FOLDING SIMPLE CHAOTIC GRAPHS WITH DENSITY VARIATION

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ABSTRACT

Chaotic graph is a graph which carries physical characters with density variation; the density of chaotic graphs can be fixed and unique or different, according to this the representation of the chaoticgraphs by matrices is different to normal chaotic graphs. Firstly, we will discuss the idea of chaotic graphs with density variation showing how to obtain the adjacency and incidence matrix for each different case, and then we will discuss the idea of folding for simple chaotic graphs with density variation showing two types of folding chaotic graphs, the first type of folding is known as topological folding; the second type of folding concerns folding a vertex into another vertex and folding physical characteristics into their selves, in each case we will discuss the decrease or increase the degree of density.

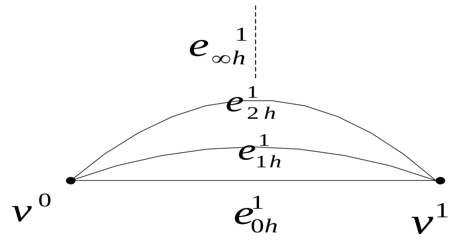
Keywords: Geometric graph,chaotic graphs, density, adjacency and incidence matrix, folding, topological folding.

INTRODUCTION

There are many physical systems whose performance depends not only on the characteristics of the components but also on the relative locations of the elements. An obvious example is an electrical network. If we change a resistor to a capacitor, generally some of the properties (such as an input impedance of the network) also change. This indicates that the performance of a system depends on the characteristics of the components. If, on the other hand, we change the location of one resistor, the input impedance again may change, which shows that the topology of the system is influencing the system's performance. There are systems constructed of only one kind of component so that the system's performance depends only on its topology. An example of such a system is a single-contact switching circuit. Similar situations can be seen in nonphysical systems such as structures of administration. Hence it is important to represent a system so THAT ITS TOPOLOGY CAN BE VISUALIZED CLEARLY.

One simple way of displaying a structure of a system is to draw a diagram consisting of points called "vertices" and line segments called "edges" which connect these vertices so that such vertices and edges indicate components and relationships between these components. Such a diagram is called a "Linear graph" whose name depends on the kind of physical system we deal with. This means that it may be called a network, a net, a circuit, a graph, a diagram, a structure, and so on.

Instead of indicating the physical structure of a system, we frequently indicate its mathematical model or its abstract model by a "Linear graph". Under such a circumstance, a linear graph is referred to as a flow graph, a signal flow graph, a flow chart, a state diagram, an organization diagram, and so forth. The generalization of this graph is the “fuzzy graph” and the most generalization of them is the “chaotic graph”, which applied in many uncertain circuits, resonance, perturbation theory and many other applications. More advanced applications using the more complicated graphs are the chaotic graphs. Generally, a chaotic graph is a geometric graph that carries many other graphs or physical characters, these geometric graphs might have similar properties or different. Considers a simple chaotic graph $G_h(v_{0h}^0 v_{0h}^1)$, this graph consists of v_{0h}^0, v_{0h}^1 vertices, the geometric edge e_{0h}^1 and smooth chaotic edges e_{ih}^1, $i = 1,2,3,...,\infty$. See Fig. (I).



$Fig.(I)$

The matrix representation of a geometric simple graph is simple, see Fig (I); the adjacent and incidence matrices of this chaotic graph are respectively:

$$A(G_h) = \begin{bmatrix} 0_{(012...\infty)h} & 1_{(012...\infty)h} \\ 1_{(012...\infty)h} & 0_{(012...\infty)h} \end{bmatrix}, I(G_h) = \begin{bmatrix} 1_{(012...\infty)h} \\ 1_{(012...\infty)h} \end{bmatrix}.$$

This paper the physical character is presented by density, the density might be constant everywhere or vary from place to another place, so we will consider three cases, the first case is when all chaotic edges have the same physical characters (i.e. fixed density) such that all $e_{0h}^{i}, e_{1h}^{i}, e_{2h}^{i},...e_{\infty h}^{i}$, $i = 1,2,3,....,\infty$, has fixed density; for example the color of a plant leaves is a perfect green, or magnetic field waves have the same velocity, and the second case is when chaotic edges have various densities such that e_{0h}^{1} represent degree 1 of green color, e_{1h}^{1i} represent degree 2 of green color,…and so on. The third case when even each chaotic level has various densities. We will denote the degree of each area on the chaotic graph by d_{pq} , where p denotes levels of chaotic graph, while q denotes different areas on each level of chaotic graph, When the density is constant on all chaotic graph levels, it is easily to find its adjacent and incidence matrices, because the adjacent and incidence matrices are a special case from the general case when the density is different, the density of each chaotic level is unit and the same for other chaotic levels, so this implies that $d_{pq} = d$ and $d$takes any real value, since $p \,\&\, q$ are both constant, so its adjacent and incidence matrices are respectively:

$$A(G_h) = \begin{bmatrix} 0_{(0123\ldots\infty)_d\, h} & 1_{(0123\ldots\infty)_d\, h} \\ 1_{(0123\ldots\infty)_d\, h} & 0_{(0123\ldots\infty)_d\, h} \end{bmatrix}, \quad I(G_h) = \begin{bmatrix} 1_{(0123\ldots\infty)_d\, h} \\ 1_{(0123\ldots\infty)_d\, h} \end{bmatrix}.$$

In the case of unique and constant density for each level of chaotic graph and different to other chaotic levels, $q$is fixed since the density on each chaotic graph level is unique and unit, but it is different to the other chaotic edges, so $p = 0,1,2,\ldots,\infty,\ d_{pq} = d_p,\ p = 0,1,2,3,\ldots,\infty.$ the adjacent and incidence matrices representing this type of chaotic graph are respectively:

$$A(G_h) = \begin{bmatrix} 0_{(012\ldots\infty)_{d_p}\, h} & 1_{(012\ldots\infty)_{d_p}\, h} \\ 1_{(012\ldots\infty)_{d_p}\, h} & 0_{(012\ldots\infty)_{d_p}\, h} \end{bmatrix},$$

$$I(G_h) = \begin{bmatrix} 1_{(012\ldots\infty)_{d_p}\, h} \\ 1_{(012\ldots\infty)_{d_p}\, h} \end{bmatrix}.$$

In the case of different density for each area in each chaotic graph, we denote density by $d_{pq},\ p = 0,1,2,3,\ldots,\infty\ ,\ q = 1,2,3,\ldots,\infty,$ p & q are as before. It’s adjacent and incidence matrices are respectively‘:

$$A(G_h)=\begin{bmatrix}0_{(012\ldots\infty)_{d_{pq}}h} & 1_{(012\ldots\infty)_{d_{pq}}h}\\ 1_{(012\ldots\infty)_{d_{pq}}h} & 0_{(012\ldots\infty)_{d_{pq}}h}\end{bmatrix},\ I(G_h)=\begin{bmatrix}1_{(012\ldots\infty)_{d_{pq}}h}\\ 1_{(012\ldots\infty)_{d_{pq}}h}\end{bmatrix}.$$

1. Folding simple chaotic graphs with density variation

There are three fundamental types of folding of any graph, especially chaotic graphs:

- Topological folding, this type of folding folds the graph into its self, so it reduce the distance of the graph.
- The other type concerns with folding of a vortex to a vortex of geometric graph.
- The last type of folding is folding chaotic edges and geometric edge into each other.

1.1 Topological Folding

Generally topological folding can be defined as:

Let $F:G\to\overline{G}$ be a map between any two graphs G and $\overline{G}$ (not necessarily to be simple) such that if $(u,v)\in G,(f(u),f(v))\in\overline{G}$; then f is called a "topological folding" of G and $\overline{G}$ provided that $d(f(u),\ f(v))\le d(u,v)$.

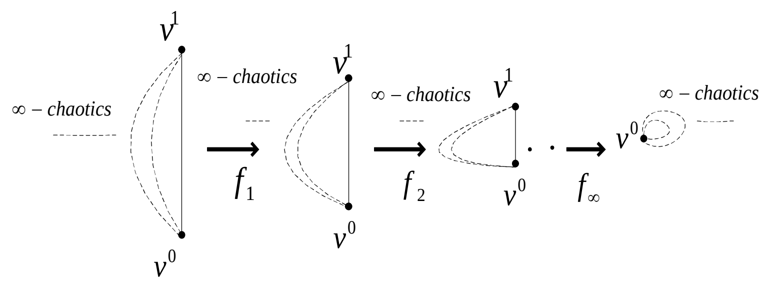
So we can generalise this to chaotic graph as:

If $F:G\to\overline{G}$ be a map between any two chaotic graphs G_h and $\overline{G}_h$ (not necessarily to be simple) such that if

$(v_{ih}^{i}, v_{ih}^{i+1}) \in G_h, i, j = 0,1,2,..,(f(v_{ih}^{i}), f(v_{ih}^{i+1})) \in \overline{G}_h$; then f is called a "topological folding" of G_h and $\overline{G}_h$ provided that $d(f(v_{ih}^{i}), f(v_{ih}^{i+1})) \le d(v_{ih}^{i}, v_{ih}^{i+1})$.

For this type of folding there are two types of folding as follows:

- The first type of folding is restricted on the geometric graph only, but not on the chaotic edges, so the end limit of successive folding sequence is a geometric vertex overlapped by infinitely chaotic edges (i.e. no geometric loop, semi multiple graph) and each chaotic edge keeps its own density as before folding, while the geometric edge changed by folding into one vertex with higher density whatever if the density is constant or varies on the geometric edge, in all cases the density will increase. (See figure (1.1.1)).



$$G_h(Simple) \xrightarrow{f_1} G_h^1(Simple) \xrightarrow{f_2} G_h^2(Simple) \longrightarrow \bullet\bullet \xrightarrow{f_\infty} G_h^\infty(null-chaotic)$$

Figure (1.1.1)

The incidence matrix representing the original chaotic graph G_h is as follows I_h :

$$I(G_h) = \begin{bmatrix} 1_{(012\ldots\infty)_{d_{pq}}h} \\ 1_{(012\ldots\infty)_{d_{pq}}h} \end{bmatrix}.$$

While the incidence matrix representing the chaotic graph G_h^{∞}, induces from the end limit of successive folding sequence of the original simple chaotic graph G_h is I_{∞} as follows ¤ .

The second type of topological folding is to fold both the geometric and the chaotic edges, in this case the end limit of successive folding is one vertex has greater density with no chaotic edges (i.e. null graph), so the density has increased more than in the previous case, so if we want to increase rate of density, it is preferred to choose this kind of folding rather than the previous folding, because the rate of density increases each time we fold a chaotic edge, not only when we fold the geometric edge (i.e. more density, less distance for the graph). See figure (1.1.2).

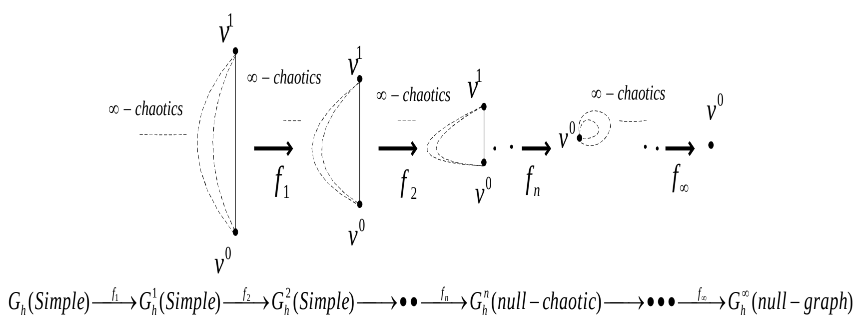


Figure (1.1.2)

incidence matrix representing the original chaotic simple graph G_h is I_h :

$$I(G_h) = \begin{bmatrix} 1_{(012\ldots\infty)_{d_{pq}}h} \\ 1_{(012\ldots\infty)_{d_{pq}}h} \end{bmatrix}.$$

And the incidence matrix representing the chaotic graph G_h^∞ induces from the end limit of successive folding sequence of the original simple chaotic graph G_h is I_∞ as follows ¤ .

Since the final graph resulted from folding is the null graph, according to this the matrix is incidence matrix representing null graph is the zero matrix, but this should be corrected, because the incidence matrix is a matrix that dimension $n \times m$, where n is number of vertices and m is number of edges, and since the null graph has no edges, so it cannot have a matrix that dimension of 1×0 and this is the reason of having ¤ matrix instead of zero matrix.

Theorem (1.1.1)

The end limit of topological folding to the geometric graph and chaotic edges is the null graph with great density and the incidence matrix representing this graph is the ¤ matrix.

Proof:

Consider the chaotic graph with density variation G_h , which consists of the geometric edge e_{0h}^1 and the chaotic edges $e_{ih}^1, i = 1,2,3,\ldots\infty$

Let $f:G_h \longrightarrow \overline{G}_h$ be a topological folding for the geometric edge e^1_{0h} and chaotic edges $e^1_{ih}, i=1,2,3,...\infty$, such that:

$$f_1:G_h \longrightarrow G_{h1},\ f_2:G_{h1} \longrightarrow G_{h2},\ f_3:G_{h2} \longrightarrow G_{h3}, ...,\ \lim_{n\to\infty} f_n(G_{h(n-1)}) = G_{hn}$$

Each folding reduce the length of the graph and increase its density, and each time we repeat the process, the graph is reduced more and the density increases more than before, until we reach the end limit of folding the geometric edge and all chaotic edges and both vertices folded on each other, so we end up with one vertex has greater density than before and this exactly the null graph, see figure (1.1.2).

Also by matrix:

$$A(G_h) = \begin{bmatrix} 0_{(012...\infty)_{d_{pq}}h} & 1_{(012...\infty)_{d_{pq}}h} \\ 1_{(012..\infty)_{d_{pq}}h} & 0_{(012..\infty)_{d_{pq}}h} \end{bmatrix} \xrightarrow{f_1}$$

$$\begin{bmatrix} 0_{(012...\infty)_{d_{pq}}h} & 1_{(012...\infty)_{d_{pq}}h} \\ 1_{(012..\infty)_{d_{pq}}h} & 0_{(012..\infty)_{d_{pq}}h} \end{bmatrix} \xrightarrow{f_2} .....$$

$$\xrightarrow{f_n} \begin{bmatrix} 0_{(012...\infty)_{d_{pq}}h} & 1_{(012...\infty)_{d_{pq}}h} \\ 1_{(012..\infty)_{d_{pq}}h} & 0_{(012..\infty)_{d_{pq}}h} \end{bmatrix} ...... \xrightarrow[n\longrightarrow\infty]{\lim f_n} .....[0_d].$$

And for incidence:

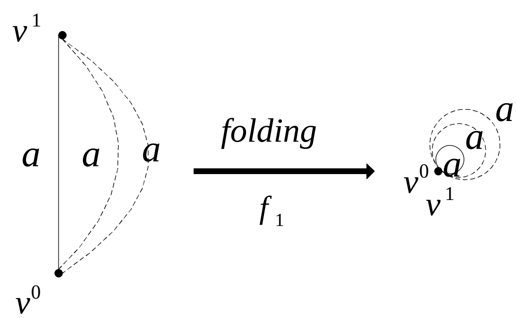
$$I(G_h) = \begin{bmatrix} 1_{(012...\infty)_{d_{pq}}h} \\ 1_{(012...\infty)_{d_{pq}}h} \end{bmatrix} \xrightarrow{f_1} \begin{bmatrix} 1_{(012...\infty)_{d_{pq}}h} \\ 1_{(012...\infty)_{d_{pq}}h} \end{bmatrix} \xrightarrow{f_2} ..... \begin{bmatrix} 1_{(012...\infty)_{d_{pq}}h} \\ 1_{(012...\infty)_{d_{pq}}h} \end{bmatrix} ......$$

$$\xrightarrow{f_n} \begin{bmatrix} 1_{(012...\infty)_{d_{pq}}h} \\ 1_{(012...\infty)_{d_{pq}}h} \end{bmatrix} ...... \xrightarrow[n\longrightarrow\infty]{\lim f_n} ¤ \quad .$$

1.1. The folding of vertex into another vertex

The folding of a vertex into another vertex of a geometric graph is a loop carries density characteristics on each chaotic level; this does not induce folding of all chaotic edges, because it does not reduce the length of the graph it only concerns folding a vertex into another vertex. Moreover, it does not effect on the density character.

When the chaotic graph has fixed density everywhere, the folding does not make any change to the density value, for example supposethat the chaotic graph density is constant everywhere on the graph and equal to ($d = a$), where (a) is a constant, her chaotic graph changes into loops, and each loop has density equal to a, sothe density still fixed does not change over all the loops. See Fig. (1.2.1).



$Fig(1.2.1)$

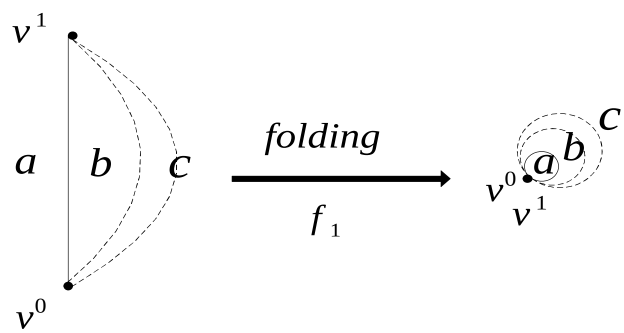
The incidence matric representing the original chaotic graph G_h is I_h :

$$I_h = \begin{bmatrix} 1_{(012)_a h} \\ 1_{(012)_a h} \end{bmatrix}.$$

While the incidence matrix representing chaotic graph G_h^{∞}, induces from folding a vertex into another vertex is I_{∞} as follows:

$$I_h = \begin{bmatrix} 1_{(012)_a h} \\ \end{bmatrix}.$$

In the case of the geometric edge and each chaotic edges have fixed density and varies to the next edge, the same result obtained, the folding does not make any changes on the chaotic graph, see figure (1.2.2)



$Fig(1.2.2)$

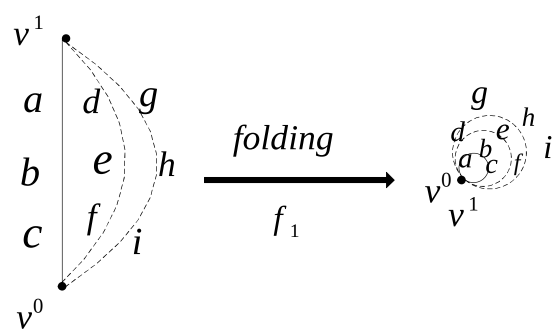
The incidence matrix representing the original chaotic graph G_h and the chaotic graph G_h^{∞} induced from folding a vertex into another vertex are respectively I_h, I_{∞} :

$$I_h = \begin{bmatrix} 1_{(0_a 1_b 2_c) h} \\ 1_{(0_a 1_b 2_c) h} \end{bmatrix}, \; I_h = \begin{bmatrix} 1_{(0_a 1_b 2_c)\ h} \\ \end{bmatrix}.$$

The same result obtained for the case when the density varies over each edge on the graph, for example suppose that each chaotic level have three different densities, so each chaotic level is divided by three densities, the densities are as follows:

$\{(d_{00} = a, d_{01} = b, d_{02} = c), d_{01} = d, d_{11} = e, d_{12} = f),$ (

$d_{20} = g, d_{21} = h, d_{22} = i)\}$, her ($d$ not equal q). See Fig (1.2.3).



Fig(1.2.3)

Result (1.2.1)

The folding of a vertex into another vertex for chaotic graphs having density character does not effect on the density, it only changes the shape of the graph into loops, see figure (1.2.3).

1.2. The folding of all chaotic edges and the geometric edge into their self with fixing all the vertices:

The folding of chaotic edges is different to the folding of a vertex into another vertex, folding chaotic edges with fixed vertices does not effect on the length of the graph, moreover; folding of chaotic edges effects on the shape of graph and on density of each level, discussion is below.

- Firstly; folding chaotic edges having fixed density increases the density, suppose the density is equal to a and constant everywhere on the graph, we deduce that the density increases, since the density is always positive, so the density on the resulting chaotic edge is bigger than the density on both chaotic edges before folding and suppose the resulting density from first folding is equal to $\bar{a}$, an $\bar{\bar{a}}$ represent the density on the resulting chaotic edge from the 2nd folding. We can deduce that ($a < \bar{a} < \bar{\bar{a}}$). See Fig. (1.3.1).

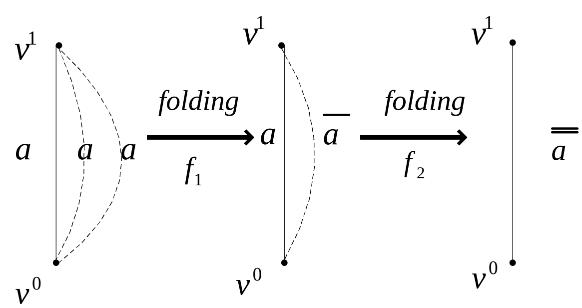
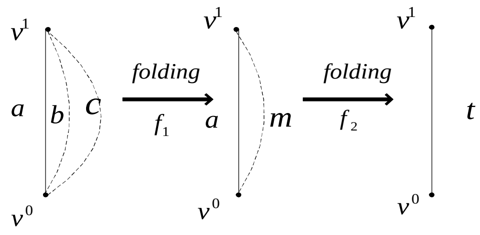


Fig.(1.3.1)

- Folding chaotic edges having fixed density on each chaotic level and vary from level into another level increases density too, the same discussion used in the point above, because when we fold a chaotic edge into another chaotic edge, the total density on every folded chaotic edge is bigger than the density on both chaotic edges before folding. See figure (1.3.2), we can see that ($m > b \& c,\ t > m \& a$), where ($a > b > c$), in this case the density increases too.



Fig(1.3.2)

- Folding chaotic edges having density variation on each chaotic level and vary from level into another level increases density too, the same discussion used in the points above.

See figure (1.3.3) where ($h > c \& e,\ n > d \& g,\ u > a \& h,\ y > b \& n$), ($b < n < y$), ($a > c > e$), and ($b > d > g$).

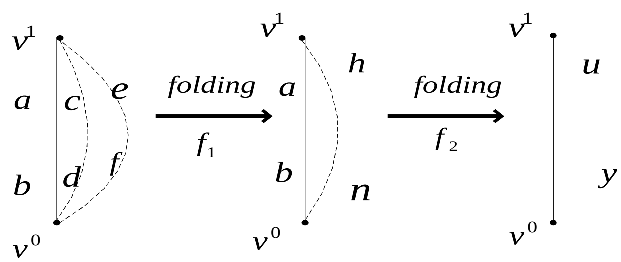


Fig.(1.3.3)

So the incidence matrix for original chaotic graph G_h and the final folded graph are as follows respectively:

$$I(G_h)=\begin{bmatrix}1_{(0_{a_1b_2}1_{c_1d_2}2_{e_1f_2})\ h}\\1_{(0_{a_1b_2}1_{c_1d_2}2_{e_1f_2})\ h}\end{bmatrix},\ I(G_h)=\begin{bmatrix}1_{(0)_{u_1y_2}\ h}\\1_{(0)_{u_1y_2}\ h}\end{bmatrix}.$$

Result (1.3.1)

Folding all chaotic edges on the geometric graph of a simple chaotic graph changes the chaotic graph into a simple graph and it increases their densities as well.

RESULTS

- Folding a simple chaotic graph with density variation topologically increase the density of the graph and reduce the length of the graph too, moreover the end limit of successive topological folding sequence to the whole graph is the null graph and this an effective method of increasing density character.

- Folding a vertex into another vertex of any simple chaotic graph have density character is a loop with multiple of chaotic loops (i.e. results multiple graph) carries density characteristics, consider any chaotic simple graph with density characters G_h, then the end limit of successive folding their vertices on each other is a geometric vertex overlapped on by different chaotic loops and each loop has its own density characters.

- The folding of all density characters into their selves does not necessarily induce folding of the geometric graph. As it is not necessarily mean folding the vertices and it increases the rate of density too, but not effected like the topological folding.

- The limit of topological folding for the whole chaotic graph induces the same result of folding all chaotic levels followed by a topological folding to the geometric edge, which gives the null graph with great density, see figure (1.4.1).

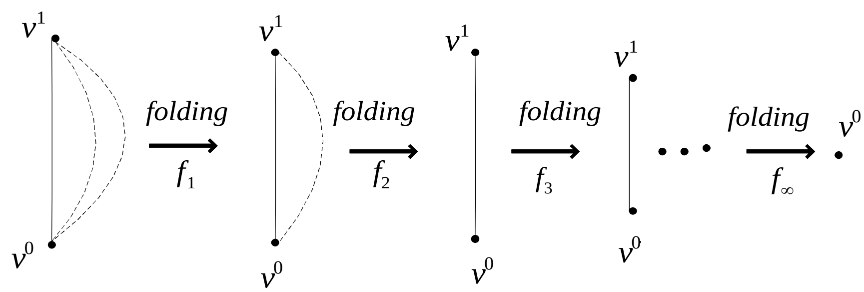


Fig .(1.4.1)

- Folding a vertex into another vertex of simple chaotic graph is a multiple chaotic graph.
- Folding all chaotic edges on the geometric graph changes the chaotic graph into a simple graph.

CONCLUSION

This paper discussed the idea of chaotic graph with density variation; the incidence and adjacency matrix were obtained. The folding of simple chaotic graph with density variation was discussed; three types of folding were studied, topological folding, folding vertices on each other and chaotic edges on the geometric edge. The topological folding increases the density of graph and reduces the length of the graph. The limit of successive folding a vertex into another vertex is a geometric vertex overlapped on by different chaotic loops and each loop has its own density characters, while folding chaotic edges induces a geometric graph with their basic edges, and we deduce that density increases.

As a future study, we can extend the idea of folding to the multiple graphs, which is a more complicated. Another idea can be studied is "unfolding", which is the inverse process of this type of folding.

APPLICATIONS

- Folding a plant leaves, most of plant leaves have variation of green color, according to this chaotic graphs can present the variation of green color of the leaves according to the density character.
- Folding a balloon, the density of the balloon color increase, while the length ofthe balloon decrease**.**
- An effective example of chaotic graph with density variations is the nerve system human body such that the nerve system in the body carries many different signals a very such a different signal represents a 1-chaotic graph, where the signals are different and depends on the mission it carries.
- The perturbation of magnetic field waves and the resonance of the waves are the chaotic graphs, since every single wave of magnetic field has different wavelength and speeds, and the wave length varies on the periodic time.

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