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ORIGINAL RESEARCH

E-differentiable minimax programming under *E*-convexity

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Abstract
In this paper, a new class of minimax programming problems is considered in which the functions involved are E-differentiable. The so-called parametric and nonparametric necessary E-optimality conditions are derived for the considered E-differentiable minimax programming problem. Further, sufficient optimality conditions are established for such nondifferentiable extremum problems under E-convexity hypotheses. Moreover, the example of a nonsmooth minimax programming problem with E-differentiable functions is given to illustrate the aforesaid results. Furthermore, the so-called Mond-Weir E-dual problem and Wolfe E-dual problem are defined for the considered E-differentiable minimax programming problem and several E-duality theorems are established also under appropriate E-convexity hypotheses.

Keywords Minimax programming · E-differentiable function · E-optimality conditions · E-duality · E-convex function

1 Introduction

Optimization problems, in which both a maximization and a minimization process are performed, are known in the area of mathematical programming as minimax problems. Extremum problems of this type occur frequently in many important areas like economics, financial planning, engineering, game theory, minimum risk problems, goal programming, facility location, Chebyshev approximation (see, for example, Cherkaev and Cherkaev 2008; Deng et al. 2005; Danskin 1967; Demyanov and Malozehon 1974; Du and Pardalos 1995; Rivaz and Yaghoobi 2013; Wang et al. 2003; Žaković and Rustem 2003; and others). Due to a growing number of theoretical and practical applications, minimax programming has recently become one of the most substantial research areas in applied mathematics and operations research. Many authors investigated, therefore, optimality conditions and duality theorems

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