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Discrete Adomian Decomposition Method for Solving Burger's-Huxley Equation

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Abstract

In this paper, the discrete Adomian decomposition method (DADM) is applied to a fully implicit scheme of the generalized Burger's–Huxley equation. The numerical results of two test problems are compared with the exact solutions. The comparisons reveal that the proposed method is very accurate and effective for this kind of problems.

Keywords: Discrete Adomian decomposition method, Finite difference scheme, Generalized Burger's–Huxley equation

1 Introduction

Nonlinear partial differential equations (NLPDEs) are encountered in various fields of science. Generalized Burger's–Huxley equation (GBH) being one of the most famous NLPDE is of high importance for describing the interaction between reaction mechanisms, convection effects, and diffusion transports. Since there exists no general technique for finding analytical solutions of nonlinear diffusion equations so far, numerical solutions of nonlinear differential equations are of great importance in physical problems.

The generalized Burger's–Huxley equation investigated by Satsuma [16] in 1987 is of the form

$$\frac{\partial u}{\partial t}+\alpha u^{\delta}\frac{\partial u}{\partial x}-\frac{\partial^2 u}{\partial x^2}=\beta u(1-u^{\delta})(u^{\delta}-\gamma), \qquad a\le x\le b,\ t\ge 0 \tag{1}$$

Subject to the initial condition

$$u(x,0)=f(x), \qquad a<x<b \tag{2}$$

and the boundary conditions

$$u(a,t)=f_1(t),\ u(b,t)=f_2(t),\ t>0 \tag{3}$$

where α,β,γ and δ are parameters, $\beta\ge 0$, $\delta>0$, $\gamma\in(0,1)$.

The GBH equation has received a great deal of attention by a wide variety of researchers. In Wang et al. [17] the solitary wave solutions of the GBH equation is studied. The non-classical symmetries and the singular modified solutions of the Burger and Burger's–Huxley equation by Estevez [7]. Recently, various powerful methods have been applied to solve the GBH equation such as spectral methods [6,11], Adomian decomposition method [9,10], variational iteration method [1], homotopy analysis method [13], differential transform method [2], differential quadrature method [14], finite difference methods [3,12,15], Exp-function method [8], Haar wavelet method [5] and many others.

The discrete version of Adomian decomposition method was first proposed by Bratsos et al. [4] applied to discrete nonlinear Schrödinger equations. Zhu et al. [18] have developed the DADM to 2D Burgers' difference equations. In this study, the DADM is implemented to nonlinear difference scheme of generalized Burger's–Huxley equation. The obtained results of two test problems are compared with the exact solutions to verify the efficiency and accuracy of the proposed method.

The paper layout is as follow: Section 2 deals with the application of the DADM to nonlinear difference scheme of the GBH equation. In Section 3, we present two test examples of the GBH equation with numerical illustrations. Section 4 concludes the paper.

2 Discrete Adomian decomposition method

To apply the DADM to Eq. (1) with initial condition (2), we formulate the following fully implicit scheme:

$$\begin{aligned}&\frac{1}{\tau}\left(u_i^{n+1}-u_i^{n}\right)+\alpha\left(u_i^{n+1}\right)^{\delta}\frac{1}{2h}\left(u_{i+1}^{n+1}-u_{i-1}^{n+1}\right)-\frac{1}{h^2}\left(u_{i+1}^{n+1}-2u_i^{n+1}+u_{i-1}^{n+1}\right)\\&=\beta\left[(\gamma+1)\left(u_i^{n+1}\right)^{\delta+1}-\gamma u_i^{n+1}-\left(u_i^{n+1}\right)^{2\delta+1}\right]\end{aligned} \tag{4}$$

We denote the discrete approximation of $u(x,t)$ at the grid point $(ih, n\tau)$ by u_i^n $(i=0,1,2,\ldots,N; n=0,1,2,\ldots)$, where $h=1/N$ is the spatial step size and τ represents time increment.
Consider the above scheme written in an operator form as

$$D_\tau^+ u_i^n + \alpha\left(u_i^{n+1}\right)^\delta D_h u_i^{n+1} - D_h^2 u_i^{n+1} = \beta\left[(\gamma+1)\left(u_i^{n+1}\right)^{\delta+1} - \gamma u_i^{n+1} - \left(u_i^{n+1}\right)^{2\delta+1}\right] \tag{5}$$

with the initial condition

$$u_i^0 = f_i\,. \tag{6}$$

The linear operator D_τ^+ denote the forward difference approximation, i.e.,

$$D_\tau^+ u_i^n = \frac{1}{\tau}\left(u_i^{n+1} - u_i^n\right) \tag{7}$$

The first and second order central difference approximations, denoted by D_h and D_h^2, respectively, are given by

$$D_h u_i^{n+1} = \frac{1}{2h}\left(u_{i+1}^{n+1} - u_{i-1}^{n+1}\right), \quad D_h^2 u_i^{n+1} = \frac{1}{h^2}\left(u_{i+1}^{n+1} - 2u_i^{n+1} + u_{i-1}^{n+1}\right). \tag{8}$$

The inverse discrete operator $(D_\tau^+)^{-1}$ is given by [4]

$$(D_\tau^+)^{-1} w^n = \tau \sum_{m=0}^{n-1} w^m\,, \tag{9}$$

Using the above definition, we get

$$(D_\tau^+)^{-1} D_\tau^+ u_i^n = u_i^n - u_i^0. \tag{10}$$

Applying the inverse operator $(D_\tau^+)^{-1}$ to Eq.(5) yields

$$\begin{aligned} u_i^n &= u_i^0 - \alpha (D_\tau^+)^{-1} M\left(u_i^{n+1}\right) + (D_\tau^+)^{-1} D_h^2 u_i^{n+1} + \beta\Big[(\gamma+1)(D_\tau^+)^{-1} N\left(u_i^{n+1}\right) \\ &\quad - \gamma (D_\tau^+)^{-1} u_i^{n+1} - (D_\tau^+)^{-1} P\left(u_i^{n+1}\right)\Big] \end{aligned} \tag{11}$$

Following the DADM, the discrete approximation u_i^n can be decomposed into a sum of components defined by the decomposition series

$$u_i^n = \sum_{k=0}^{\infty} u_{i,k}^n \tag{12}$$

The nonlinear operators $M\left(u_i^{n+1}\right), N\left(u_i^{n+1}\right)$ and $P\left(u_i^{n+1}\right)$ are related to the nonlinear terms and can be decomposed by the infinite series of the so-called Adomian polynomials as follows

$$\begin{aligned} &M\left(u_i^{n+1}\right) = \left(u_i^{n+1}\right)^\delta D_h u_i^{n+1} = \sum_{k=0}^{\infty} A_k\,, \quad N\left(u_i^{n+1}\right) = \left(u_i^{n+1}\right)^{\delta+1} = \sum_{k=0}^{\infty} B_k\,, \\ &P\left(u_i^{n+1}\right) = \left(u_i^{n+1}\right)^{2\delta+1} = \sum_{k=0}^{\infty} C_k \end{aligned} \tag{13}$$

Where A_k, B_k and C_k are the so-called Adomian polynomials that can be generated according to the following algorithms

$$A_k = \frac{1}{k!}\left[\frac{d^k}{d\lambda^k} M\left(\sum_{\ell=0}^{\infty}\lambda^{\ell} u_{i,\ell}^{n+1}\right)\right]_{\lambda=0}, \ k \geq 0$$

$$B_k = \frac{1}{k!}\left[\frac{d^k}{d\lambda^k} N\left(\sum_{\ell=0}^{\infty}\lambda^{\ell} u_{i,\ell}^{n+1}\right)\right]_{\lambda=0}, \ k \geq 0 \tag{14}$$

$$C_k = \frac{1}{k!}\left[\frac{d^k}{d\lambda^k} P\left(\sum_{\ell=0}^{\infty}\lambda^{\ell} u_{i,\ell}^{n+1}\right)\right]_{\lambda=0}, \ k \geq 0.$$

Substituting (12) and (13) into (11) yields

$$\sum_{k=0}^{\infty} u_{i,k}^{n} = f_i - \alpha(D_\tau^+)^{-1}\sum_{k=0}^{\infty} A_k + (D_\tau^+)^{-1} D_h^2 \sum_{k=0}^{\infty} u_{i,k}^{n+1} + \beta\Big[(\gamma+1)(D_\tau^+)^{-1}\sum_{k=0}^{\infty} B_k - \gamma(D_\tau^+)^{-1}\sum_{k=0}^{\infty} u_{i,k}^{n+1} - (D_\tau^+)^{-1}\sum_{k=0}^{\infty} C_k\Big] \tag{15}$$

Each term of series (12) is given by the recurrence relation

$$u_{i,0}^{n} = f_i$$

$$u_{i,k+1}^{n} = -\alpha(D_\tau^+)^{-1}A_k + (D_\tau^+)^{-1}D_h^2 u_{i,k}^{n+1} + \beta\Big[(\gamma+1)(D_\tau^+)^{-1}B_k - \gamma(D_\tau^+)^{-1}u_{i,k}^{n+1} - (D_\tau^+)^{-1}C_k\Big], \ k \geq 0 \tag{16}$$

So, the practical solution for the ℓ-term approximation is

$$\varphi_\ell = \sum_{k=0}^{\ell-1} u_{i,k}^{n}, \ \ell \geq 1, \tag{17}$$

and the exact solution is

$$u_i^n = \lim_{\ell\to\infty}\varphi_\ell = \sum_{k=0}^{\infty} u_{i,k}^{n} \ . \tag{18}$$

The first three components of Adomian polynomials A_k, B_k and C_k read

$$A_0 = \left(u_{i,0}^{n+1}\right)^{\delta} D_h u_{i,0}^{n+1}$$

$$A_1 = \delta\left(u_{i,0}^{n+1}\right)^{\delta-1} u_{i,1}^{n+1} D_h u_{i,0}^{n+1} + \left(u_{i,0}^{n+1}\right)^{\delta} D_h u_{i,1}^{n+1}$$

$$A_2 = \left(u_{i,0}^{n+1}\right)^{\delta} D_h u_{i,2}^{n+1} + \delta\left(u_{i,0}^{n+1}\right)^{\delta-1} u_{i,1}^{n+1} D_h u_{i,1}^{n+1} + \delta\left(u_{i,0}^{n+1}\right)^{\delta-1} D_h u_{i,0}^{n+1} u_{i,2}^{n+1} + \frac{1}{2}\delta(\delta-1)\left(u_{i,0}^{n+1}\right)^{\delta-2} u_{i,1}^{n+1} D_h u_{i,0}^{n+1}$$

$$B_0 = \left(u_{i,0}^{n+1}\right)^{\delta+1}$$

$$B_1 = (\delta+1)\left(u_{i,0}^{n+1}\right)^{\delta} u_{i,1}^{n+1}$$

$$B_2 = (\delta+1)\left(u_{i,0}^{n+1}\right)^{\delta} u_{i,2}^{n+1} + \frac{1}{2}\delta(\delta+1)\left(u_{i,1}^{n+1}\right)^{2}\left(u_{i,0}^{n+1}\right)^{\delta-1}$$

$$C_0 = \left(u_{i,0}^{n+1}\right)^{2\delta+1}$$

$$C_1 = (2\delta+1)\left(u_{i,0}^{n+1}\right)^{2\delta} u_{i,1}^{n+1}$$

$$C_2 = (2\delta+1)\left(u_{i,0}^{n+1}\right)^{2\delta} u_{i,2}^{n+1} + \delta(2\delta+1)\left(u_{i,1}^{n+1}\right)^{2}\left(u_{i,0}^{n+1}\right)^{2\delta-1}$$

3 Numerical experiments

In this section, we will give two test examples of the GBH equation to verify the efficiency and measure the accuracy of the DADM solutions in comparison with the exact solution and we will use Maple 16 software to obtain the numerical results.

Consider the GBH equation [9]:

$$\frac{\partial u}{\partial t} + \alpha u^{\delta}\frac{\partial u}{\partial x} - \frac{\partial^2 u}{\partial x^2} = \beta u(1-u^{\delta})(u^{\delta}-\gamma), \qquad 0 \le x \le 1,\ t \ge 0 \tag{19}$$

with the initial condition

$$u(x,0) = \left[\frac{\gamma}{2} + \frac{\gamma}{2}\tanh(\sigma\gamma x)\right]^{1/\delta}, \tag{20}$$

and the boundary conditions

$$u(0,t) = \left[\frac{\gamma}{2} + \frac{\gamma}{2}\tanh\left\{\sigma\gamma\left(\left\{\frac{-\gamma\alpha}{1+\delta} + \frac{(1+\delta-\gamma)(\rho-\alpha)}{2(1+\delta)}\right\}t\right)\right\}\right]^{1/\delta}, \tag{21}$$

$$u(1,t) = \left[\frac{\gamma}{2} + \frac{\gamma}{2}\tanh\left\{\sigma\gamma\left(1-\left\{\frac{\gamma\alpha}{1+\delta} - \frac{(1+\delta-\gamma)(\rho-\alpha)}{2(1+\delta)}\right\}t\right)\right\}\right]^{1/\delta}, \tag{22}$$

where $\sigma = \delta(\rho-\alpha)/4(1+\delta)$ and $\rho = \sqrt{\alpha^2 + 4\beta(1+\delta)}$
and the exact solution is given by

$$u(x,t) = \left[\frac{\gamma}{2} + \frac{\gamma}{2}\tanh\left\{\sigma\gamma\left(x-\left\{\frac{\gamma\alpha}{1+\delta} - \frac{(1+\delta-\gamma)(\rho-\alpha)}{2(1+\delta)}\right\}t\right)\right\}\right]^{1/\delta}. \tag{23}$$

Example 1. We consider Eq. (19) with $\gamma = 0.001, \alpha = \beta = 1, \tau = 10^{-4}$ and $h = 0.1$ The absolute errors for various values of δ, t and x are given in Table 1. We plot in Fig. 1 the absolute error for the values $\gamma = 0.001,\ \alpha = \beta = \delta = 1,\ \tau = 10^{-4}$ and $h = 0.1$.

Table 1: The absolute errors using φ_3 for various values of δ, t and x with $\gamma = 0.001, \alpha = \beta = 1$

t	x	$\delta = 1$	$\delta = 2$	$\delta = 4$	$\delta = 8$
	0.1	1.87406E–08	8.74970E–07	6.13597E–06	1.66019E–05
0.05	0.5	1.87406E–08	8.74894E–07	6.13465E–06	1.65954E–05
	0.9	1.87406E–08	8.74818E–07	6.13333E–06	1.65888E–05
	0.1	3.74812E–08	1.74995E–06	1.22721E–05	3.32047E–05
0.1	0.5	3.74812E–08	1.74980E–06	1.22695E–05	3.31916E–05
	0.9	3.74812E–08	1.74964E–06	1.22668E–05	3.31784E–05
	0.1	3.74812E–07	1.75012E–05	1.22751E–04	3.32195E–04
1	0.5	3.74812E–07	1.74997E–05	1.22724E–04	3.32063E–04
	0.9	3.74812E–07	1.74982E–05	1.22698E–04	3.31932E–04

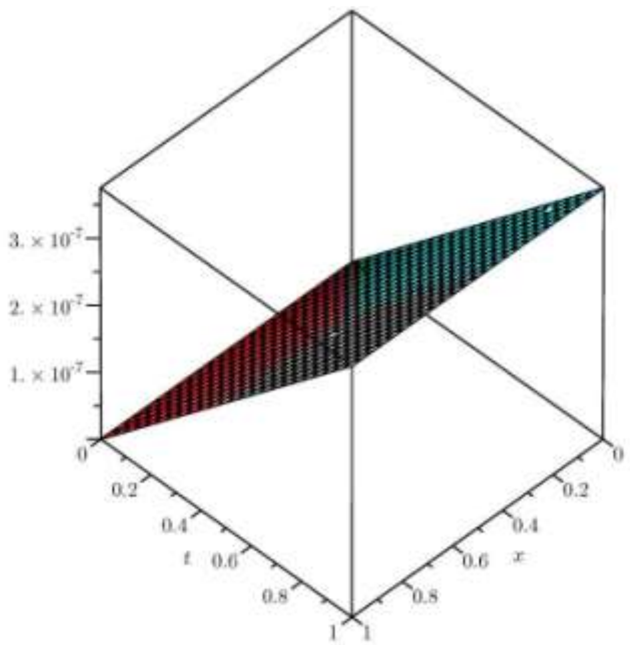

Fig. 1 The absolute error of φ_3 with $\gamma = 0.001,\ \alpha = \beta = \delta = 1,\ \tau = 10^{-4}$ and $h = 0.1$

Example 2. We consider Eq. (19) with $\gamma = \alpha = 0.1, \beta = 0.001, \tau = 10^{-4}$ and $h = 0.1$. The absolute errors for various values of δ, t and x are given in Table 2. We plot the absolute error in Fig. 2 for the values $\gamma = \alpha = 0.1$, $\beta = 0.001, \delta = 1, \tau = 10^{-4}$ and $h = 0.1$.

Table 2: The absolute errors using φ_3 for various values of δ, t and x with $\gamma = \alpha = 0.1, \beta = 0.001$

t	x	$\delta = 1$	$\delta = 2$	$\delta = 4$	$\delta = 8$
	0.1	1.36075E–07	6.45516E–07	1.46880E–06	2.31838E–06
0.05	0.5	1.36075E–07	6.45412E–07	1.46816E–06	2.31632E–06
	0.9	1.36075E–07	6.45308E–07	1.46751E–06	2.31425E–06
	0.1	2.72151E–07	1.29103E–06	2.93761E–06	4.63678E–06
0.1	0.5	2.72151E–07	1.29082E–06	2.93632E–06	4.63265E–06
	0.9	2.72151E–07	1.29062E–06	2.93503E–06	4.62851E–06
	0.1	2.72151E–06	1.29105E–05	2.93769E–05	4.63706E–05
1	0.5	2.72151E–06	1.29084E–05	2.93640E–05	4.63293E–05
	0.9	2.72151E–06	1.29063E–05	2.93511E–05	4.62879E–05

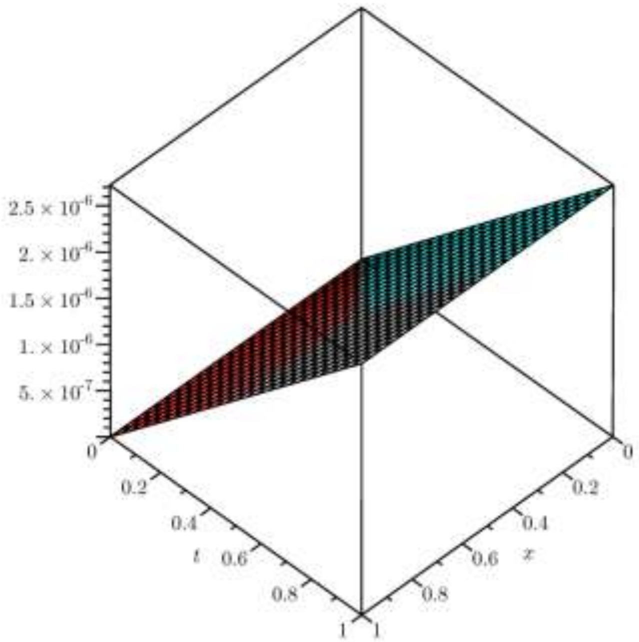

Fig. 2 The absolute error of φ_3 with $\gamma = \alpha = 0.1$, $\beta = 0.001, \delta = 1, \tau = 10^{-4}$ and $h = 0.1$

4 Conclusions

In this paper, the DADM is proposed for the generalized Burger's–Huxley equation. The obtained results, reported in Tables 1 and 2, reveal that the proposed method is very reliable, accurate and simple tool to solve the GBH equation and can be applied to other nonlinear problems.

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