):-



Fig(3): The time dependence of the normalize energy

The integral of E(t) normalized over t from t=0.0 to t= 0.8 (m sec) must equal to 3 (total laser energy) i.e.

$\int_{0.0}^{0.8} E_{normalized} dt = 3$ therefore there exist a real number P such that $\int_{0.0}^{0.8} PE_{normalized} dt = 3$, that implies P=6.8241 and

$$E(t)= \int_{0.0}^{0.8} PE_{normalized} dt \quad .....(7)$$

The integral of laser flux density I=I (t) over t from t=0.0 to t=0.8 (m sec) must equal to (I_0=7.6 X10^6 W/cm^2) therefore

$$\int_{0.0}^{0.8} I(t)dt = I_0 Dt \quad ..... (8)$$

Where Dt put to balance the units of equation (8)

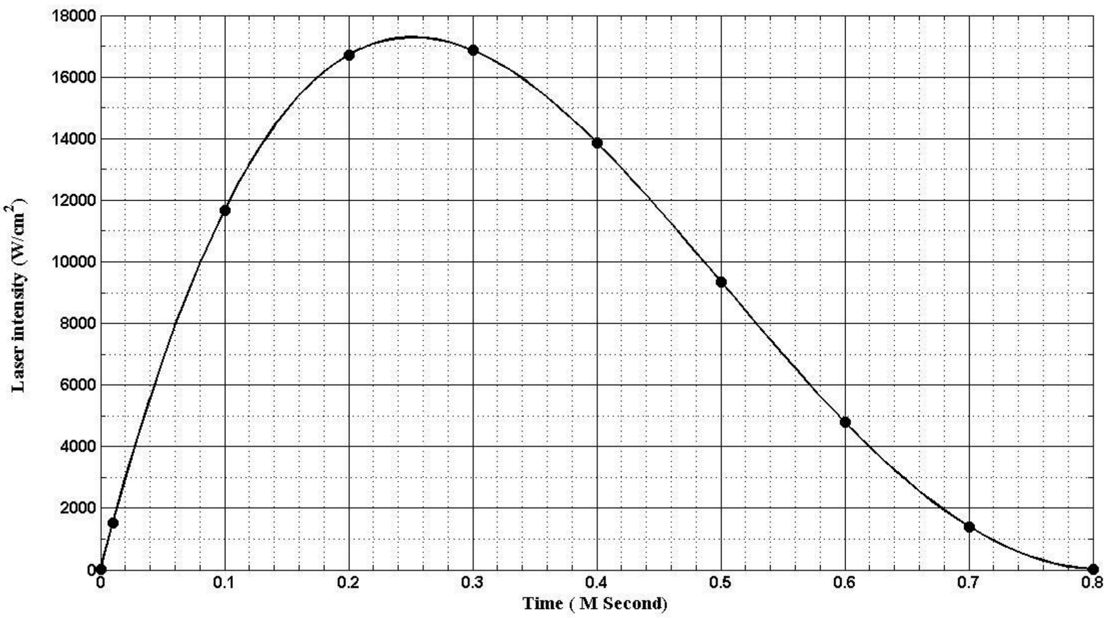
But in general $I = \frac{E}{A}$ [12] .... (9)

And from equations (7), (8) and (9) we have

$$\int_{0.0}^{0.8} I(t)dt = z.\left(\frac{\int_{0.0}^{0.8} P.E_{normalized} dt}{A.Dt} \right) \quad ..... (10)$$

Where z =3.95 and its put to balance the magnitude of two sides of equation (10)
Therefore

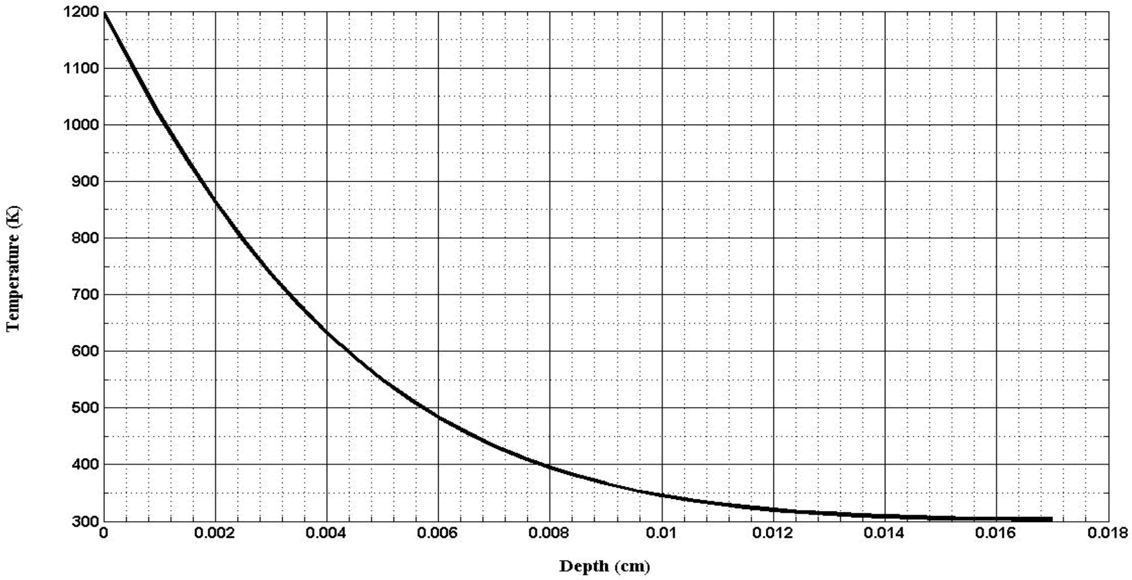
$$I(t) = \frac{z.P.E_{normalized}}{A.Dt} \quad ..... (11)$$ as shown in Fig(4):-



Fig(4):the time dependence of laser power density

Numerical Solution with variable laser power density (I=I (t)) and constant thermal properties

With all constant thermal properties of lead metal as in article (3) and I=I(t) ,we have deduced the numerical solution of heat transfer equation as in equation (3) with boundary and initial condition as in equation (2) , and the depth penetration is shown in Fig(5):-



Fig(5):Depth dependence of the temperature with laser power density I=I(t) and constant thermal properties , tv=1.143 E-4 Millie second

Evaluation the thermal properties: conductivity, density and specific heat as functions of temperature

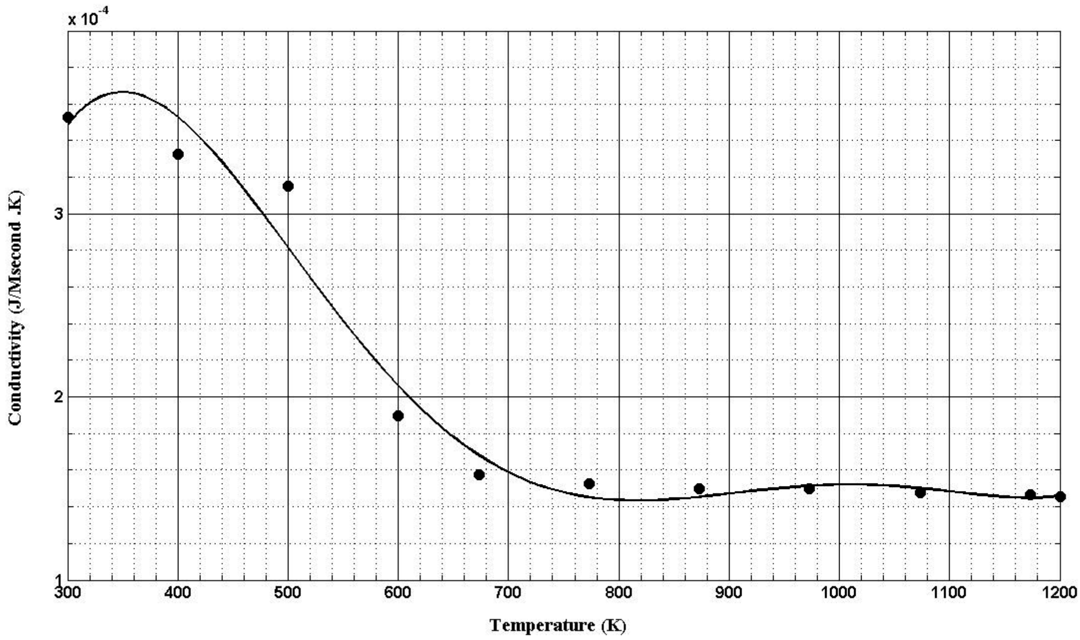
a) The conductivity K(T) , From the following data [8] :

T (K^0):-	300	400	500	600	673
K (J/msec.Cm K^0):-	35.3E-05	33.2E-05	31.5E-05	19.E-05	15.75E-05
T (K^0):-	773	873	973	1073	1173
K (J/msec.Cm K^0):-	15.2 E-05	15.E-05	15. E-05	14.75E-05	14.67E-05
T (K^0):-	1200				
K (J/msec.Cm K^0):-	14.55E-05				

By using matlab program, the pest polynomial agrees with above data was:-

$$K(T)=-1.7033\ E\text{-}3+1.6895\ E\text{-}5\ T -5.0096\ E\text{-}8\ T^2 +6.6920\ E\text{-}11\ T^3 -4.1866\ E\text{-}14\ T^4+1.0003\ E\text{-}17\ T^5 \quad \ldots\ldots(12)$$

As shown in Fig(6)



Fig(6):Temperature dependence of the thermal conductivity

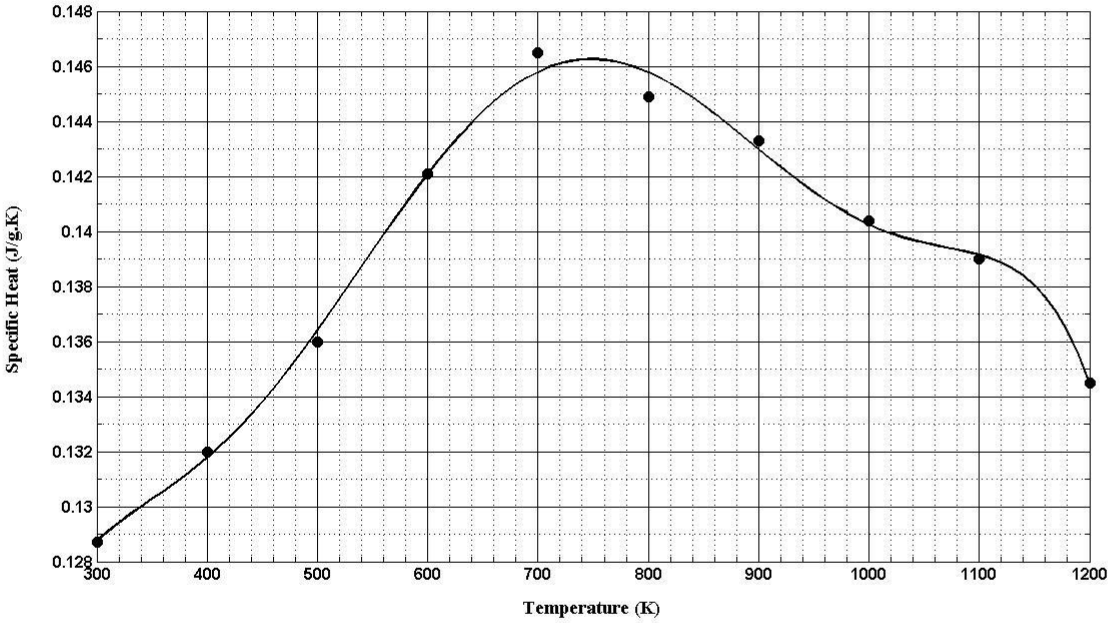
b) The specific heat C(T), from the flowing data[8] :-

$T(K^0)$:	300	400	500	600	700
$C(J/g.K^0)$:	0.1287	0.132	0.136	0.1421	0.1465
$T(K^0)$:	800	900	1000	1100	1200
$C(J/g.K^0)$:	0.1449	0.1433	0.1404	0.1390	0.1345

The best polynomial was:-

$$C(T)=-4.6853\text{ E-}2+1.9426\text{ E-}3\ T-8.6471\text{ E-}6\ T^2+1.9546\text{ E-}8\ T^3-2.3176\text{ E-}11\ T^4+1.3730\text{ E-}14\ T^5-3.2083\text{ E-}18\ T^6 \quad \ldots\ldots(13)$$

As shown in Fig(7)



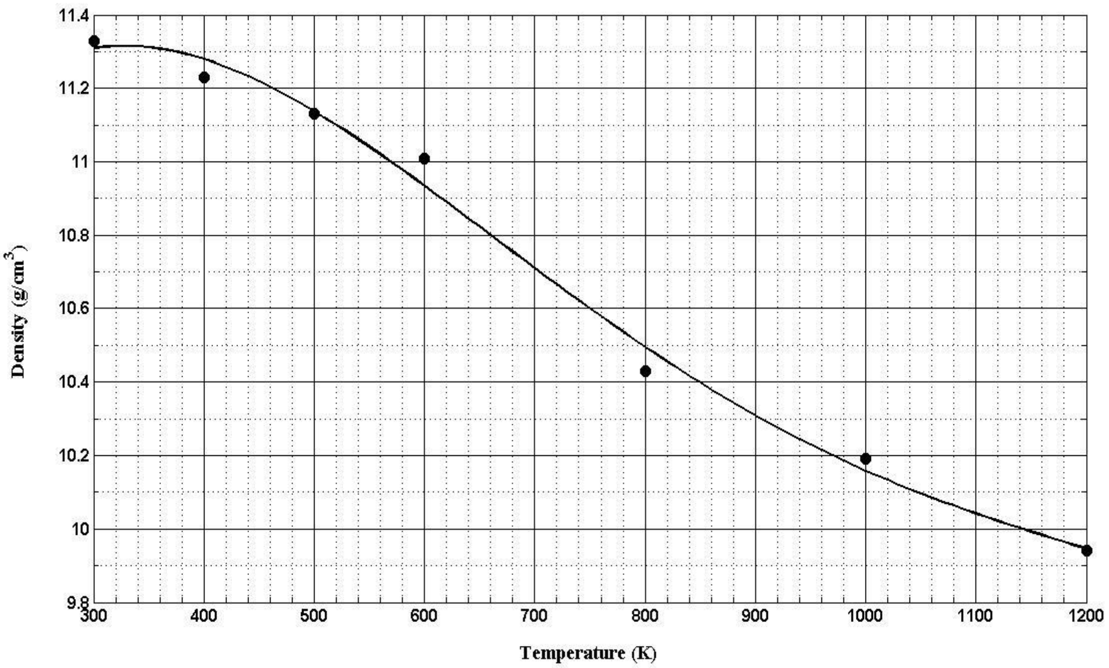
Fig(7):Temperature dependence of the specific heat

c) the density $\rho(T)$, from the flowing data [8] :-

$T(K^0)$:	300	400	500	600	800	1000	1200
ρ (J/Cm^3):	11.330	11.230	11.130	11.010	10.430	10.190	9.940

The best polynomial was:-

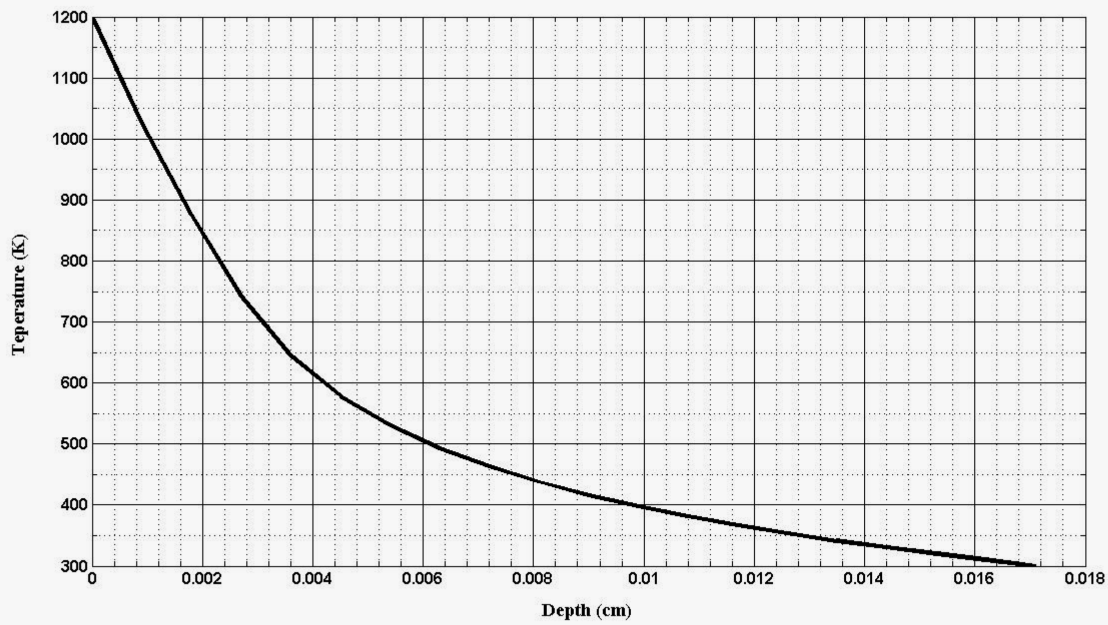
ρ (T)= 10.047 +9.121 E-3 T -2.1284 E-3 T^2+1.67 E-8 T^3 -4.5158 E-12 T^4 …..(14)
As shown in Fig(8)



Fig(8):Temperature dependence of the density

Numerical Solution with variable laser power density (I=I (t)) and variable thermal properties (K=K(T) ,C=C(T) , $\rho = \rho(T)$).

We have deduced the solution of equation (4) with initial and boundary condition as in equation (2) using the function of I(t) as in equation (11) and the functions of K(T) ,C(T) and $\rho(T)$ as in equations (12 ,13 and 14) respectively ,then by using matlab program , the depth penetration is shown in Fig(9):-



Fig(9) :Depth dependence of the temperature with the thermal properties as functions of temperature and laser power density as function of time ,tv=1.9218 E-5 Millie second

Conclusions

a) From previous studies we observe that : all equations that represent laser heat transfer were formulated to be linear i.e. the thermal properties such as (conductivity ,specific heat , density and thermal diffusion) assumed to be constant .also the laser flux density assumed to be constant .

b) when I=I(t) ,K=K_0, $\rho = \rho_0$ C=C_0 we observe that vaporization time is greater than vaporization time when I=I_0, K=K_0, $\rho = \rho_0$ C=C_0 .about (3000) times

c) when I=I(t) ,K=K(T) , $\rho = \rho(T)$,C=C(T) , the vaporization time is grater than the vaporization time when I=I_0, K=K_0, $\rho = \rho_0$ C=C_0.about (500) times

e) when I=I(t) ,K=K_0, $\rho = \rho_0$ C=C_0, the vaporization time is grater than the vaporization time when I=I(t) ,K=K(T) , $\rho = \rho(T)$,C=C(T). about(6) times

f) when I=I(t) ,K=K_0, $\rho = \rho_0$ C=C_0 we observe that the penetration depth is greater than the penetration depth when I=I_0, K=K_0, $\rho = \rho_0$ C=C_0 .about (190) times.

g) when I=I(t) ,K=K_0, $\rho = \rho_0$ C=C_0 we observe that the penetration depth is approximately similar to the penetration depth when I=I(t) ,K=K(T) , $\rho = \rho(T)$,C=C(T)

the matlab programs

a) This program calculates the laser energy as function of time

```
clear
clc
t=[0 .01 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8];
E=[0 0.02 0.17 0.22 0.24 0.2 0.12 0.07 0.02 0];
format short e
u=polyfit(t,E,5)
plot(t,E,'*')
grid on
hold on
i=0:0.02:0.8;
 E1=polyval(u,i);
 plot(i,E1,'-')
 xlabel('time in mille second')
 ylabel('Energy (J)')
 title(' Fig(     ) The time dependence of the energy')
 hold off
```

b) This program calculates the normalized laser energy as a function of time

```
clear
clc
t=[0 .01 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8];
E=[0 0.02 0.17 0.22 0.24 0.2 0.12 0.07 0.02 0];
```

```
u=polyfit(t,E,5)
i=0:0.01:0.8;
 E1=polyval(u,i);
 m=max(E1)
unormal=u/m
E2=polyval(unormal,i);
plot(i,E2,'-')
grid on
xlabel('time (Millie second)')
ylabel('Energy (J)')
title('Fig (   ) The time dependence of the normalize energy')
Enorm=E/m;
hold on
plot(t,Enorm,'*')
```

c) This program calculate the laser intensity as a function of time

```
clear
clc
I0=7.6*10^3      % laser intensity J/Msec cm^2
p=6.8241         % this number comes from '' p* integration of E-normal=3 ==>
                   p=3/integration of E-normal
z=3.395          % this number comes from the fact z*Int(I(t))=I0 in magnitude
A= 1.34*10^-3    % this number represents the area through heat flow
Dt=1             % This number comes from "integral of I(t)=I0*Dt"
                 % its comes from equality of units
 t=[0 .01 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8];
E=[0 0.02 0.17 0.22 0.24 0.2 0.12 0.07 0.02 0];
Energy=polyfit(t,E,5);     % this step calculate the energy as a function of time
i=0:0.01:0.8;              % loop of time
 E1=polyval(Energy,i);
 m=max(E1)
Enormal=Energy/m     % this step to find normal energy
format short g
I=z*((p*Enormal)/(A*Dt))
E2=polyval(I,i);
plot(i,E2)
E2=polyval(I,t);
hold on
plot(t,E2,'*')
grid on
xlabel('time (millie second)')
ylabel('Laser intensity (W/cm^2)')
title('Fig (   ) The time dependence of the Laser Intensity')
```

d) This program calculate the thermal conductivity as a function of temperature

```
clear
clc
T=[300 400 500 600 673 773 873 973 1073 1173 1200];
K=10^-5*[35.3 33.25 31.5 19.0 15.75 15.25 15.0 15.0 14.75 14.67 14.55] ;
```

```
format short e
KT=polyfit(T,K,5)
i=300:10:1200;
KT1=polyval(KT,i);
plot(T,K,'*')
grid on
hold on
xlabel('Temperature(Kelven)')
ylabel('Conductivity(J/Mellie second .K')
plot(i,KT1,'-')
```

e) This program calculates the specific heat as a function of temperature

```
clear
clc
format short
T=[300 400 500 600 700 800 900 1000 1100 1200];
C=[.1287 .132 .136 .1421 .1465 .1449 .1433 .1404 .1390 .1345];
format short e
u=polyfit(T,C,6)
format short
i=300:10:1200;
uu=polyval(u,i);
plot(T,C,'*')
hold on
plot(i,uu,'-')
grid on
xlabel('Temperature (K)')
ylabel('Specific Heat (J/g.K)')
```

f) This program calculates the density as a function of temperature

```
clear
clc
format short
T=[300 400 500 600 800 1000 1200];
P=[11.330 11.230 11.130 11.010 10.430 10.190 9.940];
format short e
u=polyfit(T,P,4)
format short
i=300:10:1200;
uu=polyval(u,i);
plot(T,P,'*')
hold on
plot(i,uu,'-')
grid on
xlabel('Temperature (K)')
ylabel('Density (g/cm^3)')
```

h) This program calculate the vaporization time and depth penetration when laser intensity as a function of time and thermal properties are constant i.e.I(t)=Io,K(T)=K_0,rho(T)=rho_0=rho,C(T)=C_0 .

```
clc
clear
a=0;b=0.0001;         % a represents the first edge of lead plate and b represents the
                      second edge
h=0.000005;            % h=x_{i+1}-x_i
N=round((b-a)/h);     %  N: refer to the number of points that we can calculate the
                      %temperature on its the function " round " use to obtain the
                      %integer number
Tv=1200               % the vaporization temperature of lead
dt=0;                 % this iteration for time
x=-h;                 % this iteration for depth
t1=0;
t=0
K=22.506*10^-5;       % conductivity of lead with unit J/mSec.cm.K^o
alpha=1;              % absorption coefficient since the surface of metal is opaque solid
                      slap that implies alpha=1
I0=7.6*10^3;          % laser intensity with unit J/mSec cm^2
C=0.14016;            % specific heat of lead  with unit J/g.K^o
rho=10.751;           % density of lead with unit g/cm^3
du=K/(rho*C)          % thermal diffusion with unit cm^2 /mSec
format short g
r1=(2*alpha*I0*h)/K;        % this element comes from finite difference method at
                            %boundary conditions
for i= 1:N

   T1(i)=300;               %the initial value of temperature  where 300 in Kelvin
end
T1


for t=1:100                       % this Loop for time in Millie second
   format short g
    t1=t1+1;
  dt=dt+0.0000000002                  % to increase the time
x=x+h;                            % to increase the penetration
x1(t1)=x;                         % calculate the penetration as a matrix
r=(dt*du)/h^2                     % this element comes from finite difference method

for i=1:N                         % this Loop calculate the temperature at second point of
                                  %penetration depending on initial temperature T=300

   if i==1 T2(i)=T1(i)+2*r*(T1(i+1)-T1(i)+(r1/2));   % this equation to calculate the
                                                     %temperature at x=a
   elseif i==N T2(i)=T1(i)+2*r*(T1(i-1)-T1(i));      % this equation to calculate the
                                                     temperature at x=N
   else
```

```
    T2(i)=T1(i)+r*(T1(i+1)-2*T1(i)+T1(i-1));     % this equation to calculate the
                                                    %temperature at all other points
     end

  if abs(Tv-T2(i))<=0.8 break       % to compare the calculated Temperature with
                                    %vaporization Temperature
   end

  end
  if abs(Tv-T2(i))<=0.8 break
   end
  T2

  dt=dt+.0000000002
  x=x+h;
  t1=t1+1;
  x1(t1)=x;
  r=(dt*du)/h^2
for i=1:N                  % This Loop to calculate the next Temperature depending on %
                              pervious temperature
     if i==1 T1(i)=T2(i)+2*r*(T2(i+1)-T2(i)+(r1/2));
     elseif i==N T1(i)=T2(i)+2*r*(T2(i-1)-T2(i));
     else
        T1(i)=T2(i)+r*(T2(i+1)-2*T2(i)+T2(i-1));
     end
     if abs(Tv-T1(i))<=0.8 break
   end
  end
  if abs(Tv-T1(i))<=0.8 break
   end
  T1

  end
  x1;
  for i=i:N        % this Loop to make the penetration and Temperature as matrices % to
                      plot them
    T(i)=T1(i);

     x2(i)=x1(i);

  end
  x2;
  plot(x2,T,'k');
  grid on
  xlabel('Depth (cm)')
  ylabel('Teperature (K)')
```

i) This program calculates the vaporization time and depth penetration when laser intensity and thermal properties are constant i.e. I(t)=I(t),K(T)=K_0, rho(T)=rho_0,C(T)=C_0 ,where K,rho,C represent the thermal conductivity, density, specific heat respectively

```
clc
clear
a=0;b=0.0175;            % a represents the first edge of lead plate and b represents the
                                 second    edge
h=.0005;                   % h=x(i+1)-x(i)
N=round((b-a)/h)          %  N:  refer to the number of points that we can calculate the
                         %temperature on its , the function " round " use to obtain the integer
                         number

Tv=1200                    % the vaporization temperature of lead
dt=0;                      % this iteration for time
x=-h;                      % this iteration for depth
t1=0;
K=22.506*10^-5;            % conductivity of lead with unit J/mSec. cm. K
alpha=1;                   % absorption coefficient since the surface of metal is opaque solid
                              slap that      implies alpha=1
C=0.14016;                 % specific heat of lead  with unit J/g.K^o
rho=10.751;              % density of lead with unit g/cm^3
du=K/(rho*C)            % thermal diffusion with unit cm^2 /mSec
format short g
for i= 1:N

   T1(i)=300;           % the initial value of temperature  where 300 K^o
end
T1


for t=1:1200          %  this Loop for time in Millie second
   format short g
   t
    t1=t1+1;
  dt=dt+.00000005        % to increase the time
x=x+h;                     % to increase the penetration
x1(t1)=x;                  % calculate the penetration as a matrix

% I as a function of time I=I(t)
I=26.694+1.5525*(10^5)*(dt)-4.1419*(10^5)*(dt^2)+2.6432*(10^5)*(dt^3)+71510*(dt^4)-
72426*(dt^5)

r=(dt*du)/h^2                % this element comes from finite difference method
r1=(2*alpha*I*h)/K         % this element comes from finite difference method at boundary
                             condition
for i=1:N                      % This Loop to calculate the next Temperature depending on
                                  initial %temperature T=300
```

```
   if i==1 T2(i)=T1(i)+2*r*(T1(i+1)-T1(i)+(r1/2)); % this equation to calculate the
                                                         temperature at x=a
   elseif i==N T2(i)=T1(i)+2*r*(T1(i-1)-T1(i));      % this equation to calculate the
                                                         temperature at x=N
   else

       T2(i)=T1(i)+r*(T1(i+1)-2*T1(i)+T1(i-1));      % this equation to calculate the
                                                     %temperature at all other points
   end

if abs(Tv-T2(i))<=.4 break   % to compare the calculated Temperature with vaporization
                             %Temperature
 end

end
if abs(Tv-T2(i))<=.4 break
 end
T2


dt=dt+.00000005
x=x+h;
t1=t1+1;
x1(t1)=x;
I=26.694+1.5525*(10^5)*(dt)-4.1419*(10^5)*(dt^2)+2.6432*(10^5)*(dt^3)+71510*(dt^4)-
  72426*(dt^5)
r=(dt*du)/h^2
r1=(2*alpha*I*h)/K
for i=1:N                    % This Loop to calculate the next Temperature depending on
                             previous % Temperature
   if i==1 T1(i)=T2(i)+2*r*(T2(i+1)-T2(i)+(r1/2));
   elseif i==N T1(i)=T2(i)+2*r*(T2(i-1)-T2(i));
   else
      T1(i)=T2(i)+r*(T2(i+1)-2*T2(i)+T2(i-1));
   end
   if abs(Tv-T1(i))<=.4 break
 end
end
if abs(Tv-T1(i))<=.4 break
 end
T1

end
x1;
for i=i:N          % this Loop to make the penetration and Temperature as matrices to plot
them
  T(i)=T1(i);
```

```
   x2(i)=x1(i);

end
x2
format long
plot(x2,T,'k');
grid on
xlabel('Depth (cm)')
ylabel('Temperature (K)')
```

j) This program calculate the vaporization time and depth penetration when laser intensity and thermal properties are variable i.e. I=I(t),K=K(T),rho=rho(T),C=C(T) .

```
 clc
clear
a=0;b=.018;             % a represents the first edge of lead plate and b represents the
                            second  edge
h=.0009;                % h=xi+1-xi
N=round((b-a)/h)        %  N :refer to the number of points that we can calculate the
                          %temperature on its, the function " round " use to obtain the
                          integer %number
Tv=1200                  % the vaporization temperature of lead
dt=0;                    % this iteration for time
x=-h;                    % this iteration for depth
t1=0;
alpha=1;                   % absorption coefficient since the surface of metal is opaque
                               solid slap that implies alpha=1
format short g
for i= 1:N

   T1(i)=300;                  %the initial value of temperature  where 300 in Kelvin
end
T1

for t=1:11000                          % this Loop for time in Millie second
   format short g
   t
    t1=t1+1;
  dt=dt+.0000000009                      % to increase the time
x=x+h ;                               % to increase the penetration
x1(t1)=x;                               % calculate the penetration as a matrix
% I as a function of time I=I(t)
I(t1)=26.694+1.5525*(10^5)*(dt)-
4.1419*(10^5)*(dt^2)+2.6432*(10^5)*(dt^3)+71510*(dt^4)-72426*(dt^5);
for i=1:N                            % This Loop to calculate the next Temperature depending
                                        on %initial temperature T=300


% the following equations represent the thermal conductivity, specific heat and density
as  %functions of temperature
```

```
K(i)=-1.7033*(10^-3)+1.6895*(10^-5)*(T1(i))-5.0096*(10^-8)*(T1(i)^2)+6.6920*(10^-11)*(T1(i)^3)-4.1866*(10^-14)*(T1(i)^4)+1.0003*(10^-17)*(T1(i)^5);

 Kdash(i)=(1.6895)*(10^-5)-(2*5.0096)*(10^-8)*(T1(i))+(3*6.6920)*(10^-11)*(T1(i)^2)-(4*4.1866)*(10^-14)*(T1(i)^3)+(5*1.0003)*(10^-17)*(T1(i)^4);

C(i)=-4.6853*(10^-2)+1.9426*(10^-3)*(T1(i))-8.6471*(10^-6)*(T1(i)^2)+1.9546*(10^-8)*(T1(i)^3)-2.3176*(10^-11)*(T1(i)^4)+1.3730*(10^-14)*(T1(i)^5)-3.2089*(10^-18)*(T1(i)^6);

rho(i)=1.0047*10+9.2126*(10^-3)*(T1(i))-2.1284*(10^-5)*(T1(i)^2)+1.6700*(10^-8)*(T1(i)^3)-4.5158*(10^-12)*(T1(i)^4);

du(i)=K(i)/(rho(i)*C(i));

r(i)=dt/((h^2)*rho(i)*C(i));

r1(i)=((2*alpha*I(t1)*h)/K(i));

% this equation to calculate the temperature at x=a
   if i==1 T2(i)=T1(i)+r(i)*[K(i)*[2*T1(i+1)-2*T1(i)+r1(i)]+((r1(i)^2)*Kdash(i)/4)];
   elseif i==N T2(i)=T1(i)+2*r(i)*K(i)*[(T1(i-1)+T1(i))]; % this equation to calculate
                                                            %the temperature at x=N
   else

% this equation to calculate the temperature at all other points
  T2(i)=T1(i)+r(i)*[K(i)*[T1(i+1)-2*T1(i)+T1(i-1)]+(Kdash(i)/4)*[T1(i+1)-T1(i-1)]^2];
   end

if abs(Tv-T2(i))<=.4 break  % to compare the calculated Temperature with vaporization Temperature
 end

end
if abs(Tv-T2(i))<=.4 break
end
for i=1:N
end
T2
dt=dt+.0000000009
x=x+h;
t1=t1+1;
x1(t1)=x;
I(t1)=26.694+1.5525*(10^5)*(dt)-4.1419*(10^5)*(dt^2)+2.6432*(10^5)*(dt^3)+71510*(dt^4)-72426*(dt^5);
for i=1:N  % This Loop to calculate the next Temperature depending on initial
              temperature T=300
 K(i)=-1.7033*(10^-3)+1.6895*(10^-5)*(T2(i))-5.0096*(10^-8)*(T2(i)^2)+6.6920*(10^-11)*(T2(i)^3)-4.1866*(10^-14)*(T2(i)^4)+1.0003*(10^-17)*(T2(i)^5);
```

```
 Kdash(i)=(1.6895)*(10^-5)-(2)*(5.0096)*(10^-8)*(T2(i))+(3)*(6.6920)*(10^-11)*(T2(i)^2)-
(4)*(4.1866)*(10^-14)*(T2(i)^3)+(5)*(1.0003)*(10^-17)*(T2(i)^4);

C(i)=-4.6853*(10^-2)+1.9426*(10^-3)*(T2(i))-8.6471*(10^-6)*(T2(i)^2)+1.9546*(10^-
8)*(T2(i)^3)-2.3176*(10^-11)*(T2(i)^4)+1.3730*(10^-14)*(T2(i)^5)-3.2089*(10^-
18)*(T2(i)^6);

rho(i)=1.0047*10+9.2126*(10^-3)*(T2(i))-2.1284*(10^-5)*(T2(i)^2)+1.6700*(10^-
8)*(T2(i)^3)-4.5158*(10^-12)*(T2(i)^4);

du(i)=K(i)/(rho(i)*C(i));

r(i)=dt/((h^2)*rho(i)*C(i));

r1(i)=((2*alpha*I(t1)*h)/K(i));

      % this equation to calculate the temperature at x=a
   if i==1 T1(i)=T2(i)+r(i)*[K(i)*[2*T2(i+1)-2*T1(i)+r1(i)]+((r1(i)^2)*Kdash(i)/4)];
   elseif i==N T1(i)=T2(i)+2*r(i)*K(i)*[(T2(i-1)+T2(i))]; % this equation to calculate the
                                                                 %temperature at x=N
   else

% this equation to calculate the temperature at all other points
T1(i)=T2(i)+r(i)*[K(i)*[T2(i+1)-2*T2(i)+T2(i-1)]+(Kdash(i)/4)*[T2(i+1)-T2(i-1)]^2];
   end

if abs(Tv-T2(i))<=.4 break  % to compare the calculated Temperature with vaporization
                                 %temperature
 end

end
if abs(Tv-T2(i))<=.4 break
end
T1
end
x1;
for i=i:N     % this Loop to make the penetration and Temperature as matrices to plot
               them
  T(i)=T1(i);

   x2(i)=x1(i);
end
x2
format short g
plot(x2,T,'k');
grid on
xlabel('Depth (cm)')
ylabel('Temperature (K)')
```

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الخلاصة:

لأشعة الليزر أهمية بالغة في مجالات واسعة ، في بحثنا هذا قمنا بدراسة تأثير هذه الأشعة على المادة في الحالة الصلبة.وقد اخذنا بدراسة الليزر النبضي ND:YAG بزمن نبضة (800 µs) وطول موجي (1.06 µm) وبطاقة بلغت (3 J) وبكثافة قدرة تبلغ (7.6×10^6 W/cm2) . لقد استخرجنا دالة رياضية لطاقة الليزر مع الزمن ومن ثم استخراج دالة رياضية تمثل كثافة القدرة مع الزمن . حيث كان المعتاد في دراسة الليزر اعتبار كثافة الليزر قيمة ثابتة.

لقد استخدمنا عنصر الرصاص في بحثنا هذا ، حيث قمنا باستخراج دوال رياضية لخصائص المادة (التوصيلية, الحرارة النوعية ، الكثافة والانتشارية الحرارية) مع درجة الحرارة .

وحلت المعادلة التفاضلية الجزئية(PDE) التي تمثل انتقال حرارة أشعة الليزر الى المادة بالبعد الواحد وعلى فرض ان كثافة قدرة الليزر ثابتة مرة ومتغيرة مرة اخرى مع الزمن($I=I_0$ و $I=I(t)$). كذلك بالنسبة الى خصائص الماد (التوصيلية, الحرارة النوعية ، الكثافة , الانتشارية الحرارية) ($K=K_0$ و $K=K(T)$, $C=C_0$ و $C=C(T)$, $\rho=\rho_0$ و $\rho=\rho(T)$, $du=du_0$ و $du=du(T)$) حيث اعتبرت ثابتة مرة ومتغيرة مرة اخرى مع درجة الحرارة.

ومن نتائج البحث ظهر ان زمن التبخر يزداد عندما تكون كثافة قدرة الليزر متغيرة مع الزمن . وكذلك يزداد زمن التبخر على اعتبار ان خصائص المادة متغيرة مع درجة الحرارة .

لقد قمنا باستخدام برنامج (Matlab 7.0) لتنفيذ كافة البرامج المتعلقة بالبحث