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Bayesian Estimation for the Generalized Logistic Distribution Type-II Censored Accelerated Life Testing

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Abstract

This paper develops Bayesian analysis for Constant Stress Accelerated Life Test (CSALT) under Type-II censoring scheme. Failure times are assumed to distribute as the three-parameter Generalized Logistic (*GL*) distribution. The inverse power law model is used to represent the relationship between the stress and the scale parameter of a test unit. Bayes estimates are obtained using Markov Chain Monte Carlo (*MCMC*) simulation algorithm based on Gibbs sampling. Then, confidence intervals, and predicted values of the scale parameter and the reliability function under usual conditions are obtained. Numerical illustration and an illustrative example are addressed for illustrating the theoretical results. WinBUGS software package is used for implementing Markov Chain Monte Carlo (*MCMC*) simulation and Gibbs sampling.

Keywords: Accelerated Life Test; Constant Stress; Type-II Censoring; Bayesian Method; Generalized Logistic Distribution; Markov Chain Monte Carlo (*MCMC*); Gibbs Samples

1 Introduction

Life data analysis involves analyzing lifetime data of a device, system, or component obtained under normal operating conditions in order to quantify their life characteristics. In many situations, and for many reasons, such data is very difficult, if not impossible, to obtain. A common way of tackling this problem is to expose the device to sufficient over stress (e.g., temperature, voltage, humidity, and so on), or forcing them to fail more quickly than they would under normal use conditions to accelerate their failures. Therefore, the failure data are analyzed in terms of a suitable physical statistical model to obtain desired information on a device under normal use conditions. This approach is called Accelerated Life Testing (ALT). The most common ALT loading is constant stress, step stress, and progressive stress (for more details, see Nelson (1990)). In CSALT, the stress is kept at a constant level of stress throughout the life of the test, i.e., each unit is run at a constant high stress level until the occurrence of failure or the observation is censored. Practically, most devices such as lamps, semiconductors and microelectronics are run at a constant stress.

Bayesian inference procedure treats unknown parameters as random variables. Through Bayesian analysis, our information, our believe, or our knowledge about the unknown parameters can be incorporating in a measurable form as a prior distribution. There is a great amount of literature on applying Bayesian approach to CSALT. Prior information is concerned with engineering facts and material properties by many authors, for example, Pathak *et al.* (1987) discussed Bayes estimation of the constant hazard rate. They assumed that the effect of acceleration was to scale up the hazard rate, and the hazard rate had the natural conjugate prior with a known mean and unknown variance. Achcar (1994) used Bayesian approach and assumed non-informative priors for the parameters of the exponential, Weibull, Birbaum-Saunders, and Inverse Gaussian distributions. He You (1996) used Bayesian approach to estimate the parameters of the exponential distribution under different priors and different censoring schemes. Aly (1997) considered natural conjugate priors for estimating the parameters of Pareto distribution. Zhong and Meeker (2007) estimated the parameters of Weibull distribution assuming log-normal prior density. Liu and Tang (2009) constructed a sequential CSALT scheme and its Bayesian inference using Weibull distribution and Arrhenius relationship. They derived closed form expression for estimating the smallest extreme value location parameter at each stress level. Unfortunately, all this work was

based on getting posterior distributions of the unknown parameters using ordinary samples. On the other hand, this paper uses Gibbs sampling to derive posterior distributions.

The GL distribution is an important and useful family in many practical situations. It includes a number of other distributions for different choices of the concerned model parameters. For example, standard Logistic, four-parameters extended GL, four-parameters extended GL type-I, two parameter GL, type-I GL, Generalized Log-logistic, standard Log-logistic, Logistic Exponential, Generalized Burr, Burr III, and Burr XII distributions. There are some who argue that the generalized logistic distribution is inappropriate for modeling lifetime data because the left-hand limit of the distribution extends to negative infinity. This could conceivably result in modeling negative times-to-failure. However, provided that the distribution in question has a relatively high scale parameter α and a relatively small scale parameter γ , the issue of negative failure times should not present itself as a problem. Nevertheless, the generalized logistic distribution has wide applications in population model been shown to be useful for modeling the log odds of moderately rare events, for graduating life data, to modeling binary response data, for the comparison of log odd of an event, in hydrological risk analysis, in environmental pollution studies, to model the data with a unimodal density, geological issues, and to analyze survival data (for more details, see Mathai and Provost (2004), Alkawasbeh and Raqab (2009), and Shabri et al. (2011)).

This paper is organized as follows: The underlying distribution and the test method are described in Section 2. Section 3 introduces Bayesian estimators of model parameters. Finally, simulation studies as well as an illustrative real Life example are addressed for illustrating the theoretical results.

2 Constant stress ALT model

The probability density function (pdf) of a three-parameter generalized logistic distribution introduced by Molenberghs and Verbeke (2011), is given by

$$f(x) = \alpha\gamma e^{\alpha x} \left(1 + \frac{\gamma}{\theta} e^{\alpha x}\right)^{-(\theta+1)}, \quad -\infty < x < \infty, \quad \alpha, \gamma, \theta > 0. \quad (1)$$

We assume the following assumptions for the CSALT procedure:

- A total of N units are divided into n_1, n_2, \dots, n_k units where $\sum_{j=1}^k n_j = N$.
- There are k levels of high stress $V_j, j = 1, \dots, k$ in the experiment, and V_u is the stress under usual conditions, where $V_u < V_1 < \dots < V_k$.

- Each n_j units in the experiment are run at a pre-specified constant stress $V_j, j = 1, \dots, k$.
- It is assumed that the stress affected only on the scale parameter of the underlying distribution.
- Assuming type-II censoring scheme, the failure times $x_{ij}, i = 1, \dots, r_j$ and $j = 1, \dots, k$ at stress levels $V_j, j = 1, \dots, k$ are the 3-parameter generalized logistic distribution with probability density function

$$f(x_{ij}, \alpha_j, \gamma, \theta) = \alpha_j \gamma e^{\alpha_j x_{ij}} \left(1 + \frac{\gamma}{\theta} e^{\alpha_j x_{ij}}\right)^{-(\theta+1)}, -\infty < x_{ij} < \infty,$$

$$\alpha_j, \gamma, \theta > 0, i = 1, \dots, r_j, j = 1, \dots, k. \tag{2}$$

- The scale parameter $\alpha_j, j = 1, \dots, k$, of the underlying lifetime distribution (2) is assumed to have an inverse power law function on stress levels, i.e.,

$$\alpha_j = CS_j^P, C, P > 0,$$

where $S_j = \frac{V^*}{V_j}, V^* = \prod_{j=1}^k V_j^{b_j}, b_j = \frac{r_j}{\sum_{j=1}^k r_j}, C$ is the constant of proportionality, and P is the power of the applied stress.

3 Bayesian Estimation

Considering the assumptions in Section (2), and assuming that the experiment is terminated at a specified number of failure units $r_j (r_j < n_j), j = 1, \dots, k$, the likelihood function will be in the following form

$$L = \prod_{j=1}^k \left\{ \frac{n_j!}{(n_j - r_j)!} \left[\prod_{i=1}^{r_j} CS_j^P \gamma e^{CS_j^P x_{ij}} \left(1 + \frac{\gamma}{\theta} e^{CS_j^P x_{ij}}\right)^{-(\theta+1)} \right] \left(1 + \frac{\gamma}{\theta} e^{CS_j^P x_{r_j j}}\right)^{-\theta(n_j - r_j)} \right\}, \tag{3}$$

Eq.(3) can be re-written as follows,

$$L(\underline{\mu}/\underline{x}) \propto C^\xi \gamma^\xi e^{C \sum_{j=1}^k \sum_{i=1}^{r_j} S_j^P x_{ij}} \left[\prod_{j=1}^k \prod_{i=1}^{r_j} \eta_{ij} \right] \left[\prod_{j=1}^k \eta_{r_j j} \right], \tag{4}$$

where $\underline{\mu} = (c, p, \gamma, \theta), \underline{x} = (x_{ij}, i = 1, \dots, r_j, j = 1, \dots, k), \xi = \sum_{j=1}^k r_j, \eta_{ij} = \left(1 + \frac{\gamma}{\theta} e^{CS_j^P x_{ij}}\right)^{-(\theta+1)},$ and $\eta_{r_j j} = \left(1 + \frac{\gamma}{\theta} e^{CS_j^P x_{r_j j}}\right)^{-\theta(n_j - r_j)}.$

Following, we present inference for the unknown parameter C when the other parameters (P, γ, θ) are known as well as inference for P when the other parameters (C, γ, θ) are known. In addition, inference for C, P when the other parameters (γ, θ) are known.

Case of unknown C

Under the assumption that the parameters P , γ , and θ are known. We assume the prior for C is gamma (λ_1, λ_2) distribution as

$$\pi(C) \propto C^{\lambda_1-1} e^{-\lambda_2 C}, \quad C > 0, \quad \lambda_1, \lambda_2 > 0. \tag{5}$$

Then posterior density function of C is given by

$$\pi(C/x) \propto C^{\xi+\lambda_1-1} e^{-(\lambda_2 - \sum_{j=1}^k \sum_{i=1}^{r_j} S_j^P x_{ij})C} \left[\prod_{j=1}^k \prod_{i=1}^{r_j} \eta_{ij} \right] \left[\prod_{j=1}^k \eta_{r_j j} \right], \quad C > 0, \quad \lambda_1, \lambda_2 > 0, \tag{6}$$

Bayesian estimate of the parameter C , the prediction of the scale parameter α and the reliability function $R(x_0)$ at the lifetime x_0 under the design stress V_u can be obtained based on Eq.(6).

Case of unknown P

Under considering that the parameters C , γ , and θ are known, and the gamma $G(\lambda_3, \lambda_4)$ is the prior density of P , that is

$$\pi(P) \propto P^{\lambda_3-1} e^{-\lambda_4 P}, \quad P > 0, \quad \lambda_3, \lambda_4 > 0. \tag{7}$$

The posterior density function of P given x under the likelihood function (4) is obtained as follows:

$$\pi(P/x) \propto P^{\lambda_3-1} e^{C \sum_{j=1}^k \sum_{i=1}^{r_j} S_j^P x_{ij} - \lambda_4 P} \left[\prod_{j=1}^k \prod_{i=1}^{r_j} \eta_{ij} \right] \left[\prod_{j=1}^k \eta_{r_j j} \right], \quad P > 0, \quad \lambda_3, \lambda_4 > 0. \tag{8}$$

Also, Bayesian estimate of P , prediction of the scale parameter α and the reliability function at the lifetime x_0 under the design stress V_u can be obtained based on Eq.(8).

Case of unknown C and P

In this case, we assume the prior density for C is gamma (λ_1, λ_2) distribution and the conditional distribution of P given C is gamma $(\lambda_3, \lambda_4 C)$, then the prior density for C and P is given by

$$\pi(C, P) \propto C^{\lambda_1+\lambda_3} P^{\lambda_3} e^{-C(\lambda_2+\lambda_4 P)}, \quad C > 0, \quad P > 0, \quad \lambda_1, \lambda_2, \lambda_3, \lambda_4 > 0. \tag{9}$$

From the likelihood function (4), we have

$$L(C, P/x) \propto C^\xi e^{C \sum_{j=1}^k \sum_{i=1}^{r_j} S_j^P x_{ij}} \left[\prod_{j=1}^k \prod_{i=1}^{r_j} \eta_{ij} \right] \left[\prod_{j=1}^k \eta_{r_j j} \right]. \tag{10}$$

Therefore, the posterior density of C and P given x based on Eq.(9) and Eq.(10) is given by

$$\pi(C, P/x) \propto C^{\lambda_1 + \lambda_3 + \xi} P^{\lambda_3} e^{-C(\lambda_2 + \lambda_4 P - \sum_{j=1}^k \sum_{i=1}^{r_j} S_j^P x_{ij})} \left[\prod_{j=1}^k \prod_{i=1}^{r_j} \eta_{ij} \right] \left[\prod_{j=1}^k \eta_{r_j j} \right],$$

$$C > 0, P > 0, \lambda_1, \lambda_2, \lambda_3, \lambda_4 > 0. \quad (11)$$

The marginal posterior density function of C , the marginal posterior density function of P , Bayesian estimate of the scale parameter α and the reliability function at the lifetime x_0 under the design stress V_u can be obtained based on Eq.(11). Similarly, numerical simulation to evaluate the value of $\tilde{\alpha}_u$ and $\tilde{R}_u(x_0)$ is used. To obtain the normalizing constants of the posterior functions and the marginal posterior densities $\pi(C/x)$, and $\pi(P/x)$ complicated integrations are often analytically intractable and sometimes even a numerical integration cannot be directly obtained. In these cases, Markov Chain Monte Carlo (MCMC) simulation is the easiest way to get reliable results [Gelman, *et al.* (2003)]. A MCMC algorithm that is particularly useful in high dimensional problems is the alternating conditional sampling called Gibbs sampling. Through the MCMC approach, a sample of the posterior distribution can be obtained. From the sample, approximations of moments and an approximation of the posterior distribution may be derived using Gibbs sampling. Gibbs sampling is used to draw a random sample of the parameters C and P from their own marginal posterior distribution $\pi(C/x)$, and $\pi(P/x)$, respectively, and then estimate the expected value of the parameters C and P using the sample mean.

Each iteration of Gibbs sampling cycles through the unknown parameters, by drawing a sample of one parameter conditioning on the latest values of all other parameters. When the number of iterations is large enough, the samples drawn on one parameter can be regarded as simulated observations from its marginal posterior distribution. Functions of the model parameters, such as α_u at the normal use condition, can also be conveniently sampled. In this paper, we use WinBUGS software, a specialized software package for implementing MCMC simulation and Gibbs sampling.

4 Numerical Illustration

4.1 Simulation Study

The following steps are used: Three accelerated stress levels $V_1 = 1, V_2 = 2, V_3 = 3$ and usual stress $V_u = 0.5$ are considered. Assume that the experiment

is terminated at a specified number of failure units r_j , $j = 1, 2, 3$, where $n_1 = n_2 = n_3 = 15, r_1 = 9, r_2 = 8, r_3 = 7$. Accelerated life data from the GL distribution are generated using MathCad software. The K-S test (Kolmogorov-Smirnov test) is used for assessing that the data set follows the GL distribution and we concluded that the data set follows it. The CSALT generated data are used for getting posterior estimation of the parameters by applying Bayesian approach. The parameters of interest are estimated as well as the scale parameter and the reliability function under usual conditions are predicted.

The case of unknown C

We start with three Markov chains with different initial values ($C = 1.0, C = 0.9, C = 0.7$), and assume that the values of the three parameters (P, γ, θ) are known. We set ($P = 0.25, \gamma = 0.05, \theta = 0.15$) and assume the prior of the parameter C is gamma distribution with parameters $\lambda_1 = 3.5$ and $\lambda_2 = 3.75$. We run 30000 iterations for each Markov chain. To check convergence, Gelman-Rubin convergence statistic, R , is introduced. R is defined as the ratio of the width of the central 80% interval of the pooled chains to the average width of the 80% intervals within the individual chains. When a WinBUGS simulation converges, R should be one, or close to one [Luo, 2004]. Figure 1 shows the Gelman-Rubin convergence statistic of C and Figure 2 shows posterior density of C (see Appendix II). One can see that Gelman-Rubin statistic is believed to be convergent. A simple summary can be generated showing posterior mean, median and standard deviation with a 95% posterior credible interval. The summary of the sampling results with respect to the unknown parameter C is displayed in Table 1 (see Appendix I). The accuracy of the posterior estimate is calculated in terms of Monte Carlo standard error (MC error) of the mean according to [Spiegelhalter *et al.* (2003)]. The simulation should be run until the MC error for each node is less than 5% of the sample standard deviation. This rule has been achieved in this paper. Table 1 shows that the estimated value of the scale parameter under usual conditions is 1.272, and the reliability decreases when the mission time x_0 increases.

The case of unknown P

In this case, we assume the values of the three parameters (C, γ, θ) are known and apply Bayesian method to determine the posterior density function of P . We set ($C = 1.0, \gamma = 0.05, \theta = 0.15$) and the parameters of the prior density of the unknown parameter P are $\lambda_1 = 3.5$ and $\lambda_2 = 3.75$. Three chains with different initials ($P = 0.25, P = 0.15, P = 0.3$) are run simultaneously in one simulation. Each chain continues for 40000 iterations. Gelman-Rubin conver-

gence statistic of P shows that the simulation is believed to have converged as shown from Figure 3. The summary for the sampling results concerning the unknown parameter P is displayed in Table 2, and shows that the estimated value of the scale parameter under usual conditions is 4.091, and the reliability decreases when the mission time x_0 increases, (see Appendix I). The posterior density of P is shown in Figure 4, (see Appendix II).

The case of unknown C and P

To apply Bayesian approach for determining the posterior density function of C and P , we assume the following points:

- The values of the two parameters (γ, θ) are $(\gamma = 0.05, \theta = 0.15)$.
- The prior of the parameter C is gamma distribution with parameters $\lambda_1 = 3.5$ and $\lambda_2 = 3.75$.
- The conditional distribution of P given C is gamma $(\lambda_3, \lambda_4 C)$ with parameters $\lambda_3 = 0.25$ and $\lambda_4 = 0.25$.
- Three chains with different initials $[(C = 0.9, P = 0.25), (C = 1.0, P = 0.3), (C = 0.7, P = 0.15)]$ are run simultaneously in one simulation.
- Each chain continues 40000 iterations.

Sampling results assume that unknown parameters C and P are displayed in Table 3 and shows the estimated value of the scale parameter under usual conditions and the reliability decreases when the mission time x_0 increases, (see Appendix I). From Figure 5, we note that the simulation is believed to be convergent. Figure 6 shows the marginal posterior density of both C and P (see Appendix II).

4.2 An Illustrative Real Life Example

This Section presents getting Bayesian estimates of the unknown parameters using a real life example based on accelerated life data given by Nelson (1970). This data represents the times to breakdown of an insulating fluid subjected to elevated voltage stress levels. For convenience, we consider only four accelerated voltage stress levels 32, 34, 36, and 38 kilovolts (KV's) and the experiment is terminated at a specified number of failure units r_j , $j = 1, \dots, 4$. The usual conditions in the experiment is considered 28 kilovolts (KV's). The failure times (in minutes) under the various stress levels are given in Table (4), (see Appendix I). Nelson's original data correspond to seven different stress levels, but some of these contains very few failures times and are therefore omitted

here. We use the K-S (Kolmogorov-Smirnov) test for assessing that the data set follows the GL distribution. Also, we got that the data set follows the GL distribution.

The case of unknown C

In this case, we assume the values of the three parameters (P, γ, θ) are known and we set $(P = 0.25, \gamma = 0.5, \theta = 0.25)$. The conjugate prior to the parameter C is assumed to be gamma distribution with the parameters $\lambda_1 = 3.5$ and $\lambda_2 = 3.75$. Five chains with different initials $(C = 0.9, C = 0.7, C = 0.5, C = 0.35, C = 0.25)$ are run simultaneously in one simulation. Each chain continues for 30000 iterations. Gelman-Rubin convergence statistic, R , indicates that the simulation is believed to have converged as shown in Figure 7. A simple summary can be generated showing posterior mean, median and standard deviation with a 95% posterior credible interval. This summary of the sampling result assuming unknown C is presented in Table 5, and shows that the estimated value of the scale parameter under usual conditions is 0.9805, and the reliability decreases when the mission time x_0 increases (see Appendix I). Figure 8 shows the posterior distribution of C (see Appendix II).

The case of unknown P

The values of the three parameters (C, γ, θ) are assumed to be known and take the values $(C = 0.5, \gamma = 0.5, \theta = 0.25)$. The prior of the parameter P is assumed to be the gamma distribution with parameters $\lambda_1 = 0.25$ and $\lambda_2 = 0.25$. Seven chains with different initials $(P = 0.8, P = 0.7, P = 0.6, P = 0.5, P = 0.4, P = 0.3, P = 0.2)$ are run simultaneously in one simulation. Each chain continues for 25000 iterations. For checking the convergence, Figure 9 shows Gelman-Rubin convergence statistic of P is converged to one. The summary of sampling results is displayed in Table 6 (see Appendix I). The mean value of the samples, as the estimate of P is shown to be 0.9314, and the estimated value of the scale parameter under usual conditions is 0.609. In addition, we note that the reliability decreases when the mission time x_0 increases. Figure 10 shows posterior of distribution of P (see Appendix II).

The case of unknown C and P

In this case, we assume the values of the three parameters (γ, θ) are known and apply Bayesian method to determine the posterior density function of C and P . We set $(\gamma = 0.5, \theta = 0.5)$ and the prior of the parameter C is the gamma distribution with parameters $\lambda_1 = 3.5$ and $\lambda_2 = 3.75$, and the conditional distribution of P given C is gamma $(\lambda_3, \lambda_4 C)$ with parameters $\lambda_3 = 3.25$ and

$\lambda_4 = 5$. Three chains with different initials $[(C = 0.5, P = 0.5), (C = 0.4, P = 0.3), (C = 0.3, P = 0.6)]$ run simultaneously in one simulation. Each chain continues for 40000 iterations. Figure 11 shows that the simulation is believed to be convergent, and Figure 12 shows the marginal posterior distributions of both C and P . The summary of the sampling results assuming C and P are unknown is displayed in Table 7, and shows that the estimated value of the scale parameter under usual conditions is 1.107, and the reliability decreases when the mission time x_0 increases (see Appendix I).

5 Conclusion

This paper presents Bayesian method for Type-II censored constant stress accelerated life test with three-parameter generalized logistic lifetime distribution and inverse power law acceleration model. The three-parameter generalized logistic distribution appears to be an important and useful family as it includes a number of other distributions for different choices of the concerned model parameters. We present Bayesian inference for three cases, the first case when the parameter C is unknown and the other parameters (P, γ, θ) are known, the second case, inference for P when the other parameters (C, γ, θ) are known, and the third case, inference for C, P when the other parameters (γ, θ) are known. Then, Bayesian analysis is conducted to estimate the point, the asymptotic confidence interval of the model parameters, prediction the scale parameter and the reliability function under the usual conditions. The use of MCMC technique and WinBUGS software enhances the flexibility of the proposed method. The simulation for Bayesian analysis has proved to be converged in this paper. We provide a numerical simulation and a real example to illustrate the proposed method. We restrict our Bayesian analysis to cases where some of the parameters are known because we are interesting to estimate the unknown parameters of the scale parameter α under k^{th} levels of stress.

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Appendices

Appendix I

Table 1: Estimates of C , α_u , and $R_u(x_0)$ based on simulated data

Parameter	Mean	S.D	MC error	2.5%	Median	97.5%
C	0.9317	0.4987	0.0016	0.2272	0.8442	2.1340
α_u	1.2720	0.6807	0.0022	0.3101	1.1520	2.9120
$R_u(0.5)$	0.9277	0.0203	0.0001	0.8753	0.9325	0.9519
$R_u(1)$	0.8838	0.05299	0.0001	0.7447	0.8976	0.9454
$R_u(3)$	0.6712	0.1620	0.0005	0.3179	0.6926	0.9122

Table 2: Estimates of P , α_u , and $R_u(x_0)$ based on simulated data

Parameter	Mean	S.D	MC error	2.5%	Median	97.5%
P	0.9313	0.4981	0.0015	0.2265	0.8443	2.1330
α_u	4.0910	5.3020	0.0146	1.3260	2.8600	14.220
$R_u(0.5)$	0.8278	0.1356	0.0003	0.4056	0.8773	0.9279
$R_u(1)$	0.6839	0.1955	0.0005	0.1396	0.7498	0.8852
$R_u(3)$	0.3182	0.1898	0.0004	0.0019	0.3255	0.6441

Table 3: Estimates of C , P , α_u , and $R_u(x_0)$ based on simulated data

Parameter	Mean	S.D	MC error	2.5%	Median	97.5%
C	0.9316	0.4981	0.0014	0.2254	0.8451	2.1360
P	0.8328	0.2367	0.0006	0.1676	0.9537	1.0000
α_u	2.7240	1.6250	0.0045	0.5465	2.4060	6.6920
$R_u(0.5)$	0.8757	0.0649	0.0001	0.7032	0.8940	0.9470
$R_u(1)$	0.7613	0.1370	0.0003	0.4319	0.7929	0.9341
$R_u(3)$	0.4199	0.2261	0.0006	0.05805	0.3992	0.8608

Table 4: Times to breakdown of an insulating fluid under various values of the stress

V (in Kilovolts)	n_j	r_j	Failure Times (in Minutes)							
			32	15	11	0.27	0.40	0.69	0.79	2.75
			13.95	15.93	27.80	82.85				
34	19	14	0.19	0.78	0.96	1.31	2.78	3.16	4.15	
			4.67	4.85	6.50	7.35	8.01	8.27	12.06	
36	15	8	0.35	0.59	0.96	0.99	1.69	1.97	2.07	
			2.58							
38	8	6	0.09	0.39	0.47	0.73	0.74	1.13		

Table 5: Estimates of C , α_u , and $R_u(x_0)$ based on the illustrative example

Parameter	Mean	S.D	MC error	2.5%	Median	97.5%
C	0.9313	0.4983	0.0012	0.2261	0.8442	2.1310
α_u	0.9805	0.5246	0.0013	0.2380	0.8888	2.2430
$R_u(0.5)$	0.6957	0.0348	0.0001	0.6118	0.7019	0.7446
$R_u(1)$	0.6316	0.06692	0.0001	0.4736	0.6426	0.7293
$R_u(3)$	0.4236	0.1365	0.0003	0.1563	0.4281	0.6660

Table 6: Estimates of P , α_u , and $R_u(x_0)$ based on the illustrative example

Parameter	Mean	S.D	MC error	2.5%	Median	97.5%
P	0.9314	0.4985	0.0011	0.2252	0.8443	2.1330
α_u	0.6090	0.0664	0.0001	0.5237	0.5949	0.7757
$R_u(0.5)$	0.7204	0.0041	0.0000	0.7094	0.7214	0.7260
$R_u(1)$	0.5034	0.1024	0.0030	0.2699	0.5180	0.6585
$R_u(3)$	0.5227	0.0234	0.0001	0.4644	0.5275	0.5539

Table 7: Estimates of C , P , α_u , and $R_u(x_0)$ based on the illustrative example

Parameter	Mean	S.D	MC error	2.5%	Median	97.5%
C	0.9316	0.4981	0.0014	0.2254	0.8451	2.1360
P	0.8328	0.2367	0.0006	0.1676	0.9537	1.0000
α_u	1.1070	0.5945	0.0017	0.2671	1.0030	2.5440
$R_u(0.5)$	0.6564	0.0279	0.0001	0.5883	0.6615	0.6952
$R_u(1)$	0.6041	0.0562	0.0001	0.4679	0.6141	0.6831
$R_u(3)$	0.2387	0.1487	0.0004	0.0220	0.2168	0.5565

Appendix II

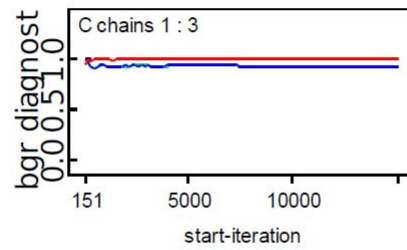


Figure 1: Gelman-Rubin Statistic of C based on simulated data

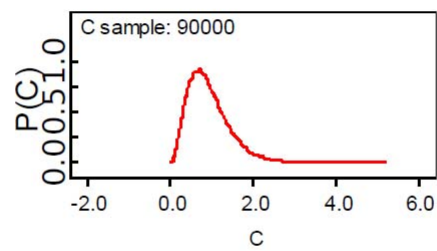


Figure 2: Posterior density plots of C based on simulated data

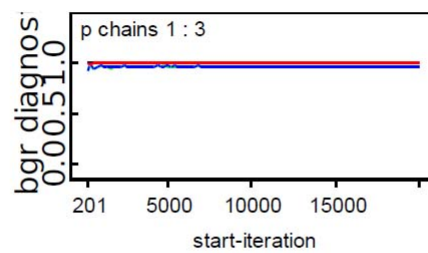


Figure 3: Gelman-Rubin Statistic of P based on simulated data

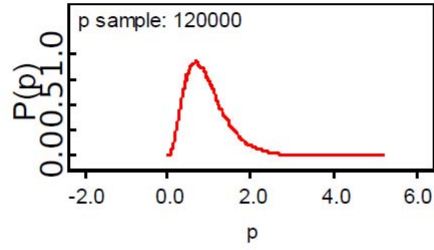


Figure 4: Posterior density plots of P based on simulated data

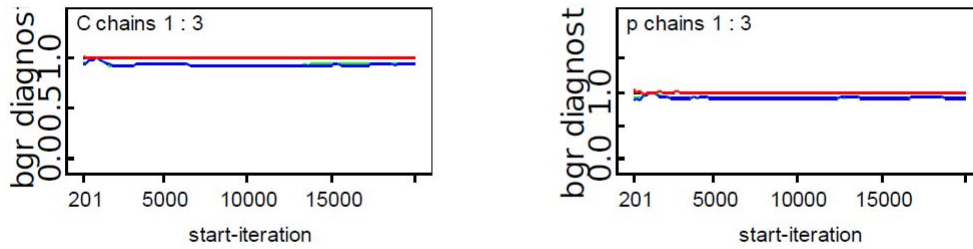


Figure 5: Gelman-Rubin Statistic of C and P based on simulated data

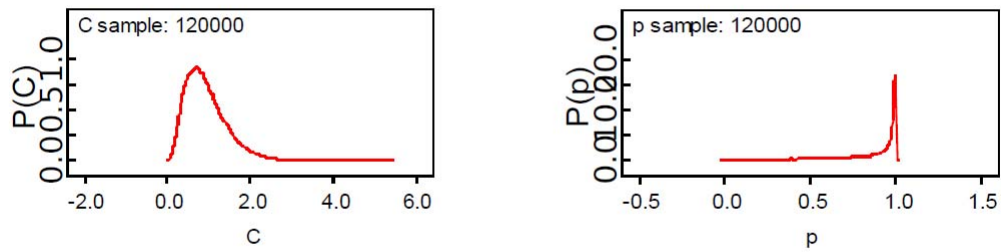


Figure 6: Posterior density plots of C and P based on simulated data

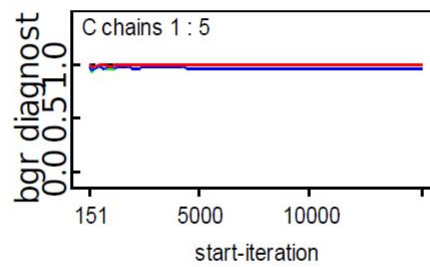


Figure 7: Gelman-Rubin Statistic of C based on the illustrative example

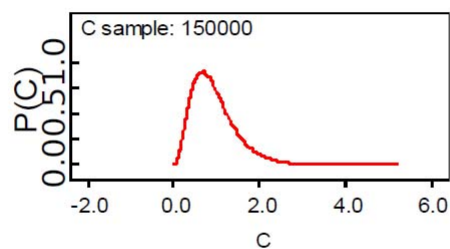


Figure 8: Posterior density plots of C based on the illustrative example

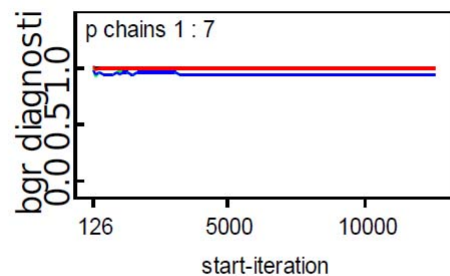


Figure 9: Gelman-Rubin Statistic of P based on the illustrative example

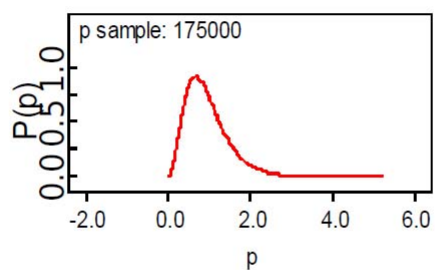


Figure 10: Posterior density plots of P based on the illustrative example

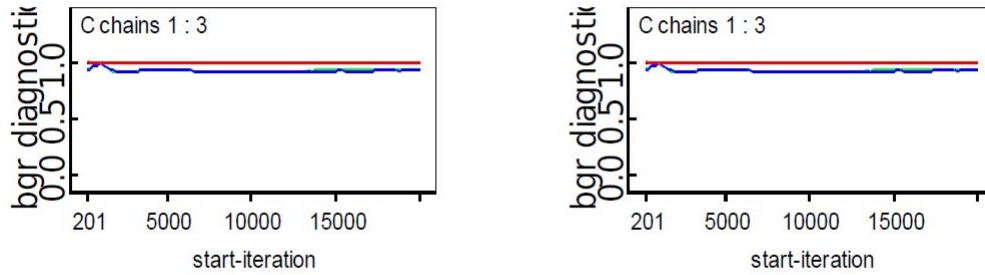


Figure 11: Gelman-Rubin Statistic of C and P based on the illustrative example

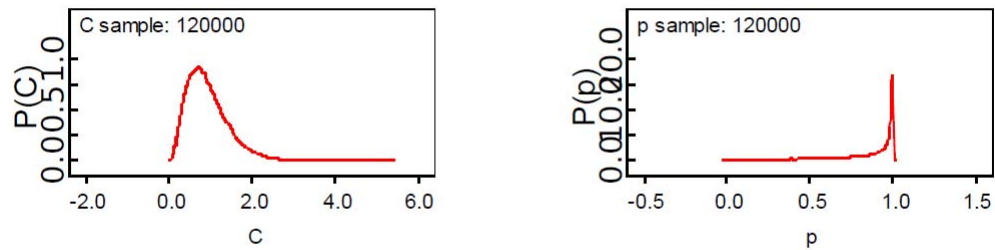


Figure 12: Posterior density plots of C and P based on the illustrative example