Finite Volume Simulation of Natural Convection in a Trapezoidal Cavity Filled with Various Fluids and Heated from the Top Wall

Ahmed Kadhim Hussein College of Engineering - Mechanical Engineering Department - Babylon University - Babylon City - Hilla – Iraq ahmedkadhim7474@gmail.com

Abstract

Laminar natural convection in a trapezoidal cavity filled with various fluids is studied numerically using a finite volume method. The cavity inclined left and right sidewalls are maintained at constant cold temperature while the top wall is maintained at constant hot temperature. The bottom wall is considered adiabatic. Fluid flow fields , isotherm patterns and the average Nusselt number are presented for the Rayleigh numbers varied as 10^3 , 10^4 , 10^5 and 10^6 while the Prandtl numbers varied as 0.07, 0.7 and 10 respectively. The inclination angle of the inclined sidewalls of the cavity, is kept constant at 35° . The effects of Rayleigh number and Prandtl number on the flow and heat transfer process in the trapezoidal cavity are investigated. Results are presented in the form of streamline and isotherm plots as well as the variation of the average Nusselt number. The results indicate that the flow circulation becomes stronger as natural convection is dominated and becomes weaker as natural convection effect is decreased. Also, the results showed a good agreement with other publications.

Key Words

Natural Convection, Finite Volume Method, Trapezoidal Cavity, Laminar Flow, Numerical Discretization

Nomenclature		
Unit	Description	Symbol
g	Gravitational acceleration	m/s^2
Н	Height of the cavity	m
Nu	Average Nusselt number	
n	Normal direction on a surface	
Р	Dimensionless pressure	
р	Pressure	N/m ²
Pr	Prandtl number	
Ra	Rayleigh number	
Т	Temperature	°C
U	Dimensionless velocity component in x-direction	
u	Velocity component in x-direction	m/s
V	Dimensionless velocity component in y-direction	
v	Velocity component in y-direction	m/s
W	Width of the cavity	m
Х	Dimensionless Coordinate in horizontal direction	
X	Cartesian coordinate in horizontal direction	m
Y	Dimensionless Coordinate in vertical direction	
у	Cartesian coordinate in vertical direction	m
Greek Symbols		

α	Thermal diffusivity	m ² /s
β	Coefficient of thermal expansion	K ⁻¹
θ	Dimensionless temperature	
Φ	Inclination angle of the cavity inclined sidewalls	degree
ν	Kinematic viscosity	m ² /s
ρ	Density	kg/m ³
Subscripts		
с	Cold	
h	Hot	
S	Side	
Т	Тор	

1.Introduction

Natural convection in closed cavities is one of the most important subjects of fluid flow and heat transfer. This subject has received much attention in last ten years due to many energy-related applications, such as cooling of electronic components, thermal insulation of buildings using air gaps, solar energy collectors, semiconductor production, furnaces and fire control in buildings [1]. The motion of the fluid in natural convection occurs due to the buoyancy forces imposed on the fluid when its density of the heat transfer surface is described as a result of thermal expansion of the fluid in a non-uniform temperature distribution. Most authors considered the flow and heat transfer inside a square or rectangular cavities because the numerical solution is not complicated and the geometry is very simple to simulate. But, natural convection in non-rectangular cavities such as triangular and trapezoidal cavities has received little attention even in the recent time. The flow inside these cavities is much more complicated to investigate, as the boundary zone and the middle core zone never have the same effect for a certain boundary condition considered. On the other side, actual or real cavities occurring in practice generally have the shapes different from square or rectangular shape.

Thus, various channels of constructions, panels of electronic equipment and solar energy collectors are considered a non-rectangular cavities. Previous studies related with the natural convection problem in trapezoidal cavities under different conditions are reported in literature by Lam et al. [2], Peric [3], Sadat and Salagnac [4], Baytas and Pop [5], Basak et al. [6], Varol et al. [7], Mamun et al. [8] and Hussein et al. [9]. Iyican et al. [10] presented an analytical study of natural convective heat transfer within a trapezoidal enclosure with parallel cylindrical top and bottom walls at different temperatures and plane adiabatic sidewalls. The flow characteristics in trapezoidal enclosures are predicted using data collected for rectangular enclosures. Iyican et al. [11] reported an experimental data for heat transfer across air-filled inclined trapezoidal enclosures for a Rayleigh number range of approximately 2×10^3 to approximately 5×10^7 . The large side was cooled to uniform temperature while the opposing small side was electrically heated. The experiments showed that conductive heat losses up the sidewalls could be very large even when the sidewalls were thermally insulated from the top and bottom surfaces. Kuyper and Hoogendoorn [12] performed a numerical simulation of laminar natural convection flow in trapezoidal enclosures to study the effect of the inclination angle on the flow and also the dependence of the average Nusselt number on the Rayleigh number.

Moukalled and Acharya [13] performed a numerical investigation to study the effects of mounting baffles to the upper inclined planes of trapezoidal cavities. The results obtained for air as a working fluid reveal a decrease in heat transfer in the presence of baffles. The decrease in heat transfer became increasingly more significant as the baffle got closer to the heated vertical wall for the bottom-cooled situation and as the baffle got closer to the symmetry line for the bottom-heated case. Moukalled and Darwish [14] reported a numerical results for natural convection heat transfer in partially divided trapezoidal cavities. The effects of Rayleigh number, Prandtl number, baffle height and baffle location

on the heat transfer were investigated. Results were displayed in terms of streamlines, isotherms, local and average Nusselt number values. The results indicated a decrease in heat transfer with the presence of baffles, with its rate generally increasing with increasing baffle height and Prandtl number.

Reynolds et al. [15] performed an experimental and numerical investigation of the heat loss characteristics of a trapezoidal cavity absorber. They concluded that the heat loss from the absorber occurred by an interaction between convection, conduction and radiation inside the cavity and then from the cavity to the surroundings. Papanicolaou and Belessiotis [16] investigated double-diffusive natural convection in an asymmetric trapezoidal enclosure. They studied unsteady behavior in the laminar and the turbulent-flow regimes. Mohamed [17] studied the laminar natural heat and mass transfer in a symmetrical trapezoidal enclosure. The base and ceiling were considered isothermal and iso-concentration surfaces, while the lateral walls were considered adiabatic and impermeable. The investigation was made for a wide range of buoyancy ratio, inclination angle, Lewis number and thermal Grashof number with fixed aspect ratio at 3.0 and Prandtl number at 0.7.

A comparison was made with the previous experimental and numerical results and explained a maximum deviation from -5 % to +12.8 %. Moukalled and Darwish [18] performed a numerical study to examine the effects of mounting two offset baffles onto the upper inclined and lower horizontal surfaces of trapezoidal cavities on the heat transfer performance. Two thermal boundary conditions were considered. In the first, the short left vertical wall was heated while the long right vertical wall was cooled. In the second, the long right vertical wall was heated while the short left vertical wall was cooled. For both boundary conditions, the highest decrease was achieved in fully partitioned enclosures. Hammami [19], presented a numerical three-dimensional study of coupled heat and mass transfer by natural convection occurring in a trapezoidal cavity. They solved the governing equations by using a finite volume technique.

The effect of the cavity dimensions on heat and mass transfer rates was examined and the results explained that as the aspect ratio increased ,multi-cellular flow patterns start to form. Basak et al. [20] investigated numerically using finite element method the natural convection within a trapezoidal enclosure for uniformly and non-uniformly heated bottom wall, insulated top wall, and isothermal sidewalls with inclination angle. Parametric study for the wide range of Rayleigh number (Ra), $10^3 \le$ Ra $\leq 10^5$ and Prandtl number (Pr) for model fluids with various tilt angles $\varphi = 45^\circ$, 30°, and 0° had been obtained. Local heat transfer rates were found to be relatively more for $\varphi = 0^{\circ}$ than those with $\varphi =$ 45° and $\phi = 30^\circ$. Average Nusselt number plots showed higher heat transfer rates for $\phi = 0^\circ$ except for the non-uniform heating of the bottom wall with Pr = 0.015 (molten metal). Natarajan et al. [21] used a finite element analysis to investigate the influence of uniform and non-uniform heating of bottom wall on natural convection flows in a trapezoidal cavity. The bottom wall was uniformly and non-uniformly heated while two vertical walls were maintained at constant cold temperature and the top wall was well insulated. They observed a symmetry while representing the flow patterns in terms of stream functions. Non-uniform heating of the bottom wall produced greater heat transfer rate at the center of the bottom wall than uniform heating case for all Rayleigh numbers but average Nusselt number showed overall lower heat transfer rate for non-uniform heating case.

Aramayo et al. [22] numerically studied the flow pattern and heat transfer in a pair of trapezoidal cavities stacked one over the other, with a separating solid plate in between. The effect of the plate physical parameters, thickness and conductivity, on the heat transfer were analyzed. Global Nusselt numbers were also calculated for the bottom side of the lower cavity. Data were obtained for air as the working fluid (Pr=0.7) in the range $10^3 < Ra_H < 10^7$. Motivated by the previous works mentioned above, the purpose of the present work is to consider the effects of Rayleigh number and Prandtl number on steady natural convection phenomena inside a trapezoidal cavity. The governing equations are transformed into a dimensionless form then solved numerically using finite volume methodology.

2. Problem Description and the Mathematical Model

The physical system under consideration, as shown schematically in (Fig.1), is a two-dimensional trapezoidal cavity. Flow and heat transport in the fluid obeys the continuity, Navier-Stokes, and energy equations. Air is considered as the fluid inside the cavity with Pr = 0.71. The temperature (T_c) is uniformly imposed on two opposing inclined sidewalls of the cavity , while the top wall is maintained at constant temperature (T_h) such that $T_h > T_c$ and the bottom wall is assumed to be thermally insulated. The fluid inside the cavity is assumed to be two-dimensional , steady, incompressible, laminar, Newtonian and has a constant thermo-physical properties except the density in the buoyancy term of the momentum equations which is treated according to Boussinesq approximation.



Fig. 1 Schematic diagram of the trapezoidal cavity (left) and Mesh Distribution (right).

The effect due to viscous dissipation is assumed to be negligible. As mentioned above, the flow and thermal fields inside the trapezoidal cavity are modeled by the continuity, Navier–Stokes and the energy equations which are given in a dimensionless form as follows [21] :-

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{2.1}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + Pr(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2})$$
(2.2)

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \Pr(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}) + Ra\Pr\theta$$
(2.3)

$$U\frac{\partial\theta}{\partial X} + V\frac{\partial\theta}{\partial Y} = \left(\frac{\partial^2\theta}{\partial X^2} + \frac{\partial^2\theta}{\partial Y^2}\right)$$
(2.4)

The above governing equations are converted into a dimensionless forms by using the following nondimensional parameters :-

$$\theta = \frac{T - T_{c}}{T_{h} - T_{c}}, \quad X = \frac{x}{H}, \quad Y = \frac{y}{H}, \quad U = \frac{uH}{\alpha}, \quad V = \frac{vH}{\alpha}, \quad P = \frac{pH^{2}}{\rho\alpha^{2}}$$
(2.5)
While the Prandtl number and Payleigh number are defined respectively as :

While the Prandtl number and Rayleigh number are defined respectively as :-

$$\Pr = \frac{\nu}{\alpha} \qquad \operatorname{Ra} = \frac{g\beta(T_h - T_c)H^3 \operatorname{Pr}}{\nu^2}$$

(2.6)

2

The rate of heat transfer is represented mathematically in terms of average Nusselt number (Nu) at the hot top wall and the cold right and left sidewalls as follows :

$$Nu_T = \int_0^H \left[\frac{\partial\theta}{\partial n}\right]_{Y=1} dn$$
(2.7)

and

$$Nu_{s} = \cos \Phi \int_{0}^{H} \left[\frac{\partial \theta}{\partial n}\right]_{Y=0,1} dn$$
(2.8)

The boundary conditions which are used in the present study can be arranged as follows :-1. The left and right inclined sidewalls of the trapezoidal cavity is maintained at constant cold

temperature (T_c) , so that :-

at
$$X = 0$$
 and $X = 1_{\theta} = 0$ (2.9)

2. The top wall of the trapezoidal cavity is maintained at constant hot temperature (T_h) so that :-

at
$$Y = 1$$
 $\theta = 1$ (2.10)

3. The bottom wall of the trapezoidal cavity is considered thermally insulated so that :

at
$$Y = 0$$
 $\frac{\partial \theta}{\partial Y} = 0$ (2.11)

4. All the cavity solid boundaries are assumed to be rigid with no-slip condition U = V = 0

3. Numerical Procedure, Grid Refinement Test and Validation

The numerical method used to solve the discretized governing equations (2.1-2.4) of the present work is the finite volume method. The numerical procedure of this method is described by Patankar [23]. The SIMPLE algorithm is used for the velocity pressure coupling. The solution is obtained by solving the dimensionless governing equations simultaneously on a non-uniform grid points. A Fortran program code is built for this purpose. To satisfy the convergence requirement an under-relaxation factor of 0.3 is used in the numerical solution. A grid clustering procedure adjacent the trapezoidal cavity edges is utilized to capture the flow and thermal fields accurately inside the trapezoidal cavity and to ensure the energy balance. In order to obtain grid independent solution, a grid refinement test is

(2.12)

performed for a trapezoidal cavity at $Ra = 10^6$, Pr = 0.7 and $\Phi = 30^\circ$. Nine combinations (80 x 80, 90 x 90, 95 x 95, 100 x 100, 120x 120, 150 x 150, 175 x 175, 200 x 200 and 225 x 225) of control volumes are used to investigate the effect of grid size on the accuracy of the computed results. Fig.2 explains the convergence of the average Nusselt number (Nu_T) at the hot top wall with grid refinement. It is observed that grid independence is achieved with combination of (120x120) control volumes where there is a slight variation in (Nu_T). This suggests that to prevent excessive computation time, a grid size of (120x120) is adequate for the present analysis.



Fig. 2 Convergence of average Nusselt number along the hot top wall of the trapezoidal cavity with grid refinement for $Ra = 10^6$, Pr= 0.7, and $\Phi = 35^\circ$.

In order to verify the present computational procedure, a comparison is carried out between the present results and the results of Natarajan et al. [21] for natural convection heat transfer in a trapezoidal enclosure with inclined cold sidewalls, adiabatic top wall and uniformly heating bottom wall for various Raleigh numbers as shown below in Table 1 and Table 2 respectively.

Table 1 Comparison of present average Nusselt number at the hot bottom wall with those of Natarajan et al.[21] for validation at Pr = 10, $\Phi = 35^{\circ}$, H/W = 1.

	Average Nusselt number at the hot bottom wall		
Ra	Present study (Finite Volume Method)	Natarajan et al. [21] (Finite Element Method)	Error (%)
10 ³	3.99857	3.98952	0.227
10^{4}	5.49225	5.48752	0.086
10 ⁵	8.43125	8.41587	0.182

	Average Nusselt number at the cold inclined sidewall		
Ra	Present study (Finite Volume Method)	Natarajan et al. [21] (Finite Element Method)	Error (%)
10 ³	1.83251	1.8112	1.176
10 ⁴	2.4556	2.4452	0.147
10 ⁵	3.6998	3.6895	0.279

Table 2 Comparison of present average Nusselt number at the cold inclined sidewalls with those of Natarajan et al. [21] for validation at Pr = 10, $\Phi = 35^{\circ}$, H/W = 1.

As it can be seen by the above comparison both average Nusselt number at the hot bottom wall and inclined cold sidewalls are in excellent agreement with a maximum discrepancy of about 1.176%. Therefore, this verification gives a good interest in the present numerical model to deal with the recent physical problem.

4. Results and Discussion

4.1 Rayleigh Number Effect

The isotherms (left) and streamlines (right) contour at various Rayleigh number ranging from 10^3 to 10^6 and $0.07 \le Pr \le 10$ are shown in Figures (3–5), when the cavity left and right sidewalls are maintained at constant cold temperatures while the bottom wall is considered adiabatic and the top wall is maintained at constant hot temperature. The flow field is represented by two symmetrical vortices with clockwise and anti-clockwise rotations inside the trapezoidal cavity. The flow circulation inside the trapezoidal cavity begins when the hot fluid rises adjacent to the hot top wall as a result of buoyancy forces, until it reaches the cold right sidewall. When the Rayleigh number is low (i.e., Ra = 10^3 and 10^4), the strength of circulation is weak and the re-circulating vortices inside the trapezoidal cavity are symmetrical to each other due to slight influence of buoyancy force.

Consequently, the nature of streamlines and flow field does not change significantly when the Rayleigh number is low. In this case the natural convection contribution in the heat transfer mechanism is slight. As the Rayleigh number increases (i.e., $Ra = 10^5$ and 10^6), the buoyancy forces become more stronger and the flow circulation inside the trapezoidal cavity is highly increases. The center of the re-circulating vortices begins to move in the downward direction and the strength of flow circulation increases while the re-circulating vortices become more irregular in comparison with the corresponding vortices when the Rayleigh number is low. In this case, the buoyancy forces are more dominant than viscous forces.

For isotherms contour, when the Rayleigh number is low (i.e., $Ra = 10^3$ and 10^4), the isotherms are symmetrical and parallel to the hot top wall, indicating that most of the heat is transferred by conduction. As the Rayleigh number increases (i.e., $Ra = 10^5$ and 10^6), the isotherm lines begin to spread out in the cavity and the isotherm shape changes from parallel shape to irregular one indicating that the convection is the dominating heat transfer mechanism in the cavity. Moreover, it can be seen that a thermal boundary layer begins to construct when the Rayleigh number increases due to increase the strength of flow circulation.



Fig. 3 Isotherms (left) and streamlines (right) contour of the trapezoidal cavity for various Rayleigh numbers with Pr = 0.07.

4.2 Prandtl Number Effect

Figures (3–5) show the isotherms (left) and streamlines (right) contour for various Prandtl and Rayleigh numbers. When the Prandtl number is low (i.e., Pr = 0.07 and 0.7), the isotherms and streamlines are governed by the effects of Rayleigh number only. The increase in the Rayleigh number leads to the concentration of the re-circulating vortices in the center of the trapezoidal cavity. No significant influence is seen for the Prandtl number on the streamlines contour when the Rayleigh number is low even when it increases to Pr = 10. This is due to the slight effect of the natural convection at this case.

From the other hand, when the effect of Rayleigh number increases the re-circulating vortices size increases. This result indicates that when the Prandtl number is high , the flow circulation has a significant role on the flow field especially when the Rayleigh number increases. Therefore, the effect of the Prandtl number is more evident as the Rayleigh number increases. For isotherms, they are generally symmetrical for all values of Prandtl number when the effect of buoyancy force is weak. As the Rayleigh number increases due to increase the buoyancy force effect , more disturbance can be seen in the isotherms when the Prandtl number increases. This is due to strong circulation which leads the isotherms to be accumulated near the hot top wall and the heat transfer occurs by convection.



Fig. 4 Isotherms (left) and streamlines (right) contour of the trapezoidal cavity for various Rayleigh numbers with Pr = 0.7.



Fig. 5 Isotherms (left) and streamlines (right) contour of the trapezoidal cavity for various Rayleigh numbers with Pr = 10.

4.3 Average Nusselt Number

Figures 6 and 7 illustrate the variation of average Nusselt number along the hot top wall and the cold inclined sidewalls respectively for various Rayleigh and Prandtl numbers. Generally, as the Rayleigh number increases , the average Nusselt number increases too. This is because the natural convection effect is significant at higher Rayleigh numbers. At low Rayleigh numbers, the average Nusselt number does not exhibit any significant variation with the Prandtl number.



Fig. 6 Variation of the average Nusselt number (Nu_T) along the hot top wall for various Rayleigh and Prandtl numbers.



Fig. 7 Variation of the average Nusselt number (Nu_s) along the cold inclined sidewalls for various Rayleigh and Prandtl numbers.

This is because of the dominance of conduction. Also, the maximum value of the average Nusselt number occurs when the Prandtl number is high. Therefore, the high average Nusselt number corresponds to the high Prandtl number and vise versa. This is due to the increase in the temperature gradient when the Prandtl number increases. This increasing leads to increase the average Nusselt number values.

5. Conclusions

The following conclusions can be detected from the results of the present work.

1-At low Rayleigh number, the circulation pattern in the trapezoidal cavity is very weak because the viscous forces are dominating over the buoyancy forces. As Rayleigh number increases, a clear confusion occurs in the shape of re-circulating vortices, since the buoyancy forces are dominating over the viscous forces.

2- At low Rayleigh number, heat transfer from the hot top wall is essentially dissipated by conductiondominated mechanism.

3- The irregularity of the isotherm lines increases when the Rayleigh number increases, where the heat is essentially dissipated by convection-dominated mechanism.

4-When the Rayleigh number increases, the thermal boundary layer thickness decreases which causes that more heat can be transferred easily in the cavity.

5-The average Nusselt number increases with the Rayleigh number at any particular Prandtl number.

6- The high average Nusselt number corresponds to the high Prandtl number and vise versa.

References

- 1. Hussain S., Hussein A. and Mahdi M., 'Natural convection in a square inclined enclosure with veecorrugated sidewalls subjected to constant flux heating from below', *Nonlinear Analysis: Modelling and Control*, 16 (2011), 152–169.
- 2. Lam S., Gani R. and Simons J., 'Experimental and numerical studies of natural convection in trapezoidal cavities', ASME Journal of Heat Transfer, 111 (1989), 372–377.
- 3. Peric M., 'Natural convection in trapezoidal cavities', Numerical Heat Transfer, Part A : Applications, 24 (1993), 213–219.
- 4. Sadat H. and Salagnac P., 'Further results for laminar natural convection in a two-dimensional trapezoidal enclosure', Numerical Heat Transfer, Part A : Applications, 27 (1995), 451-459.
- 5. Baytas A. and Pop I. , 'Natural convection in a trapezoidal enclosure filled with a porous medium', International Journal of Engineering Science, **39** (2001), 125-134.
- 6. Basak, T., Roy S., Singh A. and Pop I. 'Finite element simulation of natural convection flow in a trapezoidal enclosure filled with porous medium due to uniform and non-uniform heating', *International Journal of Heat and Mass Transfer*, **52** (2009), 70–78.
- 7. Varol Y., Oztop H. and Pop I., 'Natural convection in right-angle porous trapezoidal enclosure partially cooled from inclined wall', International Communications in Heat and Mass Transfer, **36** (2009), 6-15.
- 8. Mamun M., Islam M. and Rahman M., 'Natural convection in a porous trapezoidal enclosure with magnetohydrodynamic effect', Non-linear Analysis: Modelling and Control, 15 (2010), 159-184.
- 9. Hussein A., Ahmed S., Saha S., Hasanpour A., Mohammed H., Adegun I. and Mariani V., 'MHD natural convection inside an inclined trapezoidal porous enclosure with internal heat generation or absorption subjected to isoflux heating', *Heat Transfer-Asian Research*, **41** (2012), 498-515.
- 10. Iyican L., Bayazitoglu Y. and Witte L., 'An analytical study of natural convective heat transfer with in trapezoidal enclosure', ASME Journal of Heat Transfer, 102 (1980), 640-647.
- 11. Iyican L., Witte L. and Bayazitoglu Y., 'Natural convection and sidewall losses in trapezoidal groove collectors', ASME-Transactions, Journal of Solar Energy Engineering, Vol. 103 (1981), 167-172.
- 12. Kuyper R. and Hoogendoorn C., 'Laminar natural convection flow in trapezoidal enclosures', Numerical Heat Transfer, Part A : Applications, 28 (1995), 55-67.

- 13. Moukalled F. and Acharya S., 'Natural convection in trapezoidal cavities with baffles mounted on the upper inclined surfaces', Numerical Heat Transfer Part A : Applications, 37 (2000), 545-565.
- 14. Moukalled F. and Darwish M., 'Natural convection in a partitioned trapezoidal cavity heated from the side', Numerical Heat Transfer Part A : Applications, 43 (2003), 543-563.
- 15. Reynolds D., Jance M., Behnia M. and Morrison G., 'An experimental and computational study of the heat loss characteristics of a trapezoidal cavity absorber', Solar Energy, 76 (2004), 229–234.
- 16. Papanicolaou E. and Belessiotis V., 'Double-diffusive natural convection in an asymmetric trapezoidal enclosure: unsteady behavior in the laminar and the turbulent-flow regime', International Journal of Heat and Mass Transfer, 48 (2005), 191–209.
- 17. Mohamed T., 'Double-diffusive laminar natural convection in a symmetrical trapezoidal enclosure', International Journal of Applied Engineering Research, 1 (2006).
- 18. Moukalled F. and Darwish M., 'Buoyancy-induced heat transfer in a trapezoidal enclosure with offset baffles', Numerical Heat Transfer Part A: Applications, 52 (2007), 337-355.
- 19. Hammami M., Mseddi M. and Baccar M., 'Numerical study of coupled heat and mass transfer in a trapezoidal cavity', Engineering Applications of Computational Fluid Mechanics, 1 (2007), 216–226.
- 20. Basak T., Roy S. and Natarajan E., 'Analysis of steady convective heating for molten materials processing within trapezoidal enclosures', Industrial and Engineering Chemistry Research, 47 (2008), 8652–8666.
- 21. Natarajan E., Basak T. and Roy S., 'Natural convection flows in a trapezoidal enclosure with uniform and non-uniform heating of bottom wall', International Journal of Heat and Mass Transfer, 51 (2008), 747-756.
- 22. Aramayo A., Esteban S. and Cardon L., 'Conjugate heat transfer in a two stage trapezoidal cavity stack', Latin American Applied Research, **39** (2009), 1–9.
- 23. Patankar S., 'Numerical Heat Transfer and Fluid Flow', Hemisphere Publishing Corporation, New York, 1980.