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To cite this article: Enas Yahya Abdulla and Layla Ismet Jali 2021 *J. Phys.: Conf. Ser.* **1963** 012162

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Calculation maximum pressure of layers with using porosity mathematical model

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Abstract. In this paper, discussion maximum pressure between layers of articular cartilage for of human ankle joint in Various walking styles and design mathematical model depended on the Reynolds equation and continuity equation. Non- Newtonian synovial fluid and elastic deformation is mathematically derived. Theoretical analysis maximum pressure with different stride length for human, porosity and regular velocity in daily active we provided The stride length that accompanies human movement with different walking speed is the most important influences that protect the joint and help accomplish daily activities

Table 1: List of Terms Used

Nomenclature			
\bar{u}	Regular Velocity	μ	Dynamic viscosity
Δ	Synovial	EL	Elastic deformation
Σ	stride length	x, y, z	Coordinates along the direction
β	The Porosity	P	Pressure in thin film
x	Distance	W	Load carrying capacity
H	Film Thickness	R	Radius
θ	Permeability	μ	Dynamic viscosity
Syv	Molecular viscosity	ST	Stride length

1.Introduction

The ankle joint is one of the important and complex joints in the human body. The ankle joint (or talocrural joint) is a synovial joint located in the lower limb. It is formed by the bones of the leg (tibia and fibula) and the foot (talus). The ankle joint is a hinge type joint, with movement permitted in one plane [1]. Thus, plantarflexion and dorsiflexion are the main movements that occur at the ankle joint. eversion and inversion are produced at the other joints of the foot, such as the **subtalar joint**. As this joint bears double body weight and ground reaction during the move [2]. Ankle movement is related to gait cycle that it



describes the process of moving the foot from the moment it touches the ground to the moment the foot itself touches the ground in the next step, provided that the movement is forward in relation to the center of gravity in the body. The gait cycle is separated into two parts: first the stance phase which includes (the planter flexion and dorsiflexion) and secondly the swing phase which includes (eversion and inversion). [3-4]. In gait cycle the most important factor is the step length resulting from ankle movement that be the longest for male than female [2]. The deformation of cartilage has been studied for the most part under static loading, and the various studies have shown that cartilage exhibits viscoelastic behavior [5]. The viscoelasticity of cartilage tissue has been explained in terms of interstitial fluid flow within the cartilage matrix and of the intrinsic viscoelasticity of the matrix itself [6-7]. Also, the elastomeric distortion of the muscle fibers and the flexibility of the joint play an important role in determining the movement pattern of both sexes [8]. In this paper present a study of the maximum pressure for ankle with different movement patterns using a mathematical program (Mathematica.12) and the introduction of special parameters to obtain an analytical model of the hydrodynamic pressure of the pressure [3],

2. Basic Equation

Based on previous assumptions of quasi-static film and reducing the continuity equation and the Navier-stokes equations to:

$$\frac{\partial^2 u}{\partial z^2} = \frac{1}{\mu} \frac{\partial P}{\partial x} + \frac{1}{\mu} El \frac{\partial P}{\partial x} - Syv \frac{\partial P}{\partial x} \quad (1)$$

$$\frac{\partial^2 v}{\partial z^2} = \frac{1}{\mu} \quad (2)$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \quad (3)$$

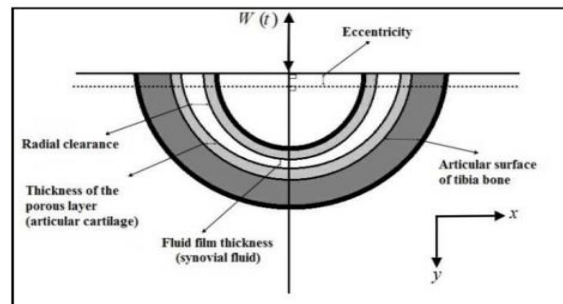


Figure (2.1) shows the physical and geometry configuration of the ankle joint

Using the integration of the Navier-Stokes equation (2) twice with respect to z , using the boundary condition of the tangential component of the velocity of the fluid in the region of the film, this was obtained by completing the solution.

$$x = 0 \quad \text{at} \quad z = h \quad , \quad x = \frac{ST}{\mu} \quad \text{at} \quad z = h$$

$$v(x, z) = \frac{1}{\mu} \frac{z^2}{2} + A_1 z + A_2 \quad (4)$$

Where A_1, A_2 be integration constant. Now, by applying the boundary condition of the tangential component of the fluid velocity in the film region we have:

$$v(x, z) = \frac{1}{\mu} \frac{z^2}{2} + \left(\frac{SL}{\mu h} - \frac{1}{\mu} \frac{h}{2} \right) z \quad (5)$$

Substituting the tangential component of fluid velocity in the porosity and film region equation (5) into the Navier- stokes equation (1) given:

$$\frac{\partial^2 u}{\partial z^2} = \frac{1}{\mu} \frac{\partial P}{\partial x} [1 + El - Syv] \quad (6)$$

$$= \frac{1}{\mu} \frac{\partial P}{\partial x} \left[1 + El - Sy \left(\frac{1}{\mu} \frac{z^2}{2} + \left(\frac{SLz}{\mu h} - \frac{1}{\mu} \frac{zh}{2} \right) \right) \right] \quad (7)$$

Integration the equation (6) twice for z , it is obtained the final form of fluid velocity between Thin films:

$$\frac{\partial}{\partial z} \left(\frac{\partial u}{\partial z} \right) = \frac{1}{\mu} \frac{\partial P}{\partial x} \left[1 + El - \frac{Sy z^2}{\mu} + \left(\frac{SySL}{\mu h} z - \frac{Sy h}{2} \frac{z}{\mu} \right) \right] \quad (8)$$

Integrating this equation twice with respect to z .

$$u(x, z) = \frac{1}{\mu} \frac{\partial P}{\partial x} \left[\frac{z^2}{2} + \frac{Elz^2}{2} - \frac{Sy z^4}{24\mu} + \frac{SySLz^3}{6\mu h} - \frac{Syhz^3}{12\mu} \right] + B_1 z + B_2 \quad (9)$$

using the boundary condition of the tangential component of the velocity of the fluid in the region of the film, this was obtained by completing the solution

$$u = 0 \quad \text{at} \quad z = 0$$

$$u = 0 \quad \text{at} \quad z = h.$$

Where B the integration constant. The relevant boundary conditions for the velocity components are:(i) at the static Permeability $z = 0$, (ii) at the dynamic (functional) Permeability $z = h$, $B = 0$

$$u(x, z) = \frac{1}{\mu} \frac{\partial P}{\partial x} \left[\frac{z^2}{2} + \frac{Elz^2}{2} - \frac{Sy z^4}{24\mu} - \frac{SySLz^3}{6\mu h} - \frac{Syhz^3}{12\mu} \right] + B_1 z \quad (10)$$

$$B_1 = \frac{-h}{2\mu} \frac{\partial P}{\partial x} \left[1 + El - \frac{Syh^2}{4} + \frac{SySL}{3\mu} \right] \quad (11)$$

$$u(x, z) = \frac{1}{2\mu} \frac{\partial P}{\partial x} \left[z^2 + Elz^2 - \frac{\delta z^4}{12\mu} + \frac{\delta \sigma z^3}{3\mu h} - \frac{\delta h z^3}{6\mu} \right] - h \left[1 + El - \frac{\delta h^2}{4} + \frac{\delta \sigma}{3h} \right] z$$

$$w(x, z) = -\frac{\partial}{\partial x} \int u(x, z) dz \quad (12)$$

using the boundary condition of the tangential component of the velocity of the fluid in the region of the film, this was obtained by completing the solution

$$w(x, 0) = 0 \quad \text{and} \quad w(x, h) = \frac{\partial h}{\partial t} .$$

$$w(x, z) = -\frac{\partial}{\partial x} \frac{1}{2\mu} \frac{\partial P}{\partial x} \int \left[z^2 + Elz^2 - \frac{\delta z^4}{12\mu} + \frac{\delta \sigma z^3}{3\mu h} - \frac{\delta h z^3}{6\mu} \right] - h \left[1 + El - \frac{\delta h^2}{4} + \frac{\delta \sigma}{3\mu} \int z dz \right] \quad (13)$$

When $x = h$ then

$$W(x, h) = -\frac{1}{2\mu} \frac{\partial^2 P}{\partial x^2} \left[\frac{h^3}{3} + El \frac{h^3}{3} - \frac{\delta h^5}{60\mu} + \frac{\sigma \delta h^3}{24\mu h} - \frac{\delta h}{24\mu} h^4 - \frac{h^3}{2} - El \frac{h^3}{2} - \frac{\delta h^5}{8} - \frac{\sigma \delta h^3}{6\mu} \right] \quad (14)$$

$$W(x, h) = \frac{1}{2\mu} \frac{\partial^2 P}{\partial x^2} \left[\frac{h^3}{6} + El \frac{h^3}{6} + 0.05833 \delta h^5 - \frac{\delta h^5}{8} + \frac{\sigma \delta h^3}{8\mu} \right] \quad (15)$$

As the modified Reynolds equation governs the hydrodynamic pressure of the distance.

To write modified Reynolds equation governing in the film pressure introducing the following dimensionless variables

$$\begin{aligned} p^* &= -\frac{pH_0^2}{\mu x_0} \frac{\partial h}{\partial t} & h^* &= \frac{h}{H_0} & L^* &= \frac{L}{L_0} & \delta &= \frac{\mu}{\mu_0} & \delta &= \frac{L}{x_0} \\ k^* &= \frac{k}{h^2} & H^* &= \frac{H}{h} & x^* &= \frac{x}{x_0} & \phi^* &= \frac{\phi}{H_0^2} & z^* &= \frac{z}{H_0} \end{aligned}$$

3. Squeeze – film characteristics

There are three types of squeeze film characteristics, which are pressure distribution, load carrying capacity and friction force.

3.1 Hydrodynamic Pressure

Stratify the above dimensionless formula in equation (15). Thus, the conclusive dimensionless form of the modified Reynolds becomes:

$$\frac{\partial h}{\partial t} = \frac{\theta h}{\mu} \frac{\partial^2 P}{\partial x^2} + \frac{1}{2\mu} \frac{\partial^2 P}{\partial x^2} \left[\frac{h^3}{6} + El \frac{h^3}{6} + 0.05833 \delta h^5 - \frac{\delta h^5}{8} + \frac{6\delta h^3}{8\mu} \right] \quad (16)$$

The pressure by using integrating twice can be found, subject to bounded condition.

$$\frac{\partial^2 P^*}{\partial x^{*2}} = \frac{2R}{6 \left[2\theta^* h_i^* + \frac{h^{*3}}{6} + El \frac{h^{*3}}{6} + 0.05833 h^{*2} \delta + \frac{6\sigma h^{*3}}{\mu 8} - \frac{B h^{*5}}{8} \right]} \quad (17)$$

To find the solution to the modified Navier-stokes equation, it has been integrated equation (17)

$$\frac{dP^*}{dx^*} = \frac{2Rx^*}{6 \left[2\theta^* h_i^* + \frac{h^{*3}}{6} + El \frac{h^{*3}}{6} + 0.05833 h^{*2} \delta + \frac{6\sigma h^{*3}}{\mu 8} - \frac{B h^{*5}}{8} \right]} + A \quad (18)$$

$$P^* = 0 \text{ at } x^* = 5 \quad \frac{dP^*}{dx^*} = 0 \text{ at } x^* = \beta \bar{u} \quad (19)$$

Where A be the integration constant, it is applied the boundary condition (19) for fluid film pressure is as follows, hence, it is obtained the integration constant is as follows:

$$A = \frac{-2R\beta\bar{u}}{6 \left[2\theta^* h_i^* + \frac{h^{*3}}{6} + El \frac{h^{*3}}{6} + 0.05833h^{*2}\delta + \frac{6\sigma h^{*3}}{\mu 8} - \frac{Bh^{*5}}{8} \right]} \quad (20)$$

Then the hydrodynamic pressure is given by:

$$P^* = \frac{r - 2r\beta\bar{u}}{6 \left[2\theta^* h^* + \frac{h^{*3}}{6} + El \frac{h^{*3}}{6} + 0.05833h^{*2}\delta + \frac{6\sigma h^{*3}}{\mu 8} - \frac{Bh^{*5}}{8} \right]} \quad (21)$$

Now with the film pressure equation known, the squeeze film characteristics can now be calculating.

3.2 Maximum Pressure

Maximum pressure between layers with different loads and speed of movement of the lubricating particles, maximum pressure is calculated based on non-dimensional pressure and it is expressed in the formula:

$$P^* = \frac{R - 2R\beta\bar{U}}{6 \left[2\theta^* h^* + \frac{h^{*3}}{6} + El \frac{h^{*3}}{6} + 0.05833h^{*2}\delta + \frac{6\sigma h^{*3}}{\mu 8} - \frac{Bh^{*5}}{8} \right]} \quad (22)$$

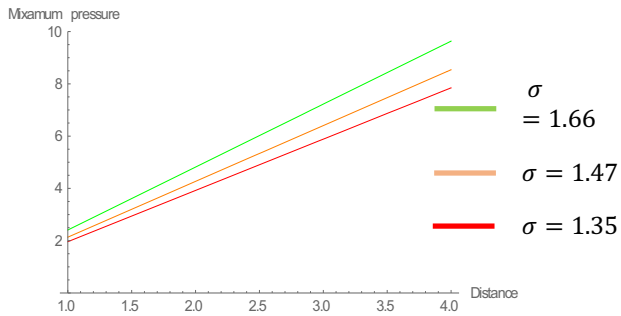
4. Results and discussion

On the basis of quasi-static squeeze film equation, this paper discusses the effectiveness of elastic deformation of articular cartilage on squeeze film characteristics in synovial human knee joint in the cases of normal and disease joint. We also determine wear layers of articular cartilage (superficial zone – middle zone – deep zone) in young and elderly.

4.1 maximum Pressure

Maximum pressure between (thin - thickness) layers consisting of atoms and molecules we illustrate by figures (2.2-2.10). The various dimensionless maximum pressure with the distance for various value stride length (σ) in stance phase and swing phase seen in figures (2.2-2.5). It is found that maximum pressure increase with increasing the stride length of a human, interpretation the greater the stride length, lead to increase the flow rate of the lubricant from porosity region male have longer strides than female, so we find a higher incidence of osteoporosis among women, so maximum pressure affected by types of gait cycle (stance phase – swing phase) where in stance phase increases the rate of production synovial fluid from synovial cells responsible for generating maximum pressure. It reaches the highest rate maximum pressure in plantar flexion (σ) to 98% in normal joint. with the various activities that the human performs, the weight doubles and the lubricant production decreases where reaches in eversion (σ). The various maximum pressure with the distance for various value dynamic viscosity of molecules (μ) is seen in figure (2.6), where we find that the viscosity of molecules (μ) generate the highest pressure between the layers of the articular cartilage, where the pressure ratio reaches to 95%. The various maximum pressure with the distance for various value film thickness (h) is seen figure (2.7). in superficial zone be film thickness high this consisting (eversion - inversion) phase, through daily active the joint is exposed to a high stress research to deep zone. The various maximum pressure with the distance for various value regular velocity is seen figure (2.8). Walking pattern is one of the most important factors affecting pressure, as the walking pattern changes from slow to fast, and accordingly, maximum pressure decreases. The various maximum pressure with the phase of velocity is seen figure (2.9). Walking pattern is one of the most important factors affecting pressure, as the walking pattern changes from slow to fast, and accordingly, maximum pressure decreases. The various maximum pressure with the distance for various

elastic deformation is seen figure (2.9). Muscle fiber elasticity gives the body a high ability to accomplish many daily activities and gives the body protection from shocks and accidents. Elastic is associated with an increase in the level of maximum pressure between the joint



Figure(2-2): The Variation of Dimensionless Max pressure ($P_{Max.}$) with a Dimensionless distance for different stride length on the stance phase/plantar flexion (σ).

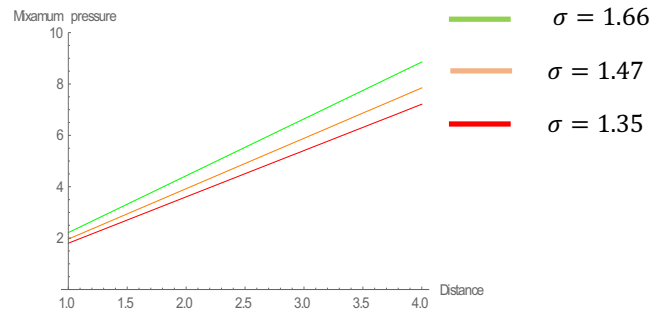


Figure (2-3): The Variation of Dimensionless Max. Pressure ($P_{Max.}$) with Dimensionless distance for different stride length on the stance phase /dorsiflexion (σ).

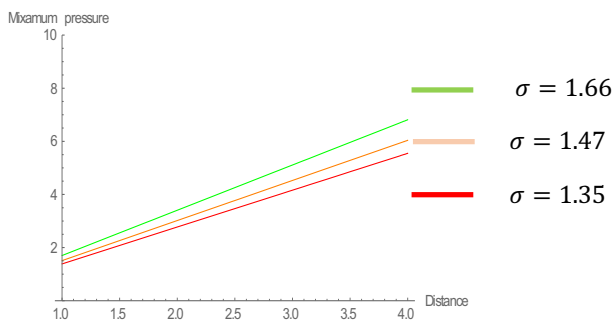
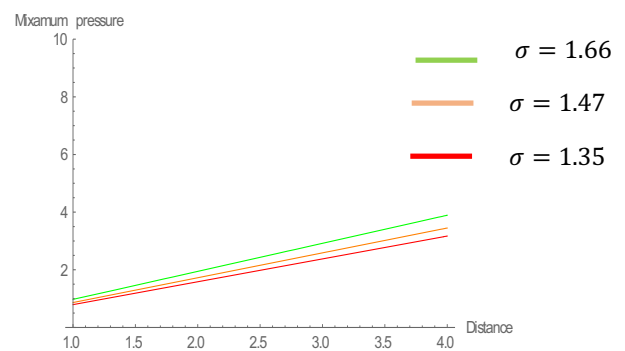


Figure (2-4): The Variation of Dimensionless Max. pressure ($P_{Max.}$) of with Dimensionless distance for different distance stride length on the swing phase/inversion (σ).



Figure(2-5):The Variation of Dimensionless Max. Pressure ($P_{Max.}$) of with a Dimensionless distance for different distance stride length on the swing phase/eversion (σ)

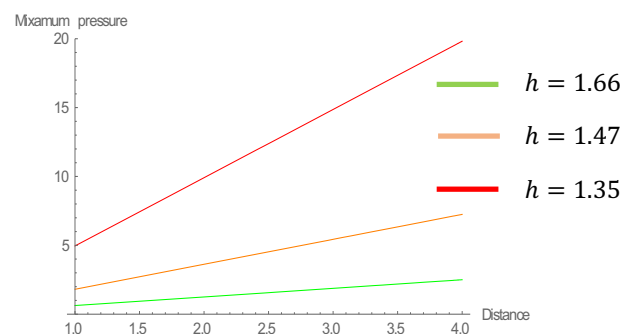
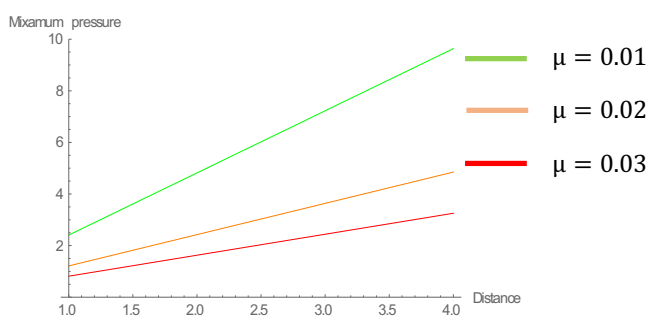
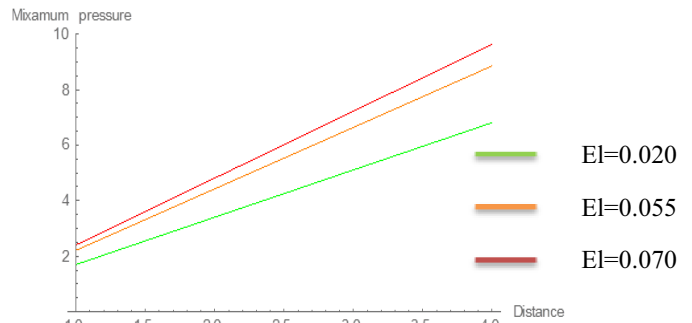
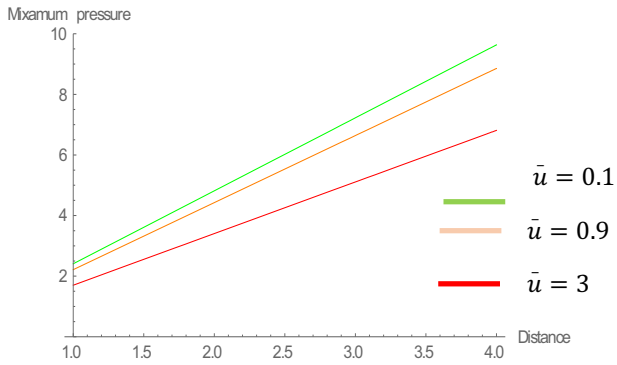


Figure (2-6):The Variation of Dimensionless Max. Pressure ($P_{Max.}$)Dimensionless distance for different viscosity (μ).

Figure (2-7):The Variation of Dimensionless Max. Pressure with ($P_{Max.}$)Dimensionless distance for different lubricant (h).



Figure(2-8) :The Variation Dimensionless Max.pressure ($P_{Max.}$) with distance for different of Elastic deformation

Figure(2-9) :The Variation Dimensionless Max.pressure ($P_{Max.}$) with distance for different of Elastic deformation

Table (2) shows the relationship between Stride length and Pressure.

Stride length (stance phase/plantar flexion) σ	1.66	1.47	1.35
Pressure p	3.44075	3.0495	2.8025
Stride length (stance phase/dorsiflexion) σ	1.66	1.47	1.35
Pressure p	3.1627	2.8031	2.5760
Stride length (swing phase/inversion) σ	1.66	1.47	1.35
Pressure p	2.4328	2.1562	1.9815
Stride length (swing phase/eversion) σ	1.66	1.47	1.35
Pressure p	1.3902	1.2321	1.1323

Table (3) shows the relationship between viscosity and pressure

Viscosity μ	Pressure p
0.01	3.4407
0.02	1.7318
0.03	1.1622

Table (4) shows the relationship between the film and pressure.

Film thickness h	Pressure p
0.35	0.8935
0.50	2.5879
0.70	7.0791

Table (5) shows the relationship between velocity & Pressure

Velocity	pressure
0.1	3.4407
0.9	3.1627
3	2.4328

Table (6) shows the relationship between Elastic deformation & Pressure

Elastic deformation	Pressure p
3	2.4328
0.9	3.1626
0.1	2.4083

Conclusions:

1. The maximum pressure of layers' joint increases in the two stages (planter flexion -dorsiflexior) since increase the lubricating fluid flow from the porosity covering the meniscus compared to stages (eversion –inversion)
2. Low viscosity increases maximum on joint because the low viscosity lead to increases load carrying capacity on joint because the low flow fluid that contain on lubricant generates a high hydrodynamic pressure, which makes the joint durability high.
3. Film thickness of layers in hydrodynamic lubrication be high Thus it results the hydrodynamic pressure between cartilage very high, it has little load bearing capacity compared to squeeze lubrication where the production of synovial cells increases in the lubricating fluid to generate higher hydrodynamic pressure to increase the bearing capacity of the joint.
4. The phase of velocity greatly affects the generate the highest pressure between layers, as we find at the beginning of the speed the endurance of the joint is high and when the speed increases, the endurance of the joint increases until it reaches its climax in the stage.
5. When the muscle fiber has a high degree of elasticity, this is reflected in the endurance of the joint to carry out daily activities.

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