Star Neutrosophic Fuzzy Topological Space

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Abstract

In this paper, we aim to develop a new type of neutrosophic fuzzy set called the star neutrosophic fuzzy set as a generalization to star neutrosophic crisp set defined in by Salama et al.[8], and study some of its properties. Adedd to, we introduce the notion of star neutrosophic fuzzy topological space as a generalization to some topological consepts as star neutrosophic fuzzy closure, and star neutrosophic fuzzy interior. Finally, we extend the concepts of fuzzy topological space, and intuitionistic fuzzy topological space to the case of star neutrosophic fuzzy sets.

Keywords: Neutrosophic logic, Neutrosophic set; Star neutrosophic fuzzy set, Neutrosophic fuzzy topology, Neutrosophic crisp set

1.Introduction

In 1983 the intuitionistic fuzzy set was introduced by Atanassov et al. [1, 2, 3] as a generalization of fuzzy sets in Zadeh's sense [12], where besides the degree of membership of each element there was considered a degree of non-membership. Smarandache [7, 8, 9], defined the notion of neutrosophic set, which is a generalization of Zadeh's fuzzy sets and Atanassov's intuitionistic fuzzy set. Neutrosophic sets have been investigated by Salama et al. [4-8]. This paper is devoted as a generalization of star neutrosophic crisp set called the fuzzy neutrosophic crisp set. The introduced set is a retraction of any triple structured fuzzy set. Where as, the star set deduced from any neutrosophic crisp set is coincide its corresponding star neutrosophic fuzzy set defined in by Salama et al in [8]. Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their classical and fuzzy counterparts, such as a neutrosophic set theory. The readers can referes the following refrences [13-15] for more informations on the application of neutrosophic theory in divers fields.

2. Preliminaries:

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [9-11], Atanassov in [1, 2, 3] and Salama in [4-8]

3. Star Neutrosophic Fuzzy Sets

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As a retraction of neutrosophic fuzzy set, we will introducethe star neutrosophic fuzzy set as a generalization of the star neutrosophiccrisp setwas introduced in [5].

3.1 Definition

For a neutrosophic fuzzy set $A = \langle T_A(x),\ I_A(x), F_A(x)\rangle$of the non-empty fixed set X,the star neutrosophic fuzzy set A^*, is defined to be the following triple structure:

$A^* = \langle\ {T_A}^*,\ {I_A}^*,\ {F_A}^*\ \rangle$;where ${T_A}^* = min(T_A, 1-(max(I_A, F_A))$, ${I_A}^* = min(I_A, 1-(max(T_A, F_A))$and

${F_A}^* = min(F_A, 1-(max(T_A, I_A))$;where ${T_A}^*,\ {I_A}^*,\ {F_A}^* : X \to [0,1\,]$.

3.1 Corollary

For any nonempty set X,the star null and the star universe sets (0_N^* and 1_N^*,respectively) are also star neutrosophic fuzzy set.

3.1 Remark

1) All types of 0_N^* and 0_N are conceded.

2) All types of 1_N^* and1_N are conceded.

3. 2 Definition

The complement of a star neutrosophic fuzzy set $A^*(\text{co}\,A^*, \text{for short})$ may be defined as one of the following two types:

c1:$co\ A^* = \langle\ 1-{T_A}^*, 1-{I_A}^*,\ 1-{F_A}^*\rangle$.

c2:$co\ A^* = \langle {F_A}^*,\ 1-{I_A}^*,\ {T_A}^*\rangle$.

3.3 Definition

The star neutrosophic fuzzy set A^* is said to be a star neutrosophic fuzzy subset of the star neutrosophic fuzzy set B^* ($A^* \subseteq B^*$), and to be defined as one of the following two types:

Type 1:$A^* \subseteq B^* \iff {T_A}^* \le {T_B}^*,\ {I_A}^* \le {I_B}^*$and ${F_A}^* \ge {F_B}^*$.

Type 2:$A^* \subseteq B^* \iff {T_A}^* \le {T_B}^*,\ {I_A}^* \ge {I_B}^*\ and\ {F_A}^* \ge {F_B}^*$.

3.4 Definition

Consider a nonempty setX, and two starneutrosophicfuzzy sets A^*, and B^*; then the star intersection and star union of any two starneutrosophic sets are defined as follows:

1. The star neutrosophic intersection of A^*, $B^*$is defined as :
   Type 1:$A^* \cap B^* = \langle min({T_A}^*, {T_B}^*), min({I_A}^*, {I_B}^*)\ ,\ max({F_A}^*, {F_B}^*)\rangle$.
   Type 2:$A^* \cap B^* = \langle min({T_A}^*, {T_B}^*), max({I_A}^*, {I_B}^*)\ ,\ max({F_A}^*, {F_B}^*)\rangle$.

2. The star neutrosophic union of A^*, $B^*$is defined as :
   Type 1:$A^* \cup B^* = \langle max({T_A}^*, {T_B}^*), max({I_A}^*, {I_B}^*)\ ,\ min({F_A}^*, {F_B}^*)\rangle$.

   Type 2:$A^* \cup B^* = \langle max({T_A}^*, {T_B}^*), min({I_A}^*, {I_B}^*)\ ,\ min({F_A}^*, {F_B}^*)\rangle$.

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3. The star neutrosophic symmetric difference of A^*, B^* is defined as :

$$A^* \ominus B^* = (A^* - B^*) \cup (B^* - A^*), \text{ or equivalently}$$

$$A^* \ominus B^* = (A^* \cup B^*) - (A^* \cap B^*).$$

3.5 Definition

The star neutrosophic difference of any two star neutrosophic fuzzy sets A^* , and $B^*$is to be defined as follows:
Type1: $A^* - B^* = \langle max(0, A^* - B^*), max(0, A^* - B^*)\ , min(1, 1 - (A^* - B^*))\rangle$.
Type2: $A^* - B^* = \langle max(0, A^* - B^*), min(1, 1 - (A^* - B^*))\ , min(1, 1 - (A^* - B^*))\rangle$.

3.1 Example

Let the neutrosophic fuzzy sets $A = < 0.7, 0.4, 0.5 >$ and $B = < 0.8, 0.6, 0.5 >$ then the star neutrosophic fuzzy sets $A^* =< 0.5, 0.3, 0.3 >$, The star neutrosophic complement of A^*, B^*may be equals two types :Type1 and Type 2 $co\ A^* = \langle\ 0.5\ , 0.7,\ \ 0.7\rangle$. or $co\ A^* = \langle\ 0.3\ , 0.7,\ \ 0.5\rangle$, $B^* =< 0.4, 0.2, 0.2 >$, $co\ B^* = \langle\ 0.6\ , 0.8,\ \ 0.8\rangle$. or $co\ B^* = \langle\ 0.2\ , 0.8,\ \ 0.4\rangle$. The star neutrosophic union of A^*, B^*may be equals two types :Type 1:$A^* \cup B^* = \langle 0.5,\ 0.3\ ,\ 0.2\rangle$ and Type 2:$A^* \cup B^* = \langle 0.5,\ 0.2\ ,\ 0.2\rangle$. It is easy to calculate other operations

* **The following figure represents the relationship between Types of Sets**

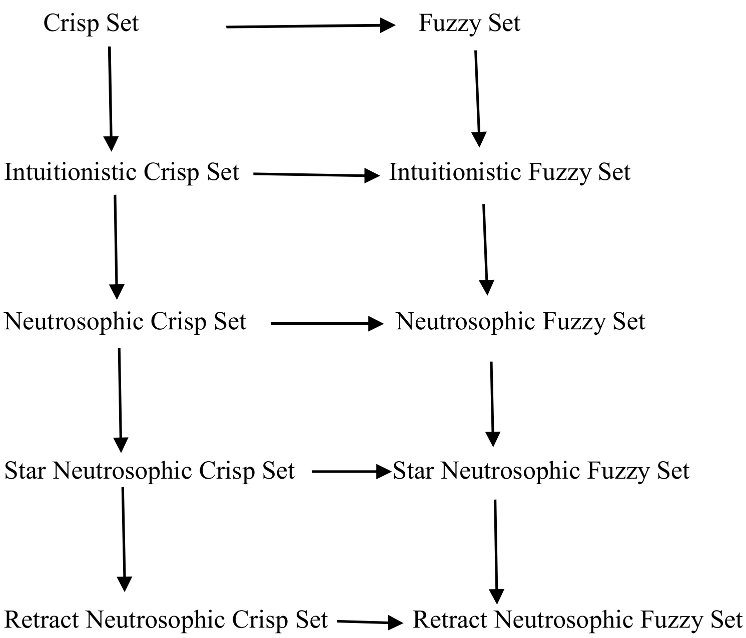

Fig. 3.1 The Relationsship between Types of Sets

4 Star Neutrosophic Topological Spaces

In this section we introduce a new type of Neutrosophic Fuzzy Topological Spaces, based on the star neutrosophic fuzzy sets.

5.1 Definition

Let X be a non empty set, and τ^*be a family of retract neutrosophic subsets of X, then τ^* is said to be a star neutrosophic topology if it satisfies the following axioms:

O1: $\emptyset_N^*\ ,\ X_N^* \in \tau^*$

O2: $A^* \cap B^* \in \tau^*, \quad \forall\, A^*, B^* \in \tau^*$

O3:$\cup_{j \in J} A_j^* \in \tau^*, \quad \forall A_j^* \in \tau^*,\ j\ \in J.$

5.2 Remarks

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Remark1: The pair (X, τ^{*}) is called a star neutrosophic topological space in X.
Remark2: The elements of $\tau^{*}$are called star neutrosophic open sets in X.
Remark3: A star neutrosophic crisp set F^{*} is said to be star neutrosophic closed set if and only if its complement, co F^{*}, is a star neutrosophic open set

5.3 Definition

Let$(X, \tau_1^{*})$and (X, τ_2^{*})be two star neutrosophic topological spaces in X. Then τ_1^{*} is contained in τ_2^{*} (symbolize ($\tau_1^{*} \subseteq \tau_2^{*}$) if $G \in \tau_2^{*}$ for each $G \in \tau_1^{*}$. In this case, we say that $\tau_1^{*}$is coarser than τ_2^{*} and that τ_2^{*} is said to be finer than τ_1^{*}.

5.4 Definition

Let (X, τ^{*})be aretract neutrosophic topological space, and A^{*}be a star neutrosophic set in X, then the star neutrosophic interior ($int(A^{*})$) of $A^{*}$and the star neutrosophic closure of A^{*} ($cl(A^{*})$) are defined by:

(a) $int(A^{*}) = \cup\{G^{*}: G^{*}$is star neutrosophic open set in X and $G^{*} \subseteq A^{*}\}$;
(b) $cl(A^{*}) = \cap\{F^{*}: F^{*}$ is star neutrosophic closed set in X and$A^{*} \subseteq F^{*}\}$;

Hence $int(A^{*})$ is a star neutrosophic open set in X, that is $int(A^{*}) \in \tau^{*}$, and $cl(A^{*})$ is a star neutrosophic closed set in X, that is $cl(A^{*}) \in \tau^{c^{*}}$.

5.5 Definition

Let X be a non empty set, and f^{*} be a family of star neutrosophic subsets of X, then f^{*} is said to be a star neutrosophic co- topology if it satisfies the following axioms:

C1: $\emptyset_N^{*}$, $X_N^{*} \in f^{*}$
C2: $A^{*} \cup B^{*} \in f^{*}, \quad \forall A^{*}, B^{*} \in f^{*}$
C3: $\cap_{j \in J} A_j^{*} \in f^{*}, \quad \forall A_j^{*} \in f^{*},\ j \in J.$

5.6 Proposition

Let (X, τ) be a neutrosophic topology spaces and A^{*}, B^{*}be two neutrosophic sets in X holding the following properties:

a) $A^{*}$is star neutrosophic open if and only if$A^{*} = int(A^{*})$;
b) $int(A^{*}) \subseteq A^{r}$, and $A^{*} \subseteq cl(A^{*})$;
c) $A^{*} \subseteq B^{*} \Rightarrow int(A^{*}) \subseteq int(B^{*})$;
d) $int(A^{*} \cap B^{*}) = int(A^{*}) \cap rint(B^{*})$;
e) $cl(A^{*} \cup B^{*}) = cl(A^{*}) \cup cl(B^{*})$;

* **The following figure represents the relationship between topological structures**

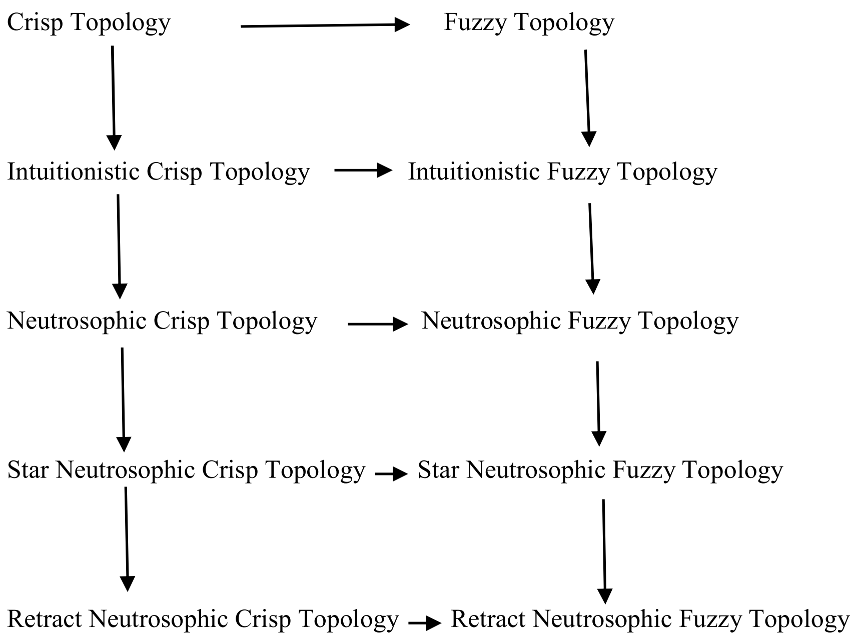

Fig. 4.1 The Relationsship between Types of Topological Structures

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Conclusion

In this paper, a new generalization of the star intuitionistic fuzzy topological space called the star neutrosophic fuzzy topological space was introduced. we've presented the concepts of star neutrosophic fuzzy topological space, star neutrosophic fuzzy cotopological space, star neutrosophic fuzzy closure, and star neutrosophic fuzzy interior.

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