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Effective stride length on load carrying capacity for ankle joint

Enas Y. Abdullah *
Layla Ismet Jalil †
Department of Mathematics
Faculty of Education
Kufa University
Kufa
Iraq

Abstract

In this paper, discussion load carrying capacity of person ankle joint in stance stage & swing stage and mathematical modeling approval on the Reynolds and continuity equation. Elastic deformation of non-Newtonian synovial fluid & speed phase is mathematically elicited. Theoretical explanation load carrying with several viscosity of synovial fluid, porosity and stride length in daily tasks we provided the elastic deformation that escorts the joint with diverse footing speed is the hardest factor determining walking pattern for both sexes.

Subject Classification: *Primary 34A45, Secondary 34A34.*

Keywords: *Stride length, Carrying capacity, Ankle joint.*

1. Introduction

The ankle hinge is one of the important & complex hinges in the person. The ankle joint (or talocrural joint) is a synovial hinge located in the undercarriage [l]. Thus, plantar flexion & dorsiflexion are the major moves that happen at the ankle joint. Un reversal and reversal are generated at the other joints of the foot [2]. Ankle shift is related to walking cycle which depicts the practicability of mobile foot from the second touching the mold to the second foot itself touching the mold in the following tread [3-4]. In gait cycle, the most prominet influencer is

* *E-mail:* inasy.abdullah@uokufa.edu.iq (Corresponding Author)
† *E-mail:* laylaismet1972@gmail.com

the step length extracted from ankle movement that be the longest for male more than female [2]. The distortion of cartilage had been researched for the most portion beneath static loading, and the various surveys has shown that gristle offers viscoelastic behavior. In this paper, we prepare a treaties about load carrying capacity for ankle with several movement mode patterns using a computer program (Mathematica.12) [3].

2. Governing Equation

Depending on supposition of quasi-calm film and curtailment the continuity and the Navier-stokes equations to:

$$\frac{\partial^2 u}{\partial z^2} = \frac{1}{\mu}\frac{\partial P}{\partial x} + \frac{1}{\mu}\ El\frac{\partial P}{\partial x} - Syv\frac{\partial P}{\partial x} \tag{1}$$

$$\frac{\partial^2 v}{\partial z^2} = \frac{1}{\mu} \tag{2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = O \tag{3}$$

Utilizing the integral of the Navier-Stokes equation (2) two times with respect to z, using the border stipulation for the tangential ingrediant to velocity of the fluid in the territory for the film, this was acquired by completing the solution.

$$x = O \text{ at } z = h,\ x = \frac{sl}{\mu} \text{ at } z = h$$

$$v(x,z) = \frac{1}{\mu}\frac{z^2}{2} + A_1 z + A_2 \tag{4}$$

Where A_1, A_2 integration constant. Now, by applying the boundary condition of the tangential component of the liqid rapidity in the overly area we have:

$$v(x,z) = \frac{1}{\mu}\frac{z^2}{2} + \left(\frac{SL}{\mu} - \frac{1}{\mu}\frac{h}{2}\right)z \tag{5}$$

Replacing the tangential motif of deltessent quickness in the porosity and film part equation (5) into the Navier- stokes equation (1) given:

$$\frac{\partial^2 u}{\partial z^2} = \frac{1}{\mu}\frac{\partial P}{\partial x}[1 + El - Syv] \tag{6}$$

$$= \frac{1}{\mu}\frac{\partial P}{\partial x}\left[1 + El - Sy(\frac{1}{\mu}\frac{z^2}{2} + \left(\frac{SLz}{\mu h} - \frac{1}{\mu}\frac{zh}{2}\right)\right] \tag{7}$$

After integrating (6) two times for z, it is gained the final form of fluid speed between tender films:

$$\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial z}\right) = \frac{1}{\mu}\frac{\partial P}{\partial x}\left[1 + El - \frac{Sy}{\mu}\frac{z^2}{2} + \left(\frac{SySL}{\mu h}z - \frac{Sy}{2}\frac{h}{\mu}z\right)\right] \tag{8}$$

And integrated (8)two time with respect to z.

$$u(x,z) = \frac{1}{\mu}\frac{\partial P}{\partial x}\left[\frac{z^2}{2} + \frac{Elz^2}{2} - \frac{Syz^4}{24\mu} + \frac{SySLz^3}{6\mu h} - \frac{Syhz^3}{12\mu}\right] + B1z + B2 \tag{9}$$

Usage border condition of the tangential strain of the velocity

u = O within z = O

u = O within z = h.

Whereas B constant of the integration. The pertinent boundary conditions for the velocity components are:

(i) On stable Permittivity $z = O$,

(ii) On functional Permittivity $z = h$, $B = O$

$$(x,z) = \frac{1}{\mu}\frac{\partial P}{\partial x}\left[\frac{z^2}{2} + \frac{Elz^2}{2} - \frac{Syz^4}{24\mu} - \frac{SySLz^3}{6\mu h} - \frac{Syhz^3}{12\mu}\right] + B_1 z \tag{10}$$

$$B1 = \frac{-h}{2\mu}\frac{\partial P}{\partial x}\left[1 + El - \frac{Syh^2}{4} + \frac{SySL}{3\mu}\right] \tag{11}$$

$$u(x,z) = \frac{1}{2\mu}\frac{\partial P}{\partial x}\left[z^2 + Elz^2 - \frac{\delta z^4}{12\mu} + \frac{\delta\sigma z^3}{3\mu h} - \frac{\delta h z^3}{6\mu}\right] - h\left[1 + El - \frac{\delta h^2}{4} + \frac{\delta\sigma}{3h}\right]z$$

$$w(x,z) = -\frac{\partial}{\partial x}\int u(x,z)\,dz \tag{12}$$

Employment the border condition of the tangential compound of the speed

$$w(x,\, O) = O \text{ and } w(x,h) = \frac{\partial h}{\partial t}.$$

$$w(x,z) = -\frac{\partial}{\partial x}\frac{1}{2\mu}\frac{\partial P}{\partial x}\int\left[z^2 + Elz^2 - \frac{\delta z^4}{12\mu} + \frac{\delta\sigma z^3}{3\mu h} - \frac{\delta h z^3}{6\mu}\right]$$
$$-h\left[1 + El - \frac{\delta h^2}{4} + \frac{\delta\sigma}{3\mu}\int zdz\right] \quad (13)$$

When $x = h$ then

$$W(x,h) = -\frac{1}{2\mu}\frac{\partial^2 P}{\partial x^2}\left[\frac{h^3}{3} + El\frac{h^3}{3} - \frac{\delta h^5}{6O\mu} + \frac{\sigma\delta h^3}{24\mu h} - \frac{\delta h}{24\mu}h^4 - \frac{h^3}{2}\right.$$
$$\left. - El\frac{h^3}{2} - \frac{\delta h^5}{8}\frac{\sigma\delta h^3}{6\mu}\right] \quad (14)$$

$$W(x,h) = \frac{1}{2\mu}\frac{\partial^2 P}{\partial x^2}\left[\frac{h^3}{6} + El\frac{h^3}{6} + O.O5833\delta h^5 - \frac{\delta h^5}{8} + \frac{\sigma\delta h^3}{8\mu}\right] \quad (15)$$

3. Hydrodynamic Pressure

To write modified Reynolds equation governing in the film pressure introducing the following dimensionless variables

$$P^* = -\frac{P\sigma_O^2}{\mu R\frac{\partial h}{\partial t}},\ B = \delta h^2,\ \ \delta = \frac{\mu}{\mu O},\ \ h^* = \frac{h}{6}$$

Applying the above dimensionless formula in equation (15)

$$\frac{\partial h}{\partial t} = \frac{\theta h}{\mu}\frac{\partial^2 P}{\partial x^2} + \frac{1}{2\mu}\frac{\partial^2 P}{\partial x^2}\left[\frac{h^3}{6} + El\frac{h^3}{6} + O.O5833\delta h^5 - \frac{\delta h^5}{8} + \frac{6\delta h^3}{8\mu}\right] \quad (16)$$

The squeeze by using integrating twice can be establish, subject to bounded condition.

$$\frac{\partial^2 P^*}{\partial x^{*2}} = \frac{2R}{6\left[2\theta^* h_i^* + \frac{h^{*3}}{6} + El\frac{h^{*3}}{6} + O.O5833h^{*2}\delta + \frac{6\sigma h^{*3}}{\mu 8} - \frac{Bh^{*5}}{8}\right]} \quad (17)$$

To get it the solution to the modifieded Navier-stokes eq, it has been integrated equation (17).

$$\frac{dP^*}{dx^*} = \frac{2Rx^*}{6\left[2\Theta^* h_i^* + \frac{h^{*3}}{6} + El\frac{h^{*3}}{6} + O.O5833h^{*2}\delta + \frac{6\sigma h^{*3}}{\mu 8} - \frac{Bh^{*5}}{8}\right]} + A \qquad (18)$$

$$P^* = 0 \quad \text{at} \quad x^* = 5\frac{dP^*}{dx^*} = 0 \quad \text{at} \quad x^* = \beta\overline{u}\} \qquad (19)$$

Such A is the constant for intgral, it is applied the terminal terms (19) for deliquescent envelope compressing is as follows, subsequently, it is gained the integration constant is as follows:

$$A = \frac{-2R\beta\overline{u}}{6\left[2\theta^* h_i^* + \frac{h^{*3}}{6} + El\frac{h^{*3}}{6} + O.O5833h^{*2}\delta + \frac{6\sigma h^{*3}}{\mu 8} - \frac{Bh^{*5}}{8}\right]} \qquad (20)$$

Then the hydrodynamic pressure is given by:

$$P^* = \frac{r - 2r\beta\overline{u}}{6\left[2\theta^* h^* + \frac{h^{*3}}{6} + El\frac{h^{*3}}{6} + O.O5833h^{*2}\delta + \frac{6\sigma h^{*3}}{\mu 8} - \frac{Bh^{*5}}{8}\right]} \qquad (21)$$

4. Load Carrying Capacity

The load-carrying ability is acquired by integrating hydrodynamic stress generated by regular movement of molecules in the liquid

$$W = \frac{2\pi\theta}{hO^2}\int_O^\delta P\,dr \qquad (22)$$

$$W = \frac{2\pi\theta}{hO^2}\int_O^\delta LP\,dr \qquad (23)$$

$$W = \frac{2\pi\theta}{hO^2}\int_O^\delta L^2 * P * \frac{1}{L}dr \qquad (24)$$

We will introduce the carrying capacity of the non-dimensional loads considering the viscosity of the fluids flowing through the layers

$$W^{*} = 2\pi * \int_{O}^{\delta} \frac{r - 2r\beta\overline{u}}{6\left[2\theta^{*}h^{*} + \frac{h^{*3}}{6} + El\frac{h^{*3}}{6} + O.O5833h^{*2}\delta + \frac{6\sigma h^{*3}}{\mu 8} - \frac{Bh^{*5}}{8}\right]} dB^{*} \quad (25)$$

$$W^{*} = 2\pi \frac{r\delta - r\delta^{2}\overline{u}}{6\left[2\theta^{*}h^{*} + \frac{h^{*3}}{6} + El\frac{h^{*3}}{6} + O.O5833h^{*2}\delta + \frac{6\sigma h^{*3}}{\mu 8} - \frac{Bh^{*5}}{8}\right]} \quad (26)$$

5. Results and Discussion

On the basis of Navier –Stokesequations this research disputed the impacts of elastic distortion of articulated gristle on stress load carrying capacity in synovial human ankle joint in the (stance - swing) phase.

6. Load Carrying Capacity

The load-bearing capacity of ankle joint is shown after applying equation (26). It is seen that these expressions depend 0n several parameters such as σ, δ and h Figures (1) to (6) represent the load-carrying capacity for ankle joint during daily active for various values of the parameters σ, δ and h respectively. It observed from these figures that the load-carrying capacity effected and changed by decreasing or increasing values of these parameters**.** The effect of stride length in the plantar flexion is seen figure (l). It is observed dimensionless load carrying capacity increases with increasing values of (σ) and the effect of stride length on the dorsiflexion σ with the variation of dimensionless load carrying capacity increases with increasing values of (σ) is seen (2). The effect of stride length (inversion) σ with the variation of dimensionless load carrying capacity is seen figure (3) .It is observed dimensionless load carrying capacity The effect of stride length eversion with the variation of dimensionless load carrying capacity is seen figure (4). It is observed dimensionless load carrying capacity increases with increasing values of (σ).) .It is observed dimensionless load carrying capacity increases with decreasing values of (μ) as is seen in figure (5) The effect).It is observed dimensionless load carrying capacity increases with decreasing values of film thickness(h) for articular cartilage is seen figure (6).

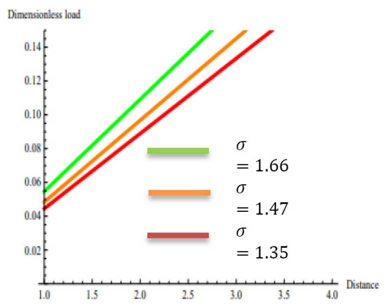

Fig. 1

The diversely dimensionless Load Carrying capacity (W*) with distance for different stride length plantreflexion σ

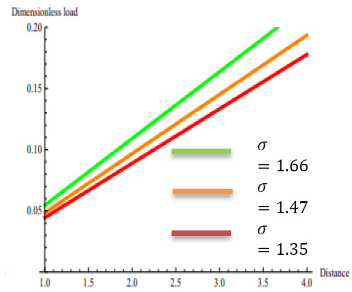

Fig. 2

Difference dimensionless load carrying capacity (W*) with distance for s tridelength (dorsiflexion).

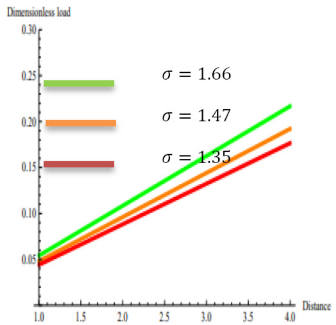

Fig. 3

The variation dimensionless load carrying capacity (W*) distance different stridelength (inversion)

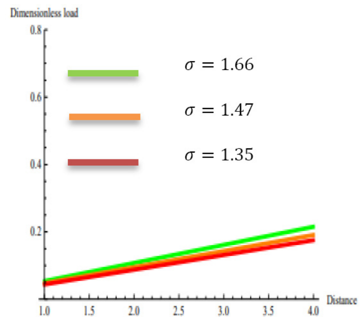

Fig. 4

The variation dimensionless load carrying capacity (W*) distance different stridelength (deversion)

7. Conclusions

The load carrying capacity of the human ankle joint increases in the two stages (planter flexion – dorsiflexion) since increase the lubricating fluid flow from the porosity covering the meniscus compared to stages (eversion –inversion) The higher the viscosity, the greater the bearing capacity, the higher because the high viscosity generates a high hydrodynamic pressure, which makes the joint durability high. Film thickness in hydrodynamic lubrication be high Thus it results the hydrodynamic pressure between

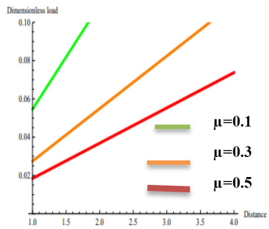

Fig. 5

The variance of dimensionless load carrying capacity (W*) with distance for different viscosity.

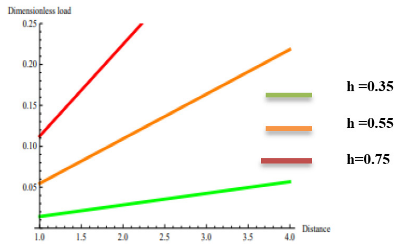

Fig. 6

The distinction of dimensionless load carrying capacity (W*) with distance for different film thickness

cartilage very high, it has little load bearing capacity compared to squeeze lubrication.

References

[1] R.L. Jaw, (1997), "Squeeze film characteristics of long partial journal bearings lubricated with couple stress fluids", Tribology International, Vol. 30, pp.53-58.

[2] Enas Y. and Hala K "A Study of Wear Articular Cartilage of Synovial Human Knee Joint Using Non-Newtonian Elastic Mathematical Model" I*raqi Journal of Science*, Vol. 61, N0. 6, pp: 1407-1418, 2020

[3] Z.M. Jin and D. Dowson, (2005) "Electro hydrodynamic Lubrication in Biological System", Proceedings of the Institution of Mechanical Engineers Part J-*Journal of Engineering Teratology* 219(J5), 367.

[4] Hartsell, H. D., and S. J. Spaulding. Effectiveness of external orthotic support 0n passive soft tissue resistance of the chronically unstable ankle. Foot Ankle Int. 18 : 144–150, 1997.

[5] Shubham Sharma & Ahmed J. Obaid (2020) Mathematical modelling, analysis and design of fuzzy logic controller for the control of ventilation systems using MATLAB fuzzy logic toolbox, *Journal of Interdisciplinary Mathematics*, 23:4, 843-849, DOI: 10.1080/09720502.2020.1727611.

[6] GinoK. Yang, Cheng-Tan Tung & Chu Hopscotch (2015) Rectangular Bus Service Zone without Capacity Constraint, *Journal of Interdisciplinary Mathematics*, 18 : 5, 649-656, DOI: 10.1080/09720502.2015.1047593.

[7] Silvio Nocera (2009) A heuristic solution for the yearly capacity of an intermodal terminal, *Journal of Interdisciplinary Mathematics*, 12:3, 367-393, D0I: 10.1080/09720502.2009.10700631.

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