VOLUME 266 JANUARY 2014 ISSN 0029-5493

Nuclear Engineering and Design

An International Journal devoted to all aspects of Nuclear Fission Energy

(Contents continued on back cover)

Available online at www.sciencedirect.com
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Nuclear Engineering and Design 266 (2014) 34–42

Contents lists available at ScienceDirect

Nuclear Engineering and Design

journal homepage: www.elsevier.com/locate/nucengdes

Viscous dissipation and radiation effects on MHD natural convection in a square enclosure filled with a porous medium

Sameh E. Ahmed[a], Ahmed Kadhim Hussein[b,*], H.A. Mohammed[c], I.K. Adegun[d], Xiaohui Zhang[e], Lioua Kolsi[f], Arman Hasanpour[g], S. Sivasankaran[h]

[a] Department of Mathematics, Faculty of Sciences, South Valley University, Qena, Egypt
[b] College of Engineering, Mechanical Engineering Department, Babylon University, Babylon City—Hilla, Iraq
[c] Department of Thermofluids, Faculty of Mechanical Engineering, University Teknologi Malaysia (UTM), 81310 UTM Skudai, Johor Bahru, Malaysia
[d] Department of Mechanical Engineering, University of Ilorin, Ilorin, Nigeria
[e] School of Physics Science and Technology, School of Energy—Soochow University, Suzhou 215006, Jiangsu, People's Republic of China
[f] Unite' de Me'trologie en Me'canique des Fluides et Thermique, Ecole Nationale d'Inge'nieurs, Monastir, Tunisia
[g] Department of Mechanical Engineering, Babol University of Technology, PO Box 484, Babol, Iran
[h] Institute of Mathematical Sciences, University of Malaya, Kuala Lumpur 50603, Malaysia

HIGHLIGHTS

- *Ha* decelerates the flow field.
- *Ha* enhances conduction.
- Magnetic field orientation is important.
- Radiation parameter important.
- Nu decreases as *Ha* increases.

ARTICLE INFO

Article history:
Received 5 May 2013
Received in revised form 13 October 2013
Accepted 16 October 2013

ABSTRACT

Numerical two-dimensional analysis using finite difference approach with "line method" is performed on the laminar magneto-hydrodynamic natural convection in a square enclosure filled with a porous medium to investigate the effects of viscous dissipation and radiation. The enclosure heated from left vertical sidewall and cooled from an opposing right vertical sidewall. The top and bottom walls of the enclosure are considered adiabatic. The flow in the square enclosure is subjected to a uniform magnetic field at various orientation angles ($\varphi = 0^\circ, 30^\circ, 45^\circ, 60^\circ$ and 90°). Numerical computations occur at wide ranges of Rayleigh number, viscous dissipation parameter, magnetic field orientation angles, Hartmann number and radiation parameter. Numerical results are presented with the aid of tables and graphical illustrations. The results of the present work explain that the local and average Nusselt numbers at the hot and cold sidewalls increase with increasing the radiation parameter. From the other side, the role of viscous dissipation parameter is to reduce the local and average Nusselt numbers at the hot left wall, while it improves them at the cold right wall. The results are compared with another published results and it found to be in a good agreement.

1. Introduction

Magneto-hydrodynamic (MHD) was originally applied to astrophysical and geophysical problems. In recent years, this subject take more attention because of its various applications in agricultural engineering and petroleum industries. Astrophysical problems include solar structure, the solar wind bathing the earth and other planets, and interstellar magnetic fields. The primary geophysical problem is planetary magnetism, produced by currents deep in the planet. Recently, considerable attention has also been focused on new applications of magneto-hydrodynamics (MHD) and heat transfer such as metallurgical processing. Also, considerable efforts have been directed towards the study of magneto-hydrodynamic (MHD) flow and heat transfer in porous and non-porous media due to the effect of magnetic fields on the boundary layer flow control and on the performance of many systems using electrically conducting fluids. In addition, this type

* Corresponding author. Tel.: +9647813769317.
E-mail addresses: sameh_sci_math@yahoo.com (S.E. Ahmed), ahmedkadhim7474@gmail.com (A.K. Hussein).

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http://dx.doi.org/10.1016/j.nucengdes.2013.10.016

Nomenclature

Symbol	description Unit
B	magnetic field (T)
g	gravitational acceleration (m/s^2)
Ha	Hartmann number
C_p	Specific heat at constant pressure (J/kg K)
K	permeability of porous media (m^2)
k	Thermal conductivity (W/m K)
L	width and height of the square enclosure m
Nu	local Nusselt number
$\overline{Nu}$	average Nusselt number
R	radiation parameter
Ra	Rayleigh number
T	temperature (K)
U	non-dimensional velocity component in X-direction
u	dimensional velocity component in x-direction (m/s)
V	non-dimensional velocity component in Y-direction
v	dimensional velocity component in y-direction (m/s)
X	non-dimensional coordinate in horizontal direction
x	Cartesian coordinate in horizontal direction (m)
Y	non-dimensional coordinate in vertical direction
y	Cartesian coordinate in vertical direction (m).

Greek Symbols

α	thermal diffusivity (m^2/s)
β	coefficient of thermal expansion (K^{-1})
β_R	extinction coefficient (m^{-1})
θ	dimensionless temperature
ε	viscous dissipation parameter
φ	magnetic field orientation angle degree
Ψ^*	stream function (m^2/s)
Ψ	dimensionless stream function
σ	Stephan–Boltzmann constant (W/m^2 K^4)
μ	dynamic viscosity (kg/m s)
ν	kinamatic viscosity (m^2/s)
ρ	density (kg/m^3)

Subscripts

c	cold
h	hot
l	left
r	right

Abbreviations

MHD	magneto-hydrodynamics

of flow finds applications in many engineering problems such as MHD generators, plasma propulsion in astronautics, nuclear reactors, and geothermal energy extractions. From the other hand, the viscous dissipation heat in the natural convective flow is important, when the flow field is of extreme size or at low temperature or in high gravitational field. Such effects are also significant in geophysical flows and also in certain industrial operations and are usually characterized by the Eckert number. Furthermore, the role of thermal radiation is of major importance in the design of many advanced energy convection systems operating at high temperature. Thermal radiation within the systems is usually the result of emission by hot walls and the working fluid. All of these important subjects have received much attention by many researchers. The problem of natural convection in a porous and non-porous mediums with viscous dissipation, radiation and MHD effects had been studied by Gebhart and Mollendorf (1969), Alam et al. (2006, 2007) and Ashish Gad and Balaji (2010). Also, Gebhart (1962) explained that the viscous dissipation effect played an important role in natural convection in various devices which were subjected to large deceleration or which operated at high rotative speeds. Mahajan and Gebhart (1989) reported the influence of viscous heating dissipation in natural convective flows, showing that the heat transfer rates were reduced by an increase in the dissipation parameter. Hossain (1992) studied the effect of viscous and Joule heating on the free convection flow of an electrically conducting and viscous incompressible fluid past a semi-infinite plate of which temperature varied linearly with the distance from the leading edge and in the presence of uniform transverse magnetic field. Chowdhury and Islam (2000) investigated MHD free convection flow of visco-elastic fluid past an infinite porous plate. Israel-Cookey et al. (2003) investigated the influence of viscous dissipation and radiation on unsteady magneto-hydrodynamic free-convection flow past an infinite vertical heated plate in a porous medium with time-dependent suction. Saeid and Pop (2004) studied numerically the viscous dissipation effect on natural convection in a porous cavity and found that the heat transfer rate at hot surface decreased with the increase of viscous dissipation parameter. Duwairi and Duwairi (2004) studied the thermal radiation heat transfer effects on the MHD-Rayleigh flow of gray viscous fluids under the effect of a transverse magnetic field. They found that increasing the magnetic field strength decreased the velocity inside the boundary layer. Hossain et al. (2005) investigated the effect of viscous dissipation on natural convection from a vertical plate placed in a thermally stratified environment. Effects of viscous dissipation and temperature stratification were also shown on the velocity and temperature distributions in the boundary layer region. Al-Mamun et al. (2005) investigated the effects of conduction and convection on magneto-hydrodynamic (MHD) boundary layer flow with viscous dissipation from a vertical flat plate. The dimensionless skin friction coefficient, the surface temperature distribution, the velocity distribution and the temperature profile over the whole boundary layer were shown graphically for different values of the magnetic parameter, the viscous dissipation parameter and the Prandtl number. Badruddin et al. (2006a) analyzed numerically using finite element method the heat transfer under the influence of radiation and viscous dissipation in a square cavity filled with saturated porous medium. The flow was assumed to follow Darcy law. Left vertical surface of the square cavity was maintained at isothermal hot temperature while the right vertical surface was maintained at isothermal cold temperature. Results were presented in terms of Nusselt number at hot and cold cavity walls for various values of viscous dissipation and radiation parameters. It was seen that the average Nusselt number at hot as well as cold walls increased with the increase in radiation parameter. Ridouane et al. (2006) studied numerically using the finite difference method coupled laminar natural convection with radiation in air-filled square enclosure heated from below and cooled from above for a wide variety of radiative boundary conditions at the sidewalls. Simulations were performed for two values of the emissivities of the active and insulated walls ($\varepsilon_1 = 0.05$ or 0.85, $\varepsilon_2 = 0.05$ or 0.85) and Rayleigh numbers ranging from 10^3 to 2.3×10^6. It was found that, for a fixed Rayleigh number, the global heat transfer across the enclosure depended only on the magnitude of the emissivity of the active walls. Badruddin et al. (2007) investigated numerically using finite element method the effect of viscous dissipation and thermal radiation on natural convection in a porous medium embedded within a vertical annular cylinder. The inner surface of the cylinder was maintained at an isothermal temperature (T_w) while the outer surface was maintained at ambient temperature (T_∞). Their study was focused to investigate the combined effect of viscous dissipation and radiation. It was observed that the viscous dissipation parameter reduced the average Nusselt number at hot surface. From the other side, the average Nusselt number increased at the cold surface due to increase viscous dissipation parameter. Al-Mamun et al. (2007) considered numerically the effects of conduction and viscous dissipation on natural convection flow of an incompressible, viscous and electrically conducting fluid in the presence of transverse magnetic field. The effects of magnetic parameter, Prandtl number, conjugate conduction parameter and viscous dissipation parameter on two-dimensional flow were discussed. Results of the velocity, temperature distributions as well as the skin friction and the rate of heat transfer were shown graphically. Zahmatkesh (2007) studied numerically the influence of thermal radiation on the development of free convective flow inside an enclosure filled with a fluid-saturated porous medium. The fluid was considered gray, absorbing-emitting, but non-scattering and the Rosseland diffusion approximation was used to describe the radiative heat flux in the energy equation. The two opposite walls of the enclosure were kept at constant but different temperatures while the other two walls were maintained adiabatic. The variations of Nusselt number on the non-adiabatic walls and temperature distribution on the adiabatic walls were investigated under different values of radiation parameter. Mezrhab et al. (2007) used a numerical hybrid lattice-Boltzmann equation finite-difference to study the radiation-natural convection phenomena in a differentially heated square cavity with partition of finite thickness and varying height located vertically at the center of the cavity. The results obtained showed that the radiation exchange produced a rise in the heat transfer. Nasrin and Alim (2009) described numerically using the implicit finite difference method the combined effects of viscous dissipation and temperature dependent thermal conductivity on MHD free convection flow with heat conduction and Joule heating along a vertical flat plate. The numerical results in terms of the skin friction coefficient, the surface temperature, the velocity and the temperature profiles over the whole boundary layer were shown graphically for different values of the Prandtl number, the magnetic parameter, the thermal conductivity variation parameter, viscous dissipation parameter and the Joule heating parameter. Shehadeh and Duwairi (2009) studied the MHD natural convection heat transfer problem with Joule and viscous heating effects inside a porous medium, in which two sides of the rectangular enclosure were adiabatic and the other two were isothermal. They found three parameters to describe the problem under consideration, i.e., the buoyancy parameter, the inclination angle parameter, and the magnetic effect parameter of the rectangular porous media-filled enclosure. Kishan and Amrutha (2010) studied non-linear MHD flow with heat and mass transfer characteristics of an incompressible, viscous, electrically conducting and Boussinesq fluid on a vertical stretching surface with thermal stratification and chemical reaction by taking into account the viscous dissipation effects. The results obtained showed that the flow field was influenced appreciably by the presence of viscous dissipation, thermal stratification, chemical reaction and magnetic field. Kishore et al. (2010) analyzed numerically using the implicit finite difference method of Crank–Nicolson's type unsteady free convection flow of an incompressible viscous fluid past an exponentially accelerated vertical plate by taking into account the heat due to viscous dissipation under the influence of a uniform transverse magnetic field. The effect of magnetic parameter, Grashof number, Eckert number, time and an acceleration parameter on velocity and temperature fields were investigated through graphs. Sharma and Singh (2010) investigated the effects of linearly varying thermal conductivity, viscous dissipation and Ohmic heating on steady free convection flow of a viscous incompressible electrically conducting liquid with a low Prandtl number along an inclined isothermal non-conducting porous plate in the presence of transverse magnetic field. The velocity and temperature distributions were discussed numerically and presented through graphs. AL-Badawi and Duwairi (2010) studied numerically the magneto-hydrodynamics (MHD) natural convection heat transfer with Joule and viscous heating effects inside an iso-flux inclined rectangular enclosure filled with porous medium. An iso-heat flux was applied for heating and cooling the two opposing walls of the enclosure while the other walls were considered adiabatic. The results showed that viscous and Joule heating effects decreased the heat transfer rates. Another useful researches have been conducted about this subject under different conditions (Raptis, 1998; Hossain et al., 2001; Badruddin et al., 2006b; Jalil and Al-Tae'y, 2007; Grosan et al., 2009; Mondal and Li, 2010). To our best knowledge, the interaction of viscous dissipation, magneto-hydrodynamic and radiation effects with natural convection flow in an enclosure filled with porous medium has not considered together up to date due to complex effects of flow and heat transfer characteristics if all these effects are taken into account. Hence, the present work is an attempt to analyze viscous dissipation and radiation effects on MHD natural convection in a square enclosure filled with a porous medium.

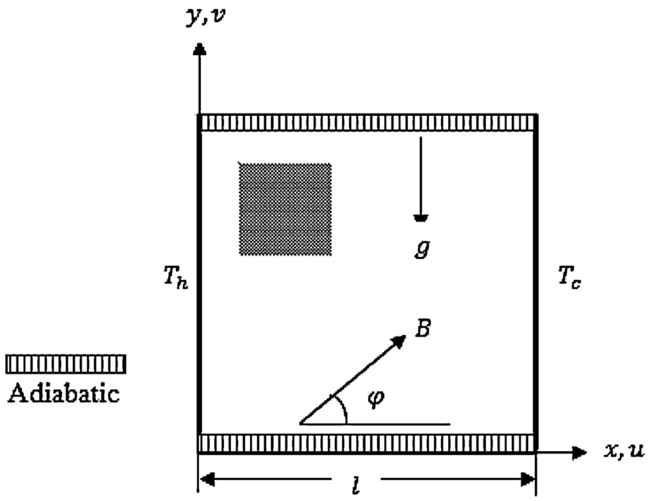

Fig. 1. Geometry of the problem and the coordinate system.

2. Mathematical formulation

2.1. Governing equations and geometrical configuration

MHD natural convective flow in the presence of thermal radiation and viscous dissipation inside a square enclosure of width and height (L) filled with fluid-saturated porous medium is considered in the present work. The geometric and the Cartesian coordinate system are schematically depicted in Fig. 1. The left sidewall of the enclosure is maintained at isothermal hot temperature (T_h) while the right sidewall of it is maintained at isothermal cold temperature (T_c). The top and bottom walls of the enclosure are considered thermally insulated. The flow in the square enclosure is subjected to a uniform magnetic field (B) at different orientation angles ($\varphi = 0°$, 30°, 45°, 60° and 90°). In order to simplify the modeling of the governing equations, the following assumptions are utilized:

(1) The flow is considered steady, two-dimensional, incompressible and laminar.
(2) Darcy's model is used for flow prediction inside the porous medium as described by Badruddin et al. (2006a).
(3) There is a local thermal equilibrium between the porous medium and the fluid.

(4) The porous medium has homogenous and isotropic permeability.
(5) The fluid physical properties are assumed constant, except the density in the body force term in the momentum equation which is treated according to Boussinesq approximation.
(6) The thermal radiation flux in y-direction is considered negligible compared with x-direction.

The range of Rayleigh number is taken as ($10 \le Ra \le 400$), the range of Hartmann number is taken as ($0 \le Ha \le 50$), the viscous dissipation parameter (ε) is varied as (0, 0.005 and 0.01) while the range of radiation parameter (R) is taken as ($0 \le R \le 15$).The dimensionless governing equations for this problem can be expressed as follows:

$$\frac{\partial^2\psi}{\partial X^2}+\frac{\partial^2\psi}{\partial Y^2}= -Ra\frac{\partial\theta}{\partial X}-Ha^2\left[\frac{\partial^2\psi}{\partial Y^2}\sin^2\varphi+2\frac{\partial^2\psi}{\partial X\partial Y}\sin\varphi\cos\varphi+\frac{\partial^2\psi}{\partial X^2}\cos^2\varphi\right] \tag{1}$$

$$\frac{\partial\psi}{\partial Y}\frac{\partial\theta}{\partial X}-\frac{\partial\psi}{\partial X}\frac{\partial\theta}{\partial Y}=\left(1+\frac{4R}{3}\right)\frac{\partial^2\theta}{\partial X^2}+\frac{\partial^2\theta}{\partial Y^2} +\varepsilon\left[\left(\frac{\partial\psi}{\partial Y}\right)^2+\left(\frac{\partial\psi}{\partial X}\right)^2\right] \tag{2}$$

These dimensionless governing equations have been obtained by employing the following non-dimensional variables as listed below (Badruddin et al., 2006a):

$$\theta=\frac{T-T_c}{T_h-T_c},\quad X=\frac{x}{L},\quad Y=\frac{y}{L},\quad \psi=\frac{\psi^*}{\alpha},\quad R=\frac{4\sigma T_c{}^3}{\beta_R k},$$
$$Ra=\frac{g\beta(T_h-T_c)KL}{\nu\alpha}$$

$$\varepsilon=\frac{\alpha\mu}{(T_h-T_c)K\rho C_p} \tag{3}$$

The corresponding dimensionless boundary conditions of the present problem are given by

1- No-slip boundary condition are applied at all enclosure boundaries, i.e., $U=V=\Psi=0$, where U and V are defined by:-

$$U=\frac{uL}{\alpha}\quad\text{and}\quad V=\frac{vL}{\alpha} \tag{4}$$

2- The left vertical sidewall (at $X=0$) is subjected to isothermal hot temperature (T_h), i.e.,

$$\text{at}\quad X=0,\theta=1 \tag{5}$$

3- The right vertical sidewall (at $X=1$) is subjected to isothermal cold temperature (T_c), i.e.,

$$\text{at}\quad X=1,\theta=0 \tag{6}$$

4- The top and the bottom walls are considered adiabatic, i.e.,

$$\text{at}\quad Y=0\quad\text{and}\quad Y=1,\quad\frac{\partial\theta}{\partial Y}=0 \tag{7}$$

In order to compute the heat transfer enhancement, the local and the average Nusselt number are represented as follows (Zahmatkesh, 2007):

$$\mathrm{Nu}=-\left[\left(1+\frac{4R}{3}\right)\frac{\partial\theta}{\partial X}\right]_{X=0,1}\quad\text{and}\quad\overline{Nu}=\int_0^1 Nu\,dY \tag{8}$$

3. Numerical procedure and validation

The numerical algorithm used to solve Eqs. (1) and (2) is based on the finite difference methodology. The central difference is used to approximate the first and second derivatives and then it transformed to the implicit line tridiagonal equations and solved in the X-direction by TDMA. The numerical method was implemented in a FORTRAN software. The unknowns θ and ψ were calculated iteratively until the following criteria of convergence was fulfilled:

$$\Sigma_{i,j}|\chi_{i,j}^{\text{new}}-\chi_{i,j}^{\text{old}}|\le 10^{-6}, \tag{9}$$

where χ is the general dependent variable. In order to ascertain the validity of the present computational procedure the average Nusselt number at the hot left sidewall $\left(\overline{Nu_l}\right)$ have been compared with the corresponding values reported by Moya et al. (1987), Baytas and Pop (1999), Misirlioglu et al. (2005) and Badruddin et al. (2006a), respectively, as shown in Table 1.The comparison is made using two values of Rayleigh number ($Ra=10$ and 100). It is found that the values obtained by the present computational procedure have a good agreement with those obtained by different authors as shown in Table 1 which validate the current computations indirectly. Therefore, the computational procedure is ready and can predict the viscous dissipation and radiation effects on MHD natural convection in a square enclosure filled with a porous medium and as a result, the previous verifications give a good interest in the present numerical model to deal with the recent physical problem.

4. Results and discussion

The problem of the two-dimensional laminar magneto-hydrodynamic natural convection in a square enclosure filled with a porous medium is solved numerically to investigate the effects of viscous dissipation and radiation. To illustrate the flow pattern and temperature distribution in the enclosure, the streamlines and the isotherms of the flow are plotted. Fig. 2 illustrates contours of streamlines (left) and isotherms (right) at $Ra=100$, $R=5$, $\varepsilon=0.005$ and $\varphi=\pi/3$ for various Hartmann number ($Ha=0$, 2, 20 and 50), respectively. The Hartmann number represents a measure of the relative importance of magneto-hydrodynamic flow. It can be observed from this figure that the flow field in the center of the enclosure is greatly affected by increasing the magnetic field strength (i.e., increasing the Hartmann number). The flow field in the enclosure can be represented by various rotating vortices which occupy all the enclosure region. These vortices are constructed due to the buoyancy force effects caused by differentially heated sidewalls. This is because the fluid adjacent the hot left sidewall moves

Table 1
Comparison of average Nusselt number at the hot left sidewall $\left(\overline{Nu_l}\right)$ with those reported by previous authors.

Author	$Ra=10$	$Ra=100$
Moya et al. (Moya et al., 1987)	1.065	2.801
Baytas and Pop (Baytas and Pop, 1999)	1.079	3.16
Misirlioglu et al. (Misirlioglu et al., 2005)	1.119	3.05
Badruddin et al. (Badruddin et al., 2006a)	1.0798	3.2005
Present (31×31)	1.081459	3.428671
Present (41×41)	1.080741	3.336948
Present (61×61)	1.080105	3.243083
Present (81×81)	1.080383	3.199452

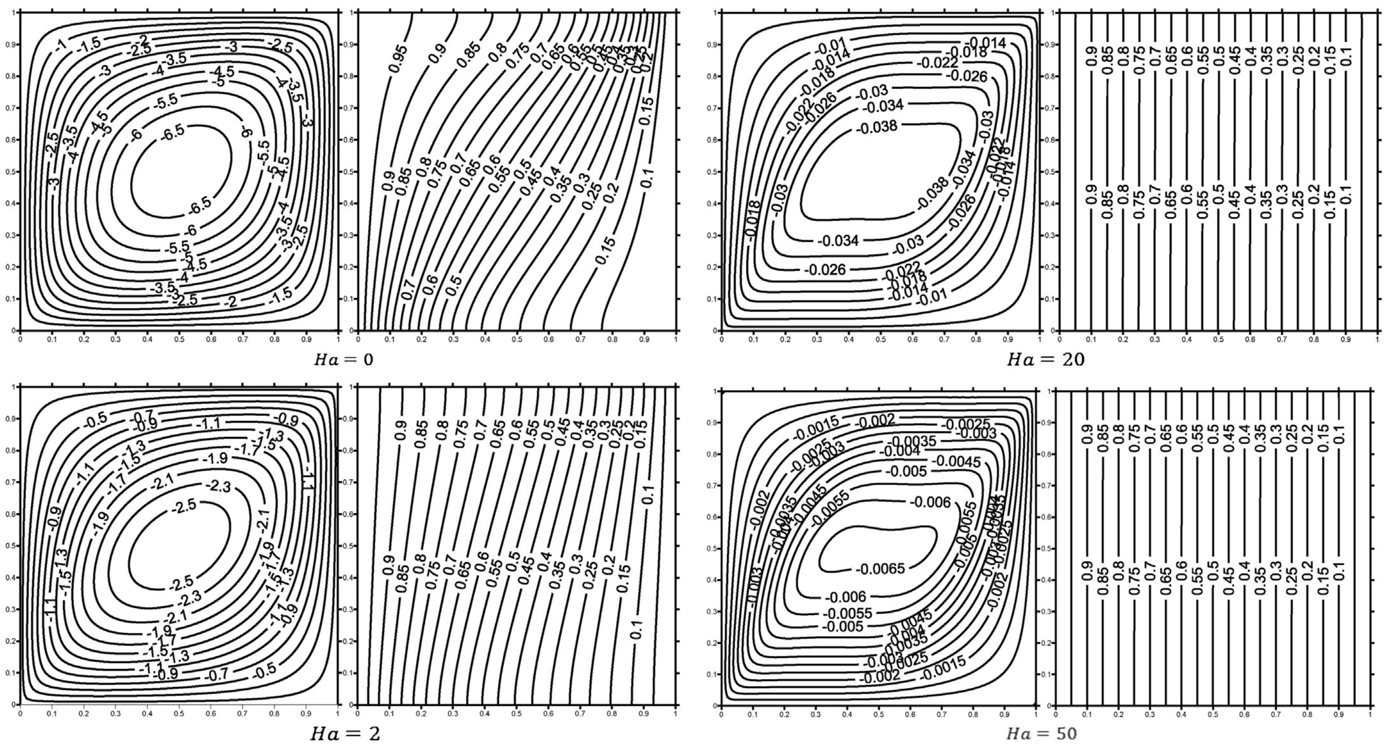

Fig. 2. Contours of streamlines (*left*) and isotherms (*right*) for various Hartmann number at $Ra = 100$, $R = 5$, $\varepsilon = 0.005$ and $\varphi = \pi/3$.

upwards, while the fluid adjacent the cold right sidewall moves in a reverse direction due to the buoyancy force effect. This repeated motion produces the flow rotating vortices. When the magnetic field effect is negligible (i.e., $Ha = 0$), the pattern of the streamlines and isotherms are similar to the corresponding pattern for the classical natural convection problem in a differentially heated porous enclosure and the strength of flow circulation is strong, since the buoyancy force due to natural convection is the only the significant force inside the enclosure. The same phenomena can be seen when the Hartmann number is low (i.e., $Ha = 2$). But, when the Hartmann number increases (i.e., $Ha = 20$ and 50), the effect of magnetic field becomes significant .Therefore, the stream function values decrease

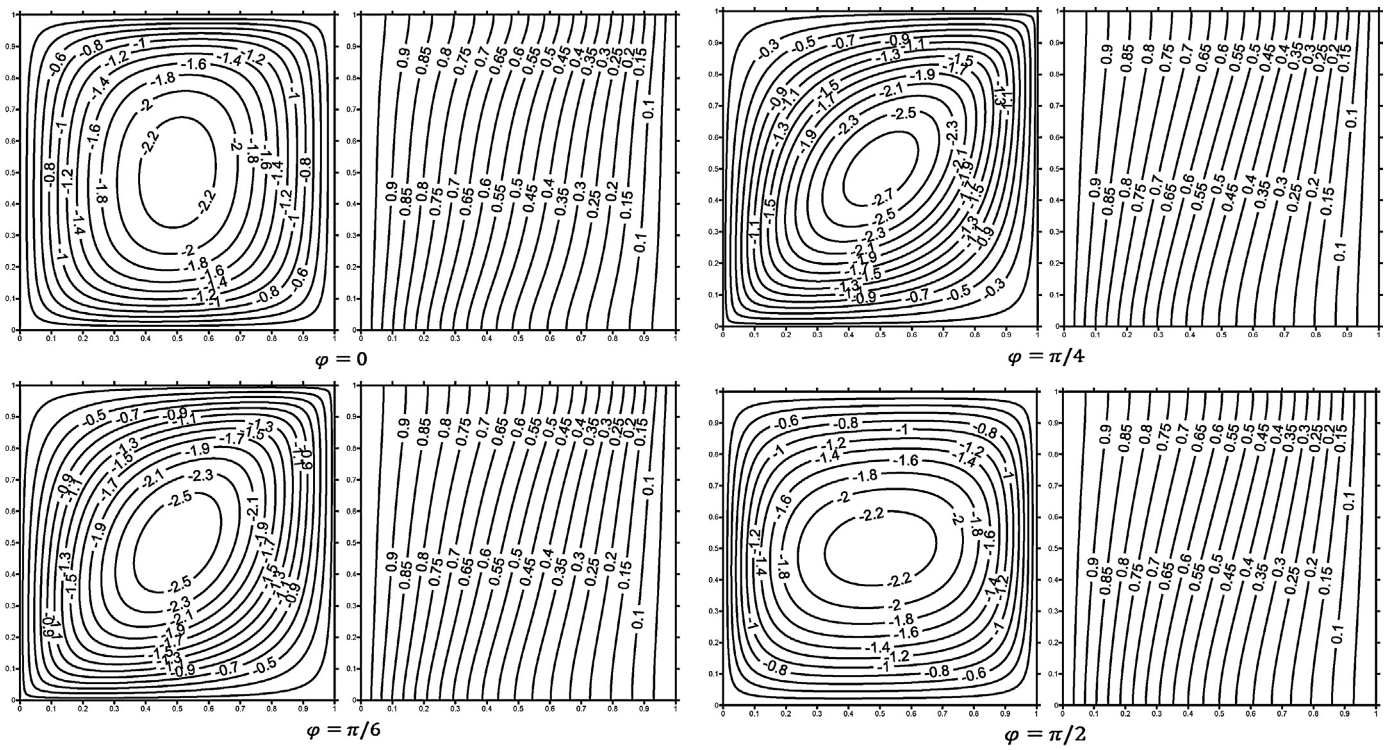

Fig. 3. Contours of streamlines (*left*) and isotherms (*right*) for various magnetic field orientation angles at $Ra = 100$, $R = 5$, $\varepsilon = 0.005$ and $Ha = 2$.

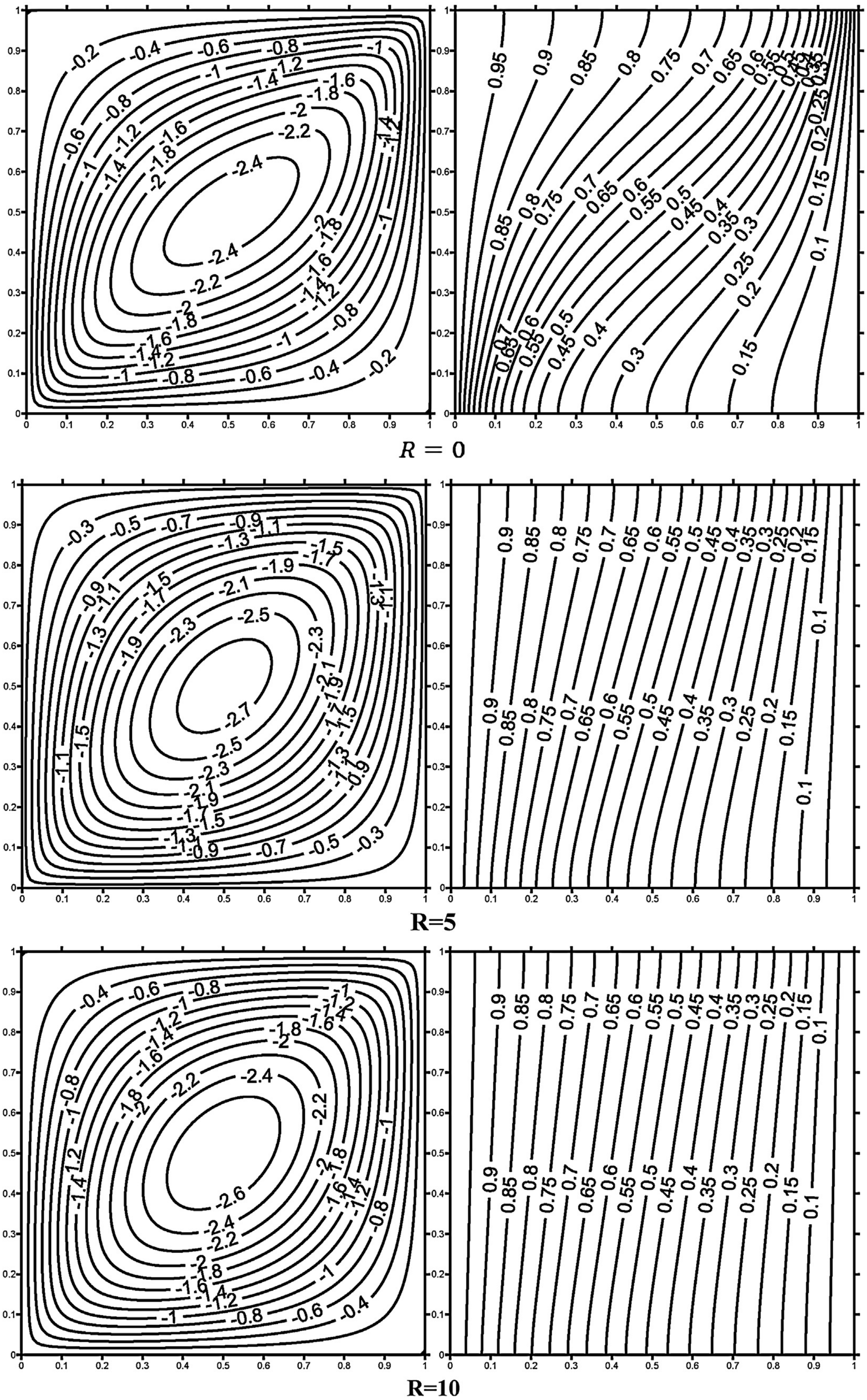

Fig. 4. Contours of streamlines (*left*) and isotherms (*right*) for various radiation parameter at $Ra = 100$, $\varphi = \pi/4$ and $Ha = 2$.

as the Hartmann number increases. Furthermore, a clear changing can be seen in the rotating vortices pattern especially in the core of the enclosure where the vortices are elongated horizontally as the Hartmann number increase. The reason of this behavior, since the effect of the buoyancy force decreases as the Hartmann number increases. With respect to isotherms, the hot fluid rises up along the hot left sidewall and descends along the cold right sidewall. When the magnetic field influence is absent ($Ha = 0$), the isotherm patterns are non-uniform and converge from each other indicating that the heat is transferred by convection. As the Hartmann number increases, the isotherm patterns begin to diverge gradually from each other and become like the symmetrical vertical lines which indicating that the heat is transferred by conduction. This behavior can be clearly observed when the Hartmann number is high (i.e., $Ha = 50$). Fig. 3 displays contours of streamlines (left) and isotherms (right) at $Ra = 100$, $R = 5$, $\varepsilon = 0.005$ and $Ha = 2$ for various magnetic field orientation angles ($\varphi = 0, \pi/6, \pi/4$ and $\pi/2$), respectively. It can be observed that, when the magnetic field orientation angle is zero (i.e., non-inclined or horizontal magnetic field case), the strength of flow circulation is high. In this case, the flow vortices pattern are similar to the elliptical shape. But, as the magnetic field orientation angle increases ($\varphi = \pi/6$ and $\pi/4$), the strength of flow circulation begins to decrease. This is because the magnetic field acts in both horizontal and vertical directions which causes to decelerate the flow circulation. For vertical magnetic field case ($\varphi = \pi/2$), the strength of flow circulation begins to increase again. Therefore, it can be concluded that when the magnetic field acts in the horizontal or in the vertical direction (i.e., $\varphi = 0$ or $\varphi = \pi/2$), the stream function values are greater than the corresponding values when it acts in inclined directions. Also, it can be seen that the core of vortices changes its direction gradually from the vertical position at ($\varphi = 0$) to the horizontal position at ($\varphi = \pi/2$).No significant change occurs in the isotherms when the magnetic field orientation angle increases. They are in general symmetrical and approximately parallel to the enclosure left and right sidewalls. Fig. 4 demonstrates contours of streamlines (left) and isotherms (right) at $Ra = 100$, $\varepsilon = 0.005$, $Ha = 2$ and $\varphi = \pi/4$ for different radiation parameter ($R = 0$, 5 and 10), respectively. It is evident from this figure that, the flow circulation increases as the radiation parameter increases. Moreover, the streamlines begin to take the enclosure geometry. With respect to isotherms, in the absence of radiation parameter ($R = 0$), the isotherms are non-uniform and cover most area of the enclosure indicating that the convection is the dominant mode of heat transfer. But, as the radiation parameter increases, the isotherms are changed to the nearly linear shape and the conduction is the dominant mode of heat transfer. It can be concluded from this result that, the radiation parameter enhances significantly the conduction mode and reduces the convection mode. Therefore, the thermal radiation has a significant effect on the establishment of the flow and thermal fields inside the enclosure. Also, it can be observed that with the presence of the radiation effect, the flow field is approximately symmetrical about the middle plane of the enclosure. Fig. 5 shows Nusselt number variations at the hot left (Nu_l) and cold right (Nu_r) sidewalls at $Ra = 100$, $R = 5$, $\varepsilon = 0.005$ and ($\varphi = \pi/3$) for various Hartmann number ($Ha = 0$, 2 and 20), respectively. It is found that, as the Hartmann number increases the Nusselt number at the hot left (Nu_l) and cold right (Nu_r) sidewalls decrease. Also, the average Nusselt number variations at the hot left $\left(\overline{Nu_l}\right)$ and cold right $\left(\overline{Nu_r}\right)$ decrease when the Hartmann number increases. This result can be observed in Table 2. This is due to the increase of the effect of the magnetic force when the Hartmann number increases. Thus, the increased magnetic field reduces the rate of heat transfer due to the reduction of the flow circulation and the increase in the thickness of the thermal boundary layer. Also, the maximum value of the local and average Nusselt numbers occur when the Hartmann number is zero (i.e., the magnetic field is absent).

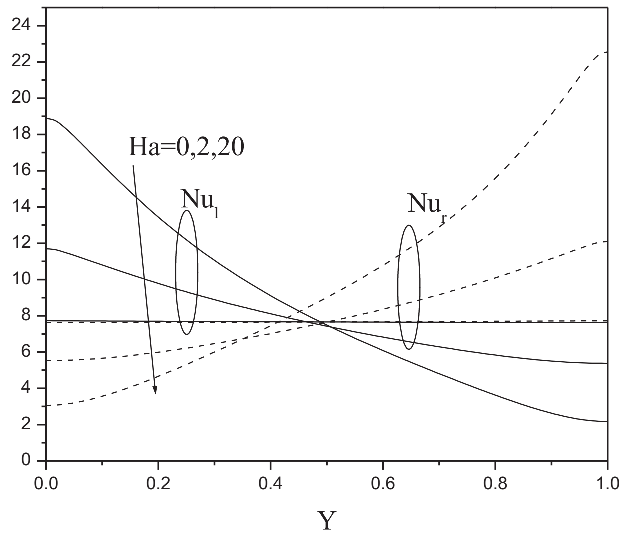

Fig. 5. Variations of Nusselt number at the hot and cold sidewalls for various Hartmann number at $Ra = 100$, $R = 5$, $\varepsilon = 0.005$ and $\varphi = \pi/3$.

Table 2
Values of average Nusselt number at the hot and cold sidewalls for different values of Hartmann number and magnetic field orientation angles at $Ra = 100$, $\varepsilon = 0.005$, $R = 5$.

φ	Ha	$\left(\overline{Nu_l}\right)$	$\left(\overline{Nu_r}\right)$
0	0	8.539859	10.22253
	5	7.675143	7.68891
	10	7.66771	7.668401
$\pi/4$	0	8.539859	10.22253
	5	7.678413	7.693625
	10	7.668669	7.669228
$\pi/2$	0	8.539859	10.22253
	5	7.671343	7.684813
	10	7.667292	7.667886

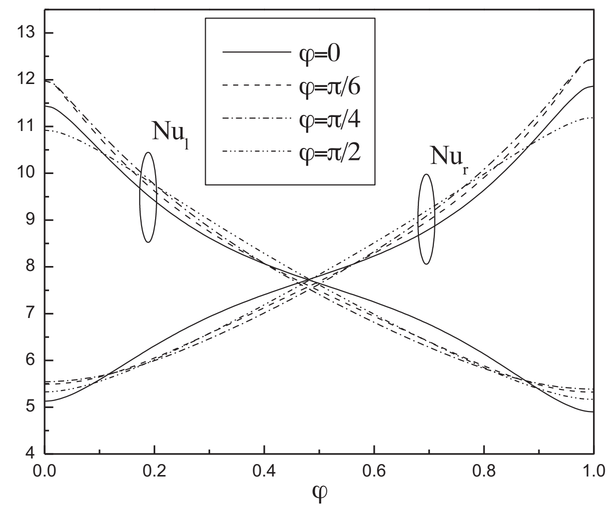

Fig. 6. Variations of Nusselt number at the hot and cold sidewalls for various magnetic field orientation angles at $Ra = 100$, $R = 5$, $\varepsilon = 0.005$ and $Ha = 2$.

Therefore, the high local and average Nusselt numbers correspond to the low Hartmann number and vice versa. This is because the effect of magnetic field becomes negligible when the Hartmann number is zero. Therefore, the flow circulation strength and the temperature gradient increase and for this reason the local and average Nusselt number increase. An opposite behavior can be found when the Hartmann number increases. Fig. 6 illustrates

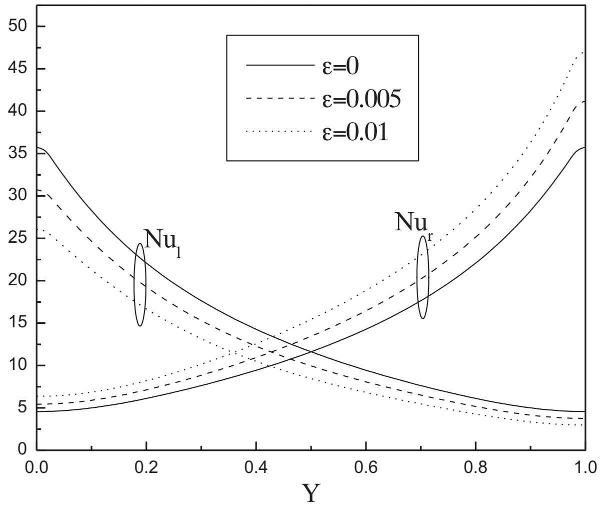

Fig. 7. Variations of Nusselt number at the hot and cold sidewalls for various viscous dissipation parameter at $Ra = 400$, $Ha = 2$, $R = 8$ and $\varphi = \pi/3$.

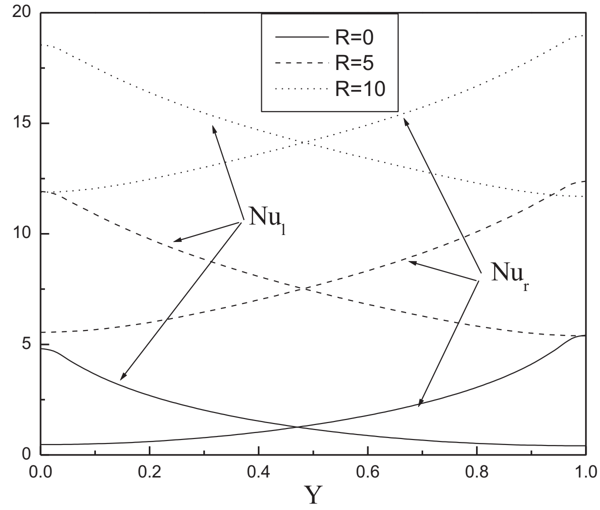

Fig. 8. Variations of Nusselt number at the hot and cold sidewalls for various radiation parameters at $Ra = 100$, $\varphi = \pi/4$, $\varepsilon = 0.005$ and $Ha = 2$.

variations of Nusselt number at the hot left (Nu_l) and cold right (Nu_r) sidewalls at $Ra = 100$, $R = 5$, $\varepsilon = 0.005$ and $Ha = 2$ for various magnetic field orientation angles ($\varphi = 0$, $\pi/6$, $\pi/4$ and $\pi/2$), respectively. It is found, that the change of magnetic field orientation angles does not have a significant influence on the heat transfer rate represented by the Nusselt number. The same results can be observed for the average Nusselt numbers at the hot left $\left(\overline{Nu_l}\right)$ and cold right $\left(\overline{Nu_r}\right)$ sidewalls as shown in Table 2.This is due to the increase of the convection effect by decreasing the Hartmann number. Also, it can be observed that for constant value of the Hartmann number (i.e., $Ha = 2$) the Nusselt number increases slightly when ($\varphi = \pi/6$ and $\pi/4$). Fig. 7 depicts variations of Nusselt number at the hot left (Nu_l) and cold right (Nu_r) sidewalls at $Ra = 400$, $R = 8$, $\varphi = \pi/3$ and $Ha = 2$ for various viscous dissipation parameter ($\varepsilon = 0$, 0.005 and 0.01), respectively. It can be noticed that the Nusselt number at the hot left sidewall (Nu_l) decreases as the viscous dissipation parameter increases. A reverse behavior can be observed for Nusselt number at the cold right sidewall (Nu_r).The same results can be found for the average Nusselt numbers at the hot left $\left(\overline{Nu_l}\right)$ and cold right $\left(\overline{Nu_r}\right)$ sidewalls as shown in Table 3. This is because the higher value of viscous dissipation parameter decelerates the fluid flow adjacent the hot left sidewall and accelerates it adjacent the cold right sidewall. Therefore, the effect of viscous dissipation parameter is to damp the heat transfer rate at the hot left wall, while it improves the heat transfer rate at the cold right wall. Fig. 8 explains variations of Nusselt number at the hot left (Nu_l) and cold right (Nu_r) sidewalls at $Ra = 100$, $\varphi = \pi/4$, $\varepsilon = 0.005$ and $Ha = 2$ for various radiation parameters ($R = 0$, 5 and 10), respectively. It is found that as the radiation parameter increases, the Nusselt number at the hot left (Nu_l) and cold right (Nu_r) sidewalls increase also. The same results can be seen for the average Nusselt numbers at the hot left $\left(\overline{Nu_l}\right)$ and cold right $\left(\overline{Nu_r}\right)$ sidewalls as shown in Table 3.This is due to enhance natural convection flow inside the enclosure when the radiation parameter increases. Furthermore, as the radiation parameter increases an additional heat by radiation is accumulated in the enclosure which causes to increase both local and average Nusselt numbers.

Table 3
Values of average Nusselt number at the hot and cold sidewalls for different values of viscous dissipation parameter and radiation parameter at $Ra = 400$, $Ha = 2$ and $\varphi = \pi/3$.

ε	R	$\left(\overline{Nu_l}\right)$	$\left(\overline{Nu_r}\right)$
0	2	7.955365	7.955449
	8	14.2922	14.29135
	15	22.634	22.63153
0.005	2	6.051797	10.15322
	8	12.32011	16.42766
	15	20.7158	24.64241
0.01	2	4.423874	12.66692
	8	10.5023	18.73966
	15	18.88656	26.75036

5. Conclusions

The following conclusions can be drawn from the results of the present work—

(1) When the Hartmann number increases, the flow field circulation intensity in the enclosure is significantly affected especially at the core of the enclosure. Moreover, the presence of the magnetic force causes to decelerate the flow circulation.
(2) When the Hartmann number is high, the isotherm lines are paralleled with enclosure sidewalls and the conduction is dominant. From the other side, when the Hartmann number is low or zero, the isotherm lines are non-uniform and converge to each other indicating that the convection is dominant.
(3) When the magnetic field orientation angle varies from horizontal direction ($\varphi = 0$) up to the vertical direction ($\varphi = \pi/2$), the flow circulation decelerates. But at ($\varphi = \pi/2$), it begins to accelerate secondly. Moreover, the core of vortices is converted gradually from the vertical position at ($\varphi = 0$) to the horizontal position at ($\varphi = \pi/2$) when the magnetic field orientation angle increases.
(4) No important effect occurs in the isotherms when the magnetic field orientation angle increases.
(5) The stream function values increase, when the magnetic field acts in the horizontal or vertical direction.
(6) It is evident that the radiation parameter has a significant effect on streamlines and isotherms. As the radiation parameter increases the role of conduction mode increases.
(7) The local and average Nusselt numbers at the hot left and cold right sidewalls decrease by increasing the Hartmann number in the enclosure.
(8) No significant change occurs in the local and average Nusselt numbers when the magnetic field orientation angles change.

(9) The role of viscous dissipation parameter is to reduce the local and average Nusselt numbers at the hot left wall, while it improves them at the cold right wall.
(10) The local and average Nusselt numbers at the hot left and cold right sidewalls increase by increasing the radiation parameter in the enclosure.

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