A COMPLEX OF CHARACTERISTIC ZERO IN THE CASE OF THE PARTITION (8,7,3)

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***ABSTRACT**:: This paper survey the complex of characteristic zero in the case of the partition (8,7,3) by employ the notion of mapping Cone and the concepts divided power of the place polarization, diagrams and Capelli identities*

Key Words: Divided power algebra, resolution of Weyl module, place polarization, mapping Cone.

1. INTRODUCTION:

Let $\mathcal{R}$ be abelian ring with, $\mathcal{F}$ be a free $\mathcal{R}$-module and $\mathcal{D}_i\mathcal{F}$ be the divided power algebra of degree i. Consider the maps

$$\partial_{21}^{(f)}: \mathcal{D}_{p+f}\mathcal{F} \otimes \mathcal{D}_{q-f}\mathcal{F} \otimes \mathcal{D}_r\mathcal{F} \longrightarrow \mathcal{D}_p\mathcal{F} \otimes \mathcal{D}_q\mathcal{F} \otimes \mathcal{D}_r\mathcal{F},$$

this map is a place polarization from place one ($\mathcal{D}_{p+f}\mathcal{F}$) to place two ($\mathcal{D}_{q-f}\mathcal{F}$) and

$$\partial_{32}^{(f)}: \mathcal{D}_p\mathcal{F} \otimes \mathcal{D}_{q+f}\mathcal{F} \otimes \mathcal{D}_{r-f}\mathcal{F} \longrightarrow \mathcal{D}_p\mathcal{F} \otimes \mathcal{D}_q\mathcal{F} \otimes \mathcal{D}_r\mathcal{F},$$

this map is a place polarization from place two ($\mathcal{D}_{q+f}\mathcal{F}$)

The complex of characteristic zero in the case of partitions (2,2,2), (3,3,3) and (4,4,3) are survey by authors in [1], [2] and [3] while the authors in [4], [5], [6] and [7] exhibit the complex of characteristic zero as a diagram in the case of partitions (3,3,2), (6,6,3), (6,5,3) and (7,6,3) by stratify the notion of mapping Cone as in [8].

The authors in [9] exhibit the terms and the exactness of the Weyl resolution in the case of partition (8,7).

In this work we discussion the complex of characteristic zero as a diagram in the case of partition (8,7,3) by employing the notion of mapping Cone in the last section after we illustrate the terms of characteristic zero complex in the same partition in the later section. The map $\partial_{ij}^{(f)}$ which mean the divided power of the place polarization ∂_{ij} where j must be less than i with its Capelli identities [10], throughout this paper we only used the pursue identities

$$\partial_{21}^{(u)} \circ \partial_{32}^{(V)} = \sum_{e\geq 0}(-1)^e\, \partial_{32}^{(V-e)} \circ \partial_{21}^{(u-e)} \circ \partial_{31}^{(e)} \qquad (1.1)$$

$$\partial_{32}^{(V)} \circ \partial_{21}^{(u)} = \sum_{e\geq 0} \partial_{21}^{(u-e)} \circ \partial_{32}^{(V-e)} \circ \partial_{31}^{(e)} \qquad (1.2)$$

$$\partial_{21}^{(1)} \circ \partial_{31}^{(1)} = \partial_{31}^{(1)} \circ \partial_{21}^{(1)} \qquad (1.3)$$

$$\partial_{32}^{(1)} \circ \partial_{31}^{(1)} = \partial_{31}^{(1)} \circ \partial_{32}^{(1)} \qquad (1.4)$$

Where ∂_{ij} is the place polarization from place j to place i.

2. The Terms of Characteristic Zero Complex in The Case of Partition (8,7,3)

The positions of the terms of the complex are determined by the length of the permutation to which they correspond in [1] and [11].

In the case of the partition (8,7,3) we reign the pursue matrix:

$$\begin{bmatrix} \mathcal{D}_8\mathcal{F} & \mathcal{D}_6\mathcal{F} & \mathcal{D}_1\mathcal{F} \\ \mathcal{D}_9\mathcal{F} & \mathcal{D}_7\mathcal{F} & \mathcal{D}_2\mathcal{F} \\ \mathcal{D}_{10}\mathcal{F} & \mathcal{D}_8\mathcal{F} & \mathcal{D}_3\mathcal{F} \end{bmatrix}$$

Then the characteristic zero complexes have the correspondence between its terms as pursues:

$\mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \ \leftrightarrow identity$
$\mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \ \leftrightarrow (12)$
$\mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \ \leftrightarrow (23)$
$\mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \ \leftrightarrow (123)$
$\mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \ \leftrightarrow (132)$
$\mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \ \leftrightarrow (13)$

Thus the characteristic zero resolution in the case of the partition (8,7,3) has the formulation:-

$$\mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \longrightarrow \begin{matrix} \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \\ \oplus \\ \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \end{matrix}$$

$$\longrightarrow \begin{matrix} \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \\ \oplus \\ \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \end{matrix} \longrightarrow \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}$$

3. The Diagram for The Complex of Characteristic Zero in The Case of Partition (8,7,3)

See the pursue diagram:

$$\begin{tikzcd}
\mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \arrow[r, "d_1"] \arrow[d, "h_1"'] \arrow[dr, phantom, "\mathcal{Q}"] & \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \arrow[r, "d_2"] \arrow[d, "h_2"'] \arrow[dr, phantom, "\mathcal{T}"] & \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \arrow[d, "h_3"] \\
\mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \arrow[r, "g_1"] & \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \arrow[r, "g_2"] & \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}
\end{tikzcd}$$

If we acquaint

$d_1: \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \longrightarrow \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_2\mathcal{F}$ as

$d_1(v) = \partial_{32}(v) \quad ;where\ \ v \in \mathcal{D}_{10} \otimes \mathcal{D}_7 \otimes \mathcal{D}_1$

$h_1: \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \longrightarrow \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_1\mathcal{F}$ as

$h_1(v) = \partial_{21}(v) \quad ;where\ \ v \in \mathcal{D}_{10} \otimes \mathcal{D}_7 \otimes \mathcal{D}_1$

And

$h_2: \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \longrightarrow \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_2\mathcal{F}$ as

$h_2(v) = \partial_{21}^{(2)}(v) \quad ;where\ \ v \in \mathcal{D}_{10} \otimes \mathcal{D}_6 \otimes \mathcal{D}_2$

Now, we reign to acquaint the map

$g_1: \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \longrightarrow \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_2\mathcal{F}$

Which fabricate the diagram $\mathcal{Q}$ commutative, i.e.

$$g_1 \circ h_1 = h_2 \circ d_1$$

Implies that

$g_1 \circ \partial_{21} = \partial_{21}^{(2)} \circ \partial_{32}$

By employing Capelli identity (1.1) we gain:

$$\partial_{21}^{(2)} \circ \partial_{32} = \partial_{32} \circ \partial_{21}^{(2)} - \partial_{31} \circ \partial_{21}$$
$$= (\tfrac{1}{2}\partial_{32} \circ \partial_{21} - \partial_{31}) \circ \partial_{21}$$

Thus, $g_1 = \frac{1}{2}\partial_{32} \circ \partial_{21} - \partial_{31}$

If we acquaint the map

$g_2: \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \longrightarrow \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}$ as

$g_2(v) = \partial_{32}(v) \quad ;where\ \ v \in \mathcal{D}_8 \otimes \mathcal{D}_8 \otimes \mathcal{D}_2$

And

$h_3: \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \longrightarrow \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}$ as

$$h_3(v) = \partial_{21}(v) \quad ; where \;\; v \in \mathcal{D}_{10} \otimes \mathcal{D}_6 \otimes \mathcal{D}_2$$

We require acquainting d_2 to fabricate the diagram $\mathcal{T}$ commute:
$d_2: \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \longrightarrow \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}$ such that

$h_3 \circ d_2 = g_2 \circ h_2$ i.e. $\partial_{21} \circ d_2 = \partial_{32} \circ \partial_{21}^{(2)}$

By employing Capelli identity (1.2) we gain: And

$\partial_{32} \circ \partial_{21}^{(2)} = \partial_{21}^{(2)} \circ \partial_{32} + \partial_{21} \circ \partial_{31}$
$= \partial_{21} \circ (\frac{1}{2}\partial_{21} \circ \partial_{32} + \partial_{31})$
Thus, $d_2 = \frac{1}{2}\partial_{21} \circ \partial_{32} + \partial_{31}$

Now consider the pursue diagram:

$$\begin{tikzcd}
\mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \arrow[r, "d_1"] \arrow[d, "h_1"'] \arrow[dr, phantom, "\mathcal{H}"] & \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \arrow[r, "d_2"] \arrow[dr, phantom, "\mathcal{N}"] & \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \arrow[d, "h_3"] \\
\mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \arrow[r, "g_1"] \arrow[urr, "w"] & \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \arrow[r, "g_2"] & \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}
\end{tikzcd}$$

Acquaint $w: \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \longrightarrow \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}$ by

$$w(v) = \partial_{32}^{(2)}(v) \quad ; where \;\; v \in \mathcal{D}_9 \otimes \mathcal{D}_8 \otimes \mathcal{D}_1$$

Proposition (3.1):
The diagram $\mathcal{H}$ is commutative.
Proof:
To prove the diagram $\mathcal{H}$ is commutative, we require proving

$$d_2 \circ d_1 = w \circ h_1$$

$d_2 \circ d_1 = (\frac{1}{2}\partial_{21} \circ \partial_{32} + \partial_{31}) \circ \partial_{32}$
$= \partial_{21} \circ \partial_{32}^{(2)} + \partial_{31} \circ \partial_{32}$
$= \partial_{32}^{(2)} \circ \partial_{21} - \partial_{32} \circ \partial_{31} + \partial_{31} \circ \partial_{32}$
$= \partial_{32}^{(2)} \circ \partial_{21}$
$= w \circ h_1$ ∎

Proposition (3.2):
The diagram $\mathcal{N}$ is commutative.
Proof:
$g_2 \circ g_1 = \partial_{32} \circ (\frac{1}{2}\partial_{32} \circ \partial_{21} - \partial_{31})$
$= \partial_{32}^{(2)} \circ \partial_{21} - \partial_{32} \circ \partial_{31}$
$= \partial_{21} \circ \partial_{32}^{(2)} + \partial_{32} \circ \partial_{31} - \partial_{32} \circ \partial_{31}$
$= \partial_{21} \circ \partial_{32}^{(2)}$
$= h_3 \circ w$ ∎

Eventually, we acquaint the maps σ_1, σ_2 and σ_3 where:
- $\sigma_3(x) = (d_1(x), h_1(x)); \forall\, x \in \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_1\mathcal{F}$
- $\sigma_2((x_1, x_2)) = (d_2(x_1) - w(x_2), g_1(x_2) - h_2(x_1))$; $\forall\, x \in \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \;\oplus\; \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_1\mathcal{F}$
- $\sigma_1((x_1, x_2)) = (h_3(x_1) + g_2(x_2))$; $\forall\, x \in \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \;\oplus\; \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_2\mathcal{F}$

Where

$$\sigma_3: \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \longrightarrow \begin{matrix} \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \\ \oplus \\ \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \end{matrix}$$

$$\sigma_2: \begin{matrix} \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \\ \oplus \\ \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \end{matrix} \longrightarrow \begin{matrix} \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \\ \oplus \\ \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \end{matrix}$$

and

$$\sigma_1: \begin{matrix} \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \\ \oplus \\ \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \end{matrix} \longrightarrow \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}$$

Proposition (3.3):

$$0 \longrightarrow \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \xrightarrow{\sigma_3} \begin{matrix} \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \\ \oplus \\ \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \end{matrix}$$

$$\xrightarrow{\sigma_2} \begin{matrix} \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \\ \oplus \\ \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \end{matrix} \xrightarrow{\sigma_1} \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}$$

Is complex.
Proof:
From the acquaint, it is known that ∂_{21} and ∂_{32} are injective [12], then we get σ_3 is injective by Capelli identities (1.1-1.4).
Now
$(\sigma_2 \circ \sigma_3)(x) = \sigma_2(d_1(x), h_1(x))$
$= \sigma_2(\partial_{32}(x), \partial_{21}(x))$
$= \left(d_2(\partial_{32}(x)) - w(\partial_{21}(x)), g_1(\partial_{21}(x)) - h_2(\partial_{32}(x))\right)$

$d_2(\partial_{32}(x)) - w(\partial_{21}(x))$
$= \left(\frac{1}{2}\partial_{21} \circ \partial_{32} + \partial_{31}\right) \circ \partial_{32}(x) - \partial_{32}^{(2)} \circ \partial_{21}(x)$
$= (\partial_{21} \circ \partial_{32}^{(2)} + \partial_{31} \circ \partial_{32} - \partial_{32}^{(2)} \circ \partial_{21})(x)$
$= (\partial_{32}^{(2)} \circ \partial_{21} - \partial_{32} \circ \partial_{31} + \partial_{31} \circ \partial_{32} - \partial_{32}^{(2)} \circ \partial_{21})(x)$
$= 0$

$g_1(\partial_{21}(x)) - h_2(\partial_{32}(x))$
$= \left(\frac{1}{2}\partial_{32} \circ \partial_{21} - \partial_{31}\right) \circ \partial_{21}(x) - \partial_{21}^{(2)} \circ \partial_{32}(x)$
$= (\partial_{32} \circ \partial_{21}^{(2)} - \partial_{31} \circ \partial_{21} - \partial_{21}^{(2)} \circ \partial_{32})(x)$
$= (\partial_{21}^{(2)} \circ \partial_{32} + \partial_{21} \circ \partial_{31} - \partial_{31} \circ \partial_{21} - \partial_{21}^{(2)} \circ \partial_{32})(x)$
$= 0$

We reign $(\sigma_2 \circ \sigma_3)(x) = 0$
And
$(\sigma_1 \circ \sigma_2)(x_1, x_2) = \sigma_1(d_2(x_1) - w(x_2), g_1(x_2) - h_2(x_1))$
$= \sigma_1((\frac{1}{2}\partial_{21} \circ \partial_{32} + \partial_{31})(x_1) - \partial_{32}^{(2)}(x_2), (\frac{1}{2}\partial_{32} \circ \partial_{21} - \partial_{31})(x_2) - \partial_{21}^{(2)}(x_1))$
$= \partial_{21} \circ (\frac{1}{2}\partial_{21} \circ \partial_{32} + \partial_{31})(x_1) - \partial_{32}^{(2)}(x_2)) + \partial_{32} \circ ((\frac{1}{2}\partial_{32} \circ \partial_{21} - \partial_{31})(x_2) - \partial_{21}^{(2)}(x_1))$
$= (\partial_{21}^{(2)} \circ \partial_{32} + \partial_{21} \circ \partial_{31} - \partial_{32} \circ \partial_{21}^{(2)})(x_1) + (\partial_{32}^{(2)} \circ \partial_{21} - \partial_{32} \circ \partial_{31} - \partial_{21} \circ \partial_{32}^{(2)})(x_2)$
$= (\partial_{32} \circ \partial_{21}^{(2)} - \partial_{21} \circ \partial_{31} + \partial_{21} \circ \partial_{31} - \partial_{32} \circ \partial_{21}^{(2)})(x_1) + (\partial_{32}^{(2)} \circ \partial_{21} + \partial_{32} \circ \partial_{31} - \partial_{32} \circ \partial_{31} - \partial_{21} \circ \partial_{32}^{(2)})(x_2)$
$= 0$ ∎

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