Taylor Series Method for Solving Linear Fredholm Integral Equation of Second Kind Using MATLAB

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Abstract

This paper presents a new method to find the approximation solution for linear ferdholm integral equation : $y(x) = f(x) + \lambda \int_a^b k(x,t)y(t)$ by using Taylor series expansion to approximate the kernel $k(x,t)$ as a summation of multiplication functions $f_n(x)$ by $g_n(t)$ i.e. $k(x,t) = \sum_{n=1}^{N} f_n(x) g_n(t)$ then use the degenerate kernel idea to solve the fredholm integral equation .In this paper we solve the above integral equation with $a = 0$ and $b = 1$, λ is a real number, $f(x)$ and $k(x,t)$ are real continues functions .

We have deduced a MATLAB program to solve the above equation, we have used MATLAB (R2008a) to perform this program .

The presented method has high accurate when compare its results with the other analytical methods results .

الخلاصة

فــي هــذا البحــث ابتكــر البــاحثون طريقــة جديــدة لإيجــاد الحــل التقريبــي لمعادلــة فريــدهولم التكامليــة الخطيــة $y(x) = f(x) + \lambda \int_a^b k(x,t)y(t)$ وذلك باستخدام متسلسلة تايلور لتقريب النواة $k(x,t)$ وجعلها على شكل حاصل جمع من ضرب الدوال $f_n(x)$ و $g_n(t)$ بمعنى آخر تكون النواة كالآتي : $k(x,t) = \sum_{n=1}^{N} f_n(x) g_n(t)$ ومن ثم استخدام فكرة النواة المنحلة لإيجاد حل معادلة فريدهولم التكاملية . في بحثنا هذا وجدنا الحل عند $a = 0$ و $b = 1$ بحيث يكون λ عدد حقيقي وكانت كل من $f(x)$ و $k(x,t)$ دوال حقيقية مستمرة .

في هذا البحث كونا برنامج لتقريب دالة النواة ومن ثم ايجاد الحل وقد نفذنا البرنامج بواسطة MATLAB (R2008a).

وعند مقارنة نتيجة الحل التقريبي مع نتائج حلول تحليلية اخرى وجدنا ان الحل التقريبي كان عالي الدقة.

1-Introduction

Integral equations, that is, equations involving an unknown function which appear under an integral sign. Such equations occur widely in divers areas of applied mathematics ,they offer a powerful technique for using the integral equation rather than differential equations is that all of the conditions specifying the initial value problems or boundary value problems for a differential equation can often be condensed into a single integral equation . So that any boundary value problems can be transformed into fredholm integral equation involving an unknown function of only once variable.

This reduction of what may represent a complicated mathematical model of physical situation into a single equation is itself a significant step , but there are other advantages to be gained by replacing differentiation with integration ,some of these advantages arise because integration is a smooth process ,a feature which has significant implication when approximation solution are sough .

2-Importance of the work

The main purpose is to produce of this paper a new approximation solution by approximate the kernel $k(x,t)$ using Taylor series expansion for the function of two variables and making it as a degenerate kernel then finding the solution of ferdholm integral equation .

3-A Review of previous works

There are many papers deal with numerical and approximate solutions of fredholm integral equations, Akber and Omid (Zabadi & Fard, 2007) produced an approach via optimization methods to find approximation solution for non linear fredholm integral equation of first kind, while Vahidi and Mokhtari produced the system of linear fredholm integral equation of second kind was handled by applying the decomposition method(Vahidi & Mokathri, 2008). Babolian and Sadghi proposed the parametric form of fuzzy number to convert a linear fuzzy fredholm integral equation of second kind to a linear of integral equation of the second kind in crisp case (Babolian & Goghory, 2005).

Hana and others considered the problem of numerical inversion of fredholm integral equation of the first kind via piecewise interpolation(Hanna *et al.,* 2005). Maleknejad and others proposed to use the continuous legender wavelets on the interval $[0,1]$to solve the linear second kind integral equation (Maleknejad *et al.,* 2003), the numerical methods to approximate the solution of system of second kind fredholm integral equation were proposed by Debonis and Laurita (Debonis & Laurita , 2008).

Chan *et al.,* presented a scheme based on polynomial interpolation to approximate matrices A from the discretizetion the integral operators(Chan *et al.,* 2002) and cubic spline interpolations has been proposed to solve integral equations by Kumar and Sangal (Kumar and Sangal, 2004)

3- Separate or degenerate kernel

A kernel $k(x,t)$ is called separable if it can be expressed as the sum of a finite number of terms ,each of which is the product of a function of x only and a function of t only i.e.

$k(x,t)=\sum_{i=1}^{n} g_i(x)h_i(t)$ (Raisinghania, 2007).

4- Solution of ferdholm integral equation of second kind with degenerate kernel (Raisinghania, 2007)**.**

Consider the non homogenous fredholm integral equation of second kind

$$y(x)=f(x)+\lambda\int_a^b k(x,t)y(t)dt\quad\text{...(1)}$$

Since the kernel $k(x,t)$ is degenerate or separate we take

$$k(x,t)=\sum_{i=1}^{n} f_i(x)g_i(t)\quad\text{..(2)}$$

Where the functions $f_i(x)$ assumed to be linearly independent, using (2) and (1) reduces to $y(x)=f(x)+\lambda\int_a^b[\sum_{i=1}^{n} f_i(x)g_i(t)]y(t)dt$...........................(3)

or $y(x)=f(x)+\lambda\sum_{i=1}^{n} f_i(x)\int_a^b g_i(t)y(t)dt$..................(4)

using (4) ,(3) reduces to $y(x)=f(x)+\lambda\sum_{i=1}^{n} C_i f_i(x)$.................................(5)

where constants $C_i\,(i=1,2,3,\ldots\ldots,n)$ are to be determined in order to find the solution of (1) in the form given by (5) . We now proceed to evaluate $C_i\,'s$ as follows:

from (5) we have $y(t) = f(t) + \lambda\sum_{i=1}^{n} C_i f_i(t)$..................................(6)

substituting the values of $y(x)$ and $y(t)$ given in (5) and (6) respectively in (3) , we have

$$f(x) + \lambda\sum_{i=1}^{n} C_i f_i(x) = f(x) + \lambda\sum_{i=1}^{n} f_i(x)\int_a^b g_i(t)\{f(t) + \lambda\sum_{i=1}^{n} C_i f_i(t)\}dt$$

or $\sum_{i=1}^{n} C_i f_i(x) = \sum_{i=1}^{n} f(x)\{\int_a^b g_i(t)f(t)dt + \lambda\sum_{j=1}^{n} C_j \int_{j\,a}^{b} g_i(t)f_j(t)dt\}$.........................(7)

Now, let $\boldsymbol{\beta_i = \int_a^b g_i(t)f(t)dt}$ and $\alpha_{ij} = \int_a^b g_i(t)f_j(t)dt$............................(8)

Where β_i and α_{ij} are known constant, then (7) may simplify as

$\sum_{i=1}^{n} C_i f_i(x) = \sum_{i=1}^{n} f_i(x)\{\beta_i + \lambda\sum_{j=1}^{n}\alpha_{ij}C_j\}$ or $\sum_{i=1}^{n} f_i(x)\{C_i - \beta_i - \lambda\sum_{j=1}^{n}\alpha_{ij}C_j\} = 0$, but the functions $f_i(x)$ are linearly independent ,therefore $C_i - \beta_i - \lambda\sum_{j=1}^{n}\alpha_{ij}C_j = 0 \;\; i = 1,2,3,\ldots,n$ or

$C_i - \lambda\sum_{j=1}^{n}\alpha_{ij}C_j = \beta_i \;\; i = 1,2,3,\ldots,n$ ....(9)

Then we obtain the following system of linear equations to determine $C_1, C_2, \ldots, C_n$

$$\begin{aligned}
(1-\lambda\alpha_{11})C_1 - \lambda\alpha_{12}C_2 - \ldots - \lambda\alpha_{12}C_n &= \beta_1 \\
-\lambda\alpha_{21}C_1 + (1-\lambda\alpha_{22})C_2 - \ldots - \lambda\alpha_{2n}C_n &= \beta_2 \\
&\vdots \\
&\vdots \\
-\lambda\alpha_{n1}C_1 - \lambda\alpha_{n2}C_2 - \ldots + (1-\lambda\alpha_{nn})C_n &= \beta_n
\end{aligned}$$

The determinate $D(\lambda)$ of system

$$D(\lambda) = \begin{vmatrix} 1-\lambda\alpha_{11} & -\lambda\alpha_{12} & \ldots & -\lambda\alpha_{1n} \\ -\lambda\alpha_{21} & 1-\lambda\alpha_{22} & \ldots & -\lambda\alpha_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ -\lambda\alpha_{n1} & -\lambda\alpha_{n2} & \ldots & 1-\lambda\alpha_{nn} \end{vmatrix} \quad \ldots\ldots\ldots\ldots(10)$$

Which is a polynomial in λ of degree at most (n) , $D(\lambda)$ is not identically zero ,since when $\lambda = 0$, $D(\lambda) = 1$ .to discuss the solution of (1) , the following situation arise:

Situation I : when at least on right member of the system $(\beta_1),(\beta_2),\ldots,(\beta_n)$ is non zero ,the following two cases arise under this situation :

(i) if $D(\lambda) \neq 0$,then a unique non zero solution of system $(\beta_1),(\beta_2),\ldots,(\beta_n)$ exist and so (1) has unique non zero solution given by (5) .

(ii) if $D(\lambda)=0$,then the equations $(\beta_1),(\beta_2),....,(\beta_n)$ have either no solution or they possess infinite solution and hence (1) has either no solution or infinite solution.

Situation II: when $f(x)=0$,then (8) shows that $\beta_j=0$ for $j=1,2,...,n$ .Hence the equations $(\beta_1),(\beta_2),....,(\beta_n)$ reduce to a system of homogenous linear equation .The following two cases arises under this situation

(i) if $D(\lambda)\neq 0$,then a unique zero solution $C_1=C_2=...=C_n=0$ of the system $(\beta_1),(\beta_2),....,(\beta_n)$ exist and so from (5) we see that (1) has unique zero solution $y(x)=0$.

(ii) if $D(\lambda)=0$,then the system $(\beta_1),(\beta_2),....,(\beta_n)$ posses infinite non zero solutions and so (1) has infinite non zero solutions , those value of λ for which $D(\lambda)=0$ are known as the eigenvalues and any nonzero solution of the homogenous fredholm integral equation $y(x)=\lambda\int_a^b k(x,t)y(t)dt$ is known as a corresponding eigenfunction of integral equation .

Situation III: when $f(x)\neq 0$ but

$\int_a^b g_1(x)f(x)dx=0,\int_a^b g_2(x)f(x)dx=0,...,\int_a^b g_n(x)f(x)=0$ i.e. $f(x)$ is orthogonal to all the functions $g_1(t),g_2(x),...,g_n(x)$,then (8) shows that $\beta_1,\beta_2,...,\beta_n$ reduce to a system of homogenous linear equations. The following two cases arise under this situation .

(i) if $D(\lambda)\neq 0$,then a unique zero solution $C_1=C_2=...=C_n=0$ then (1) has only unique solution $y(x)=0$.

(ii) If $D(\lambda)=0$ then the system $(\beta_1),(\beta_2),....,(\beta_n)$ possess infinite nonzero solutions and (1) has infinite nonzero solutions .The solution corresponding to the eigenvalues of λ .

4-1 Example (1) : find the analytical solution of the following integral equation

$$y(x)=1+\int_0^1(1-3xt)y(t)dt$$

Solution :since $k(x,t)=1-3xt$ that mean $k(x,t)$ separated function $f_1(x)=1,f_2(x)$ $g_1(t)=1,g_2(t)=t$, $f(x)=1,\lambda=1$, from equation (6) we obtain $y(x)=1+[C_1-3xC_2]$,then

$$\begin{bmatrix}1-\lambda\alpha_{11} & -\lambda\alpha_{12}\\ -\lambda\alpha_{21} & 1-\lambda\alpha_{21}\end{bmatrix}\begin{bmatrix}C_1\\ C_2\end{bmatrix}=\begin{bmatrix}\beta_1\\ \beta_2\end{bmatrix}\Rightarrow\begin{bmatrix}1-\alpha_{11} & -\alpha_{12}\\ -\alpha_{21} & 1-\alpha_{21}\end{bmatrix}\begin{bmatrix}C_1\\ C_2\end{bmatrix}=\begin{bmatrix}\beta_1\\ \beta_2\end{bmatrix}$$

$\alpha_{11}=\int_0^1 dx=1$, $\alpha_{12}=-\int_0^1 3dx=\frac{-3}{2}$, $\alpha_{21}=\int_0^1 xdx=\frac{1}{2}$,$\alpha_{22}=-\int_0^1 3x^2dx=-1$

$\beta_1=\int_0^1 dx=1$, $\beta_2=\int_0^1 xdx=\frac{1}{2}$,then $\begin{bmatrix}0 & 3/2\\ -1/2 & 2\end{bmatrix}\begin{bmatrix}C_1\\ C_2\end{bmatrix}=\begin{bmatrix}1\\ 1/2\end{bmatrix}$ that implies $C_1=\frac{5}{3}$, $C_2=\frac{2}{3}$ and $y(x)=1+[\frac{5}{3}-2x]$.

5- Taylor series of function with two variables (Karris, 2004)

Let $f(x,y)$ is a continuous function of two variables x and y ,then the Taylor series expansion of function f at the neighborhood of any real number a with respect to the variable y is :

$$taylor(f,y,a)=\sum_{n=o}^{\infty}\frac{(y-a)^n}{n!}\frac{\partial^n}{\partial y^n}f(x,y=a)$$

and $taylor(f,y,a,m)=\sum_{n=o}^{m}\frac{(y-a)^n}{n!}\frac{\partial^n}{\partial y^n}f(x,y=a)$ that mean the m^{th} terms of Taylor expansion to the function at the neighborhood a with respect to the variable y

5-1 Examples

Example (2) : The five terms of Taylor series expansion of the function $f(x,y)=e^{xy}$ at 1) $a=0$ and 2) $a=3$ as the following :

1) $taylor(f,y,0,5)=1+xy+\frac{1}{2}y^2x^2+\frac{1}{6}y^3x^3+\frac{1}{24}y^4x^4$

2)

$$taylor(f,y,3,5)=e^{3x}+(y-3)xe^{3x}+\frac{1}{2}(y-3)^2x^2e^{3x}+\frac{1}{6}(y-3)^3x^3e^{3x}+\frac{1}{24}(y-3)^4x^4e^{3x}$$

Example (3): Compare the values of the function $f(x,y)=e^{xy}$ at the point (2,4) with its Taylor expansion of three terms .

Solution: $f(x,y)=e^{xy}$ and $f(2,4)=e^8=2980.9$

the three terms of Taylor expansion is $taylor(f,x,2,3)=e^{2y}+y(x-2)e^{y}+\frac{y^2}{2}(x-2)^2e^{2y}$,

then the Taylor expansion at (2,4) is 2981.

6-Remark: The Taylor series must be calculated at the point or close to the point that we want the value of the function at that point as shown in example (3).

7-Our work : since any continuous function $k(x,t)$ of two variables can be approximated by the Taylor expansion therefore , then this function can be separated as a summation of product terms of $f_i(x)$ by $g_i(t)$ i.e. $k(x,t)=\sum_{i=1}^{n}f_i(x)g_i(t)$

7-1 Example (4) : if $f(x,t)=e^{xt}$,then the Taylor expansion with respect the variable t at $a=0$ with five terms is $taylor(f,t,0,5)=1+tx+\frac{1}{2}t^2x^2+\frac{1}{6}t^3x^3+\frac{1}{24}t^4x^4$,that mean

$f_1(x)=1, f_2(x)=x, f_3(x)=\frac{1}{2}x^2, f_4(x)=\frac{1}{6}x^3, f_5(x)=\frac{1}{24}x^4$,and

$g_1(t)=1, g_2(t)=t, g_3(t)=t^2, g_4(t)=t^3, g_5(t)=t^4$

7-1-1 The Algorithm of separation kernel and solution of fredholm integral equation

- **a-** ***input the kernel*** $k(x,t)$
- **b-** ***input the function*** $f(x)$
- **c-** ***input the value of*** λ

d- ***input the values*** a ***and*** b

e- ***input the number of Taylor series' terms*** N

f- ***calculate the Taylor expansion of*** $k(x,t)$ ***with respect*** t ***,***

$$taylor(f,t,a,N)=\sum_{i=o}^{N}\frac{(t-a)^i}{i!}\frac{\partial^i}{\partial y^i}f(x,t=a)$$

g- ***from*** **f** ***find*** $f_i(x)$ ***and*** $g_i(t)$ ***,*** $i=0,1,\ldots,N$

h- ***calculate*** $\alpha_{ij}=\int_a^b g_i(x)f_j(x)dx\ i,j=1,2,\ldots,N$ ***and*** $\beta_i=\int_a^b g_i(x)f(x)dx$ $,j=1,2,\ldots,N$

i- ***calculate the matrix*** $A=\begin{bmatrix}1-\lambda\alpha_{11} & -\lambda\alpha_{12} & \ldots & -\lambda\alpha_{1N}\\ -\lambda\alpha_{21} & 1-\lambda\alpha_{22} & \ldots & -\lambda\alpha_{2N}\\ \vdots & \vdots & \vdots & \vdots\\ -\lambda\alpha_{N1} & -\lambda\alpha_{N2} & \ldots & 1-\lambda\alpha_{NN}\end{bmatrix}$

j- ***calculate the determinate*** $D(A)$ ***of matrix*** A

k- ***if*** $f(x)\neq 0$ ***go to step n***

l- ***if*** $D(A)=0$ ***the system has infinite number of solutions ,go to step s***

m- ***the system has unique solution*** $C_1=C_2=\ldots=C_N=0$***,go to step s***

n- ***if*** $\beta_i\neq 0$ ***go to step r***

o- ***if*** $D(A)=0$ ***, the system has infinite number of solutions ,go to step s***

p- ***the system has unique solution*** $C_1=C_2=\ldots=C_N=0$

q- ***if*** $D(A)=0$ ***,the system has no real solution, go to step s***

r- ***the solution of system is*** $[C_i]=[A_{ij}]^{-1}[\beta_i]^T$ ***then*** $y(x)=f(x)+\lambda\sum_{i=1}^{n}C_i f_i(x)$

s- ***end***

7-1-2 Numerical results

In this section we present numerical results by solve the ferdholm integral equation by our approximation solution then comparison it with analytical solution

7-1-2-1 Examples

Example (5) :the approximation solution of integral equation $y(x)=1+\int_0^1\sin(x+t)dt$ as following : $taylor(\sin(x+t),t,5)=\sin(x)+t\cos(x)-\frac{t^2}{2}\sin(x)-\frac{t^3}{6}\cos(x)+\frac{t^4}{24}\sin(x)$, that implies

$$f_1(x)=\sin(x), f_2(x)=\cos(x), f_3(x)=\frac{-1}{2}\sin(x), f_4(x)=\frac{-1}{6}\cos(x), f_5(x)=\frac{1}{24}\sin(x)$$

and

$g_1(t)=1, g_2(t)=t, g_3(t)=t^2, g_4(t)=t^3, g_5(t)=t^4$, by using the previous algorithm and the related MATLAB program the solution is $y=1+3.9878\sin(x)+2.3833\cos(x)$,

alfa =

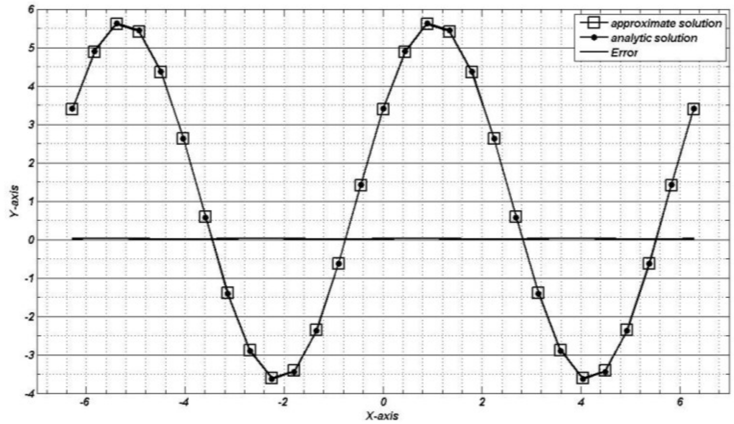
0.4597 0.8415 -0.2298 -0.1402 0.0192
0.3012 0.3818 -0.1506 -0.0636 0.0125
0.2232 0.2391 -0.1116 -0.0399 0.0093
0.1771 0.1717 -0.0885 -0.0286 0.0074
0.1467 0.1331 -0.0733 -0.0222 0.0061
beta = [1.0000 0.5000 0.3333 0.2500 0.2000]
A =
0.5403 -0.8415 0.2298 0.1402 -0.0192
-0.3012 0.6182 0.1506 0.0636 -0.0125
-0.2232 -0.2391 1.1116 0.0399 -0.0093
-0.1771 -0.1717 0.0885 1.0286 -0.0074
-0.1467 -0.1331 0.0733 0.0222 0.9939
C = [4.8387 2.6109 1.7935 1.3655 1.1020]
Y(x) =1+3.9878*sin(x)+2.3833*cos(x)
While the analytical solution by using the degenerate kernel was in Raisinghania. (2007)
$y = 1 + 4.01\sin(x) + 2.404\cos(x)$.
The following table shows the analytical and approximate results

Table (1) comparison between the analytical solution and the approximation solution of

$$y(x) = 1 + \int_0^1 \sin(x+t)dt$$

x	Analytical solution y1=1+4.01*sin(x)+2.404*cos(x)	Approximate solution y2=1+3.9878*sin(x)_2.3833*cos(x)	Error=abs(y1-y2)
-6.28318	3.404005242	3.383305213	0.020700029
-5.65487	5.30189787	5.272102379	0.029795491
-5.02655	5.55661239	5.529102297	0.027510093
-4.39823	4.07085655	4.056139771	0.014716779
-3.76991	1.412138355	1.415836197	0.003697843
-3.14159	-1.404002621	-1.383302606	0.020700015
-2.51327	-3.301896674	-3.272101186	0.029795487
-1.88496	-3.556613074	-3.529102973	0.027510101
-1.25664	-2.070858854	-2.056142058	0.014716796
-0.62832	0.587858602	0.584160778	0.003697823
0	3.404	3.3833	0.0207
0.628318	5.301895477	5.272099993	0.029795484
1.256637	5.556613759	5.529103649	0.02751011
1.884955	4.070861158	4.056144345	0.014716813
2.513274	1.412144442	1.415842246	0.003697803
3.141592	-1.403997379	-1.383297394	0.020699985
3.76991	-3.30189428	-3.2720988	0.02979548
4.398229	-3.556614443	-3.529104325	0.027510118
5.026547	-2.070863463	-2.056146632	0.014716831
5.654866	0.587852514	0.58415473	0.003697784
6.283184	3.403994758	3.383294787	0.020699971

The following figure shows comparison between of the two results



Fig(1) the analytical and approximation solutions results of integral equation

$$y(x) = 1 + \int_0^1 \sin(x+t)dt$$

Example (6) : The approximation solution of the integral equation

$y(x) = x + \int_0^1 \{xt + (xt)^{\frac{1}{2}}\}dt$ as the following:

$$k(x,t) = xt + (xt)^{\frac{1}{2}}$$

$$\Rightarrow taylor(k,t,1,5) = x + x^{\frac{1}{2}} + (x + \frac{1}{2}x^{\frac{1}{2}})(t-1) - \frac{1}{8}x^{\frac{1}{2}}(t-1)^2 + \frac{1}{16}x^{\frac{1}{2}}(t-1)^3 - \frac{5}{128}x^{\frac{1}{2}}(t-1)^4$$

That implies

$$f_1(x) = x + x^{\frac{1}{2}}, f_2(x) = x + \frac{1}{2}x^{\frac{1}{2}}, f_3(x) = \frac{-1}{8}x^{\frac{1}{2}}, f_4(x) = \frac{1}{16}x^{\frac{1}{2}}, f_5(x) = \frac{-5}{128}x^{\frac{1}{2}}$$

$$g_1(t) = 1, g_2(t) = (t-1), g_3(t) = (t-1)^2, g_4(t) = (t-1)^3, g_5(t) = (t-1)^4 .$$

By using the algorithm and the MATLAB program we obtain the solution is

$$y = 3.6601x + 2.3743x^{\frac{1}{2}}$$

alfa =

1.1667	0.8333	-0.0833	0.0417	-0.0260
-0.4333	-0.3000	0.0333	-0.0167	0.0104
0.2357	0.1595	-0.0190	0.0095	-0.0060
-0.1516	-0.1008	0.0127	-0.0063	0.0040
0.1072	0.0703	-0.0092	0.0046	-0.0029

beta =[0.5000 -0.1667 0.0833 -0.0500 0.0333]

A =

-0.1667	-0.8333	0.0833	-0.0417	0.0260
0.4333	1.3000	-0.0333	0.0167	-0.0104
-0.2357	-0.1595	1.0190	-0.0095	0.0060
0.1516	0.1008	-0.0127	1.0063	-0.0040
-0.1072	-0.0703	0.0092	-0.0046	1.0029

C =[3.0452 -1.1206 0.6055 -0.3874 0.2729]

Y = 3.6601*x+2.3743*x^(1/2)

While the analytical solution was([9] $y = \frac{96}{26}x + \frac{60}{26}x^{\frac{1}{2}}$

Table (2) comparison between the analytical solution and the approximation solution of $y(x) = x + \int_0^1 \{xt + (xt)^{\frac{1}{2}}\}dt$

x	Analytical solution y1=(90/26)x+(60/26)x^.5	Approximate solution y2=3.6601x+2.3743x^.5	Error =abs(y1-y2)
0	0	0	0
0.5	3.477938726	3.508933631	0.030994905
1	6	6.0344	0.0344
1.5	8.364795857	8.398061748	0.033265891
2	10.64818514	10.67796726	0.029782117
2.5	12.87955115	12.90434792	0.024796778
3	15.0739634	15.09270823	0.01874483
3.5	17.24037391	17.25225857	0.011884659
4	19.38461538	19.389	0.004384615
4.5	21.51073925	21.50710089	0.003638363
5	23.62169533	23.6095962	0.012099134
5.5	25.71971049	25.69877707	0.020933423
6	27.80651479	27.7764235	0.030091295
6.5	29.88348405	29.84395102	0.039533039
7	31.95173379	31.90250734	0.049226457
7.5	34.01218336	33.95303834	0.059145014
8	36.06560106	35.99633452	0.069266535
8.5	38.1126368	38.03306454	0.07957226
9	40.15384615	40.0638	0.090046154

The following figure shows the comparison between the two results

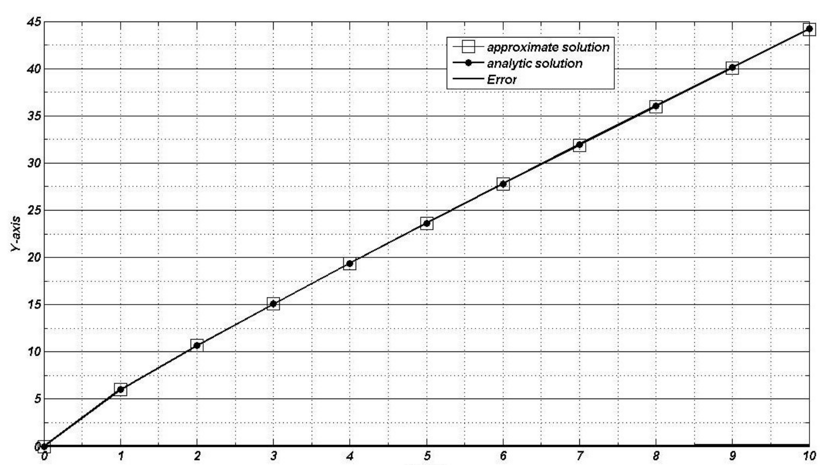


Fig (2) the analytical and approximation solutions results of integral equation

$$y(x) = x + \int_0^1 \{xt + (xt)^{\frac{1}{2}}\}dt$$

7-1-3 Remark : We find Taylor expansion of the kernel at the point $a = 1$ instead at $a = 0$ to avid the division by zero.

Conclusion and future work

The method of approximate kernel by Taylor expansion is a new method to solve the fredholm integral equation of second kind, and it has high accurate results , in this paper we have approached to solve the fredholm integral equation with integration limits from 0 to 1 just.

In future work we hope to solve the fredholm integral equation of second kind with integration limits from a to b whatever the values of a and b .

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