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# Effectiveness of Image Curvelet Transform Coefficients for Image Denoising

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**Abstract**

In this research, we investigate the effect of image curvelet transform coefficients in image denoising. The curvelet transform applies to input images, resulting in a set of curvelet coefficients that capture different frequency and directional components of the image. To improve the denoising process, we introduce an approach (CTRLC: Curvelet Transformation Remove Least Correlation) based on the correlation between the abstract coefficient and other coefficients. By analyzing the correlation values, we identify the coefficient that is least associated with the abstract coefficient and remove it from the transformed image. This selective removal allows us to attenuate noise while preserving the relevant image information. Experimental evaluations are conducted on a variety of images contaminated with different levels of noise. The results show that our proposed method effectively reduces noise and enhances the image quality. Comparative analyses with existing denoising techniques further validate the superiority of our approach in terms of noise reduction and preservation of important image details. The CTRLC method achieved a PSNR of 87.2695, compared to other methods that ranged between 23.43 and 77.5. This confirms the effectiveness of our proposed approach in image restoration after denoising. The findings of this research contribute to the field of image denoising by highlighting the significance of curvelet transform coefficients and the correlation-based coefficient removal technique. The proposed method offers a solution for effectively reducing noise in images while maintaining their visual integrity.

***Keywords***:
Images noise, curvelet transform, images coefficient, image processing, correlation coefficient.

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## I. Introduction

Image denoising is a crucial task in image processing that aims to remove unwanted noise while preserving important image details. In the images, noise is presented because of the measurement techniques and instrumentation. Image denoising is widely used in various fields of image processing and plays an important role in enhancing the quality of images. The main objective of denoising is to remove the noise from the image and restore a clean image[1]. Image denoising is removing various types of noise or distortions from an image while preserving its edges and other features. This is achieved using mathematical transformations such as the Curvelet transform, which has shown to be effective in representing the edges of an image due to its anisotropic and vectorial properties [2].

There are many denoising methods available that stem from various disciplines, including probability theory, statistics, partial differential equations, linear and nonlinear filtering, and spectral and multiresolution analysis. These methods are based on explicit or implicit assumptions about the underlying noise-free signal, aiming to effectively separate it from random noise. Transform-domain denoising methods, in particular, assume that the true signal can be well approximated by a sparse representation in the transform domain, typically a linear combination of a few basis elements. By preserving the high-magnitude transform coefficients that primarily carry the energy of the true signal and discarding the remaining coefficients associated with noise, an effective estimation of the true signal can be achieved. The sparsity of the representation depends on both the characteristics of the transform and the true signal. Multiresolution transforms excel at achieving sparsity for spatially localized details, such as edges and singularities, which are abundant in natural images and contain significant information [3].

The Curvelet transform is a modern approach for manipulating and retrieving images, which involves analyzing images at different scales and segmenting them into squares whose size depends on the scales used. This transformation is a multi-scale transformation with strong directional sensitivity. The main advantage of the Curvelet transform is its sparsity property, which represents smooth curve points except for the discontinuities along the curve with limited curvature. This representation is more effective than the wavelet transform for segmented objects and scattered representations for non-segmented objects [4][5].

The motivation behind employing curvelet transform as a tool for image denoising stems from its inherent ability to effectively capture and represent image features at multiple scales and orientations[4].

In this paper, we investigate the effect of image Curvelet transform coefficients in image denoising. Specifically, we decompose images into their coefficients using the Curvelet transform, exclude some coefficients (frequencies), and reconstruct the image. We then measured the differences between the original and reconstructed images using various metrics, including SNR, PSNR, MSE, CORR, and Energy, to study the effect of the excluded frequencies on the image.

To provide context, in Section 2, we summarize previous studies related to image denoising techniques and the use of Curvelet transform coefficients. Section 3 presents a brief explanation of the Curvelet transform and its application in image denoising and the Methods for measuring the efficiency of recovered images. In Section 4, we state the method formulation and the proposed algorithm that incorporates the coefficient removal strategy. Section 5 present the experimental results and analyzes the impact of different Curvelet transform coefficients on the denoising performance. Finally, in Section 6, we provide conclusions and discuss the implications of our findings for future research in the field of image denoising.

## 2. Literature review

Starck, Candès and Donoho[6], proposed a method for image denoising using Curvelet thresholding, which involved thresholding the Curvelet coefficients in each scale and direction. The study conducted by N. Zahra and A.H. Siddiqi [7], was to investigate investigated the application of the curvelet transform for denoising magnetic resonance imaging (MRI) images. The researchers proposed a denoising method based on the curvelet transform, which is a multiscale transform that can effectively capture the edges and textures of an image. The proposed method consisted of three steps: (1) decomposing the noisy image into curvelet coefficients, (2) applying a threshold to the curvelet coefficients to remove the noise, and (3) reconstructing the denoised image from the thresholded curvelet coefficients. To evaluate the performance of the proposed method, the researchers conducted experiments on both simulated and real MRI images. The results showed that the proposed method outperformed several other denoising methods in terms of peak signal-to-noise ratio (PSNR) and mean square error (MSE).

In study [8], the curvelet analysis method was employed for denoising fetal ultrasound images. The denoising process began with transforming the speckle-affected images into a new space. Within this space, curvelet coefficients with high signal-to-noise ratios were retained, while those with low ratios were reduced. Subsequently, the manipulated coefficients were converted back to the original space to generate the despeckled images. The curvelet decomposition process comprised subband decomposition, smooth partitioning, and renormalization to ensure optimal scale representation. Additionally, ridgelet analysis was conducted on each block using the digital ridgelet transform. By setting a threshold, noisy curvelet coefficients were eliminated. This iterative process concluded with the inverse curvelet transform, known for its efficiency in image reconstruction because of its requirement of fewer nonzero coefficients.

The study by T. Janardhan Reddy [9]. proposed a denoising method for medical images based on the curvelet transform that preserves edges. The proposed method comprised two main steps: (1) decomposing the noisy image into curvelet coefficients and applying a threshold to the coefficients to remove the noise, and (2) reconstructing the denoised image from the thresholder curvelet coefficients while preserving the edges using an edge-preserving function. To evaluate the performance of the proposed method, the researcher conducted experiments on both simulated and real medical images. The results showed that the proposed method outperformed several other denoising methods in terms of peak signal-to-noise ratio (PSNR) and visual quality.

The study [10] proposed a method for enhancing contrast in cone beam CT (CBCT) images using fast discrete curvelet transforms (FDCT) via Unequally Spaced Fast Fourier Transform (USFFT). By manipulating the Curvelet coefficients got through FDCT-USFFT and applying thresholding, the proposed method significantly improves CBCT image quality.

The study [11]. proposed a method for constructing high-pass filters based on curvelet transform coefficients. It first applied the curvelet transform to the input signal to get the curvelet coefficients, then used a thresholding technique to remove the low-frequency coefficients and keep the high-frequency coefficients. Finally, reconstructed the signal using the inverse curvelet transform to get the high-pass filtered signal. To evaluate the performance of the proposed method, the researchers conducted experiments on a dataset of signals. The study [12] introduces a denoising method for positron emission tomography (PET) images, which combines wavelet and curvelet transforms to address noise and low spatial resolution. By integrating adaptive threshold selection, the proposed approach handles both isotropic and anisotropic features.

The paper [13]. presents a method for removing speckle noise from optical coherence tomography (OCT) images. The proposed method is based on three-dimensional (3D) curvelet transform and dictionary learning, and it is shown to outperform other state-of-the-art methods in terms of quantitative and qualitative evaluation metrics. The results suggest that the proposed approach could be a useful tool for improving the diagnostic accuracy of OCT images in clinical settings.

The study [14] explores denoising techniques for multispectral remote sensing images using curvelet transform with both wrapping function and Unequally Spaced Fast Fourier Transform (USFFT) for grid construction. The method defines curvelet coefficients by scale, angle, and spatial location. The research was aimed at denoising Linear Imaging Self Scanning Sensor (LISS) III images.

## 3. Background

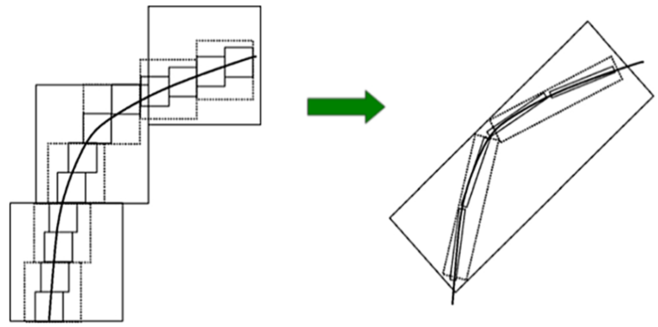
### 3.1. The Curvelet Transform and its application in image denoising

The curvelet transform is a mathematical tool used for signal processing and image analysis. It is useful for analyzing data with curvilinear features, such as edges or curves. The transform works by decomposing a signal into a set of curvelets, which are small, curved line segments that capture the signal's local behavior. By decomposing the signal into curvelets, the transform can capture both the global and local features of the signal [15].

The Curvelet Transform is a multiscale transform that has shown great promise in image processing applications, particularly in denoising images with directional features. The Curvelet Transform uses a combination of wavelets and a curvelet transform to capture both local and directional image features. This allows it to outperform other methods in denoising images with anisotropic features, such as edges, curves, and corners.

In image denoising, the Curvelet Transform is typically used in combination with thresholding techniques to remove noise while preserving image details. Several studies have showed the effectiveness of the Curvelet Transform in image denoising, including, The redundant version of Curvelet Transform provides improved denoising performance [16][17][18].

Curvelet handles noteworthy occurrences that happen along curved edges present in a 2D image. As depicted in **Fig. 1** and **2**, the curvelet requires fewer coefficients for representation and produces an edge that is smoother compared to the edge produced by wavelet [19].

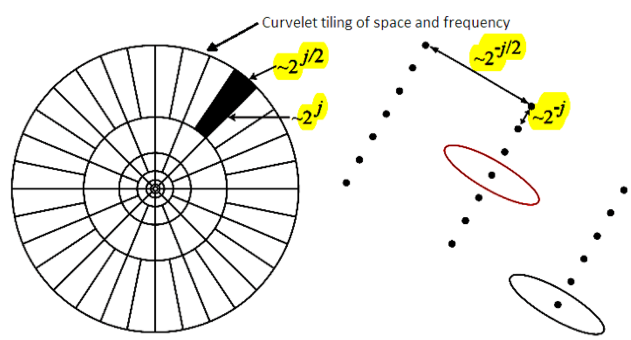


**(a)** **(b)**

**Fig. 1.** An approximating comparison between wavelet (a) and curvelet (b)[19].

The spatial field in **Fig. 2** (a) on the left side shows all the wedge piles that correspond to a specific kerflet (oval shape in the drawing) at a specific scale and a specific angle. This shows that if the inverse of the fast Fourier transform is taken for a specific pile, this will determine the kerflet coefficient at that scale and that angle. This is the main idea behind implementing a curvelet. The **Fig. 2** (b) on the right represents a curvelet in a spatial Cartesian grid at a certain scale and angle.

The multi-scale pyramid of Curvelet transform comprises shell components that are organized based on area, scale, and direction parameters, including peak components of fine scales, as noted in [7]. Curvelets exhibit temporal frequency properties similar to wavelet waves, but also possess high directivity and anisotropy. These properties can be approximated with a minimal number of coefficients, making Curvelets highly efficient [20].




**(a)** **(b)**

**Fig. 2.** Curvelet tiling of space and frequency (a).The spatial Cartesian grid associated with a given scale and orientation (b) [4].

The wrapping curvelet transform of fourier samples takes the 2D image as input in the form of a Cartesian matrix f[m,n] such that $0 < m < M$, $0 \leq n < N$ and constructs a number of curvelet coefficients indexed by the measure j, and the orientation coefficient l, and two position variables, Spatial (k1, k2) as output. Statistical operations are performed on the coefficients to form the tissue description using curvelet, and it is possible to define the discrete curvelet coefficients from

Equation 1 as [4]:

$$C^D(j,l,k1,k2) = \sum_{0\le m\le M}\sum_{0\le n\le N} f[m,n]\emptyset^D_{j,l,k1,k2}[m,n] \ldots\ldots\ldots (1)$$

Where $\emptyset^D_{j,l,k1,k2}$ :It is a digital curvelet representing the wave shape, the exponent D indicates that it is digital. This method efficiently implements a "parabolic scaling law" on subbands in the frequency domain to effectively or more efficiently detect curved edges within the image.

### 3.2. Methods for measuring the efficiency in denoising images

The standard methods used on digital images in this research focused on calculating the quality of the resulting image. The most important of these metrics are:

Signal to Noise Ratio (SNR)

$$SNR = 10\log_{10}\left[\frac{\sigma_x^2}{\sigma_z^2}\right] \cdots\cdots\cdots (2)$$

$\sigma_x^2$ It is the average square of the image.

$\sigma_z^2$ It is the difference between the mean squared of the original and recalled image[21][22][23].

- PSNR peak to noise ratio

$$PSNR = 10\log_{10}\left(\frac{R^2}{MSE}\right) \cdots\cdots\cdots (3)$$

R represents the largest value in the original image data.
MSE square error of image data.[24][25][26].

- MSE (Mean Sequence Error)

$$MSE = \frac{\sum_{m,n}[I1(m,n) - I2(m,n)]^2}{M * N} \cdots\cdots\cdots (4)$$

I1(m,n) represents the original image.
I2(m,n) represents the recovered image.[27][26][28].

- correspondence between the original image and the image retrieved Correlation[27][29].

$$corr = \frac{\sum_m\sum_n(A_{mn}-\bar{A})(B_{mn}-\bar{B})}{\sqrt{(\sum_m\sum_n(A_{mn}-\bar{A})^2)(\sum_m\sum_n(B_{mn}-\bar{B})^2)}} \cdots\cdots\cdots (5)$$

Where:

A- Original image.

B- Recalled image

$\bar{A}$ - Represents the average for A
$\bar{B}$- Represents the average for B

- Energy

$$Energy = \sum^{m}\sum^{n} p^2\,[i,j] \cdots\cdots\cdots (6)$$

Where:

P[i,j]: sum the squares of all elements

Energy: Returns the sum of the elements in the energy box determined by the number of duplicate image-point pairs, and it is expected that the energy will be high if there are a large number of image point pair[24][29].

## 4. Method

To evaluate the performance of the proposed image denoising method, four different images were selected: Barbara.jpg (gray scale image from MATLAB images, 512x512 pixel spatial resolution, referred to as IM1), Cameraman (gray scale image from MATLAB images, 256x256 pixel spatial resolution, referred to as IM2), Koala (color image from Windows images, 1024x768 pixel spatial resolution, referred to as IM3), and Penguins (color image from Windows images, 1024x768 pixel spatial resolution, referred to as IM4). The images were in JPEG format, except for Cameraman, which was in TIFF format. Before analysis, all images were resized to a uniform size of 512x512 pixels to reduce computational costs and improve the efficiency of image processing. Additionally, color images were converted to grayscale for further analysis. The goal is to use the Curvelet coefficient deletion method that has the least correlation with the image abstract, in order to refine the image and remove noise from it.

The image deconstruction process involved applying the curvelet transform at two levels to each image. The curvelet transform decomposed images into multiple coefficients organized in cells. Subsequently, the correlation coefficient was calculated between each coefficient of the transformed image cells and the coefficient of the first cell by assessing the correlation values, the cell with the lowest correlation coefficient with the first cell was identified for removal.

After removing the identified cell, the images were reconstructed using the inverse curvelet transform. This reconstruction aimed to restore the images by integrating the remaining coefficients. The quality and clarity of the reconstructed images were evaluated by comparing them with their original counterparts. The convergence between the original and reconstructed images was measured using multiple metrics, including signal-to-noise ratio (SNR), peak signal-to-noise ratio (PSNR), mean square error (MSE), correlation coefficient (CORR), and energy.

To implement the proposed algorithm, MATLAB programming language was utilized, enabling efficient execution and analysis of the denoising process. The MATLAB program facilitated image reading, grayscale conversion, scaling, curvelet transform application, correlation coefficient calculation, coefficient removal, inverse curvelet transform, and metric calculation, enabling comprehensive evaluation of the proposed method.

This methodology ensured a systematic approach to

*Special description of the title. (dispensable)

assess the performance of the image denoising method, allowing for accurate comparisons between the original images, denoised images, and the images recovered after coefficient removal.
Proposed algorithm steps:

1. Read the image.
2. Convert the image to gray-scale.
3. Perform preliminary processing by scaling the image to a size of 512x512 pixels.
4. Apply the Curvelet transform to the image, decomposing and analyzing it into several coefficients.
5. Calculate the correlation coefficient between the first cell (the abstract) and the coefficients of the other cells, each separately.
6. Find the cell with the lowest correlation coefficient with the first cell.
7. Delete the coefficients of the cell with the lowest correlation with the first cell.
8. Perform inverse Curvelet transform on the remaining image coefficients to reconstruct the image.
9. Calculate the following measures (SNR, PSNR, MSE, CORR, and ENERGY) between the original image, the denoised image, and the recovered image.

The Matlab R2018a program was used to implement the proposed algorithm.

## 5. Results and discussion

In this research, the performance of the proposed image denoising method based on the curvelet transform and coefficient removal strategy was evaluated. The experiments were conducted on a set of images (IM1, IM2, IM3, and IM4) using a two-level curvelet transform, as shown in **Fig. 3**. The images were decomposed into coefficients, and the coefficient with the least correlation with the abstract coefficient for all images was selectively removed. The images were then reconstructed, and various evaluation metrics were calculated to assess the quality of the denoised images.

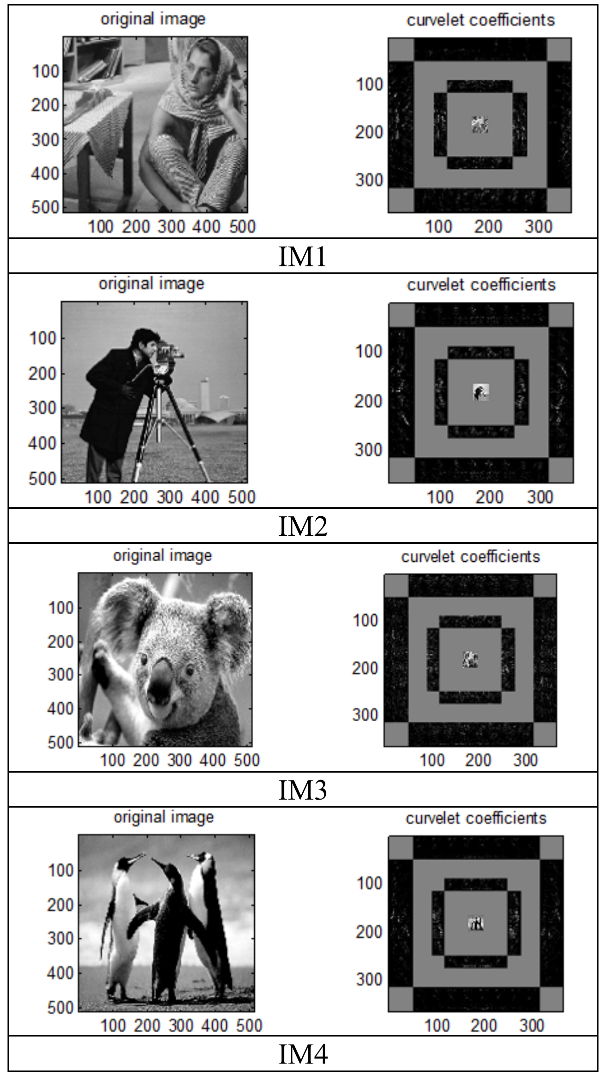



**Fig. 3.** Represented the images (IM1, IM2, IM3 and IM4) deconstructed to two levels.

The obtained results, presented in Table (1), provide insights into the effectiveness of the proposed method.

**Table 1.** Represented the metrices values (SNR, PSNR, MSE, CORR and ENERGY) when deconstructed the images (IM1, IM2, IM3 and IM4) to two levels.

| **Images** | **SNR** | **PSNR** | **MSE** | **COR** | **ENERGY** |
|---|---|---|---|---|---|
| **IM1** | 23.2963 | 75.7224 | 0.1422 | **0.9898** | 1.9378 |
| **IM2** | 24.7372 | 78.1748 | 0.1072 | **0.9942** | 1.9398 |
| **IM3** | 27.0932 | 80.6979 | 0.0802 | **0.9968** | 1.9429 |
| **IM4** | 34.2149 | 87.2695 | 0.0376 | **0.9993** | 1.9462 |

The metrics, including signal-to-noise ratio (SNR), peak signal-to-noise ratio (PSNR), mean square error (MSE), correlation coefficient (CORR), and energy, were calculated between the original images and the recovered images, upon deconstructing images (IM1, IM2, IM3, and IM4) into two levels. It is evident from the table that as the image complexity increases, there is a noticeable improvement in all evaluated metrics. Specifically, the PSNR values demonstrate a substantial increase from IM1 to IM4, indicating enhanced image quality and fidelity. Additionally, the MSE values decrease consistently across the images, indicating reduced distortion levels. Moreover, the correlation (CORR) values approach unity as the images become more refined, indicating a stronger linear relationship between the original and reconstructed images. Furthermore, the ENERGY values also exhibit a steady increase, reflecting higher image clarity and information preservation. These findings collectively suggest the effectiveness of the deconstruction process in enhancing image quality and fidelity, particularly evident in the remarkable PSNR improvement observed in IM4. Additionally, graphical charts, as shown in **Fig. 4**, were generated to visualize the patterns observed in the images when decomposed to two levels.

From the analysis of the results, several interesting observations can be made. First, it is evident that as the number of levels in the curvelet transform increases, the metrics such as correlation coefficient, SNR, PSNR, and energy also increase. This indicates that the proposed method successfully preserves and enhances the important features and details in the images, resulting in improved image quality and information content.

Furthermore, the decreasing trend in the mean square error (MSE) as the number of levels increases highlights the method's ability to effectively reduce noise and faithfully reconstruct the underlying image structure. The higher affinity ratio of 99% between the recovered images and the original images further supports the accuracy and efficiency of the adopted method in restoring the image details.

These findings suggest that the proposed method based on the curvelet transform and coefficient removal strategy holds great potential for image denoising applications. However, it is important to note that the effectiveness of the method may vary depending on different image characteristics and noise levels. Further research and experimentation are necessary to validate the robustness and generalizability of the proposed method.

Overall, the results obtained from this study provide valuable insights into the performance and capabilities of the proposed method, contributing to the field of image denoising and opening avenues for future research and advancements in this area.

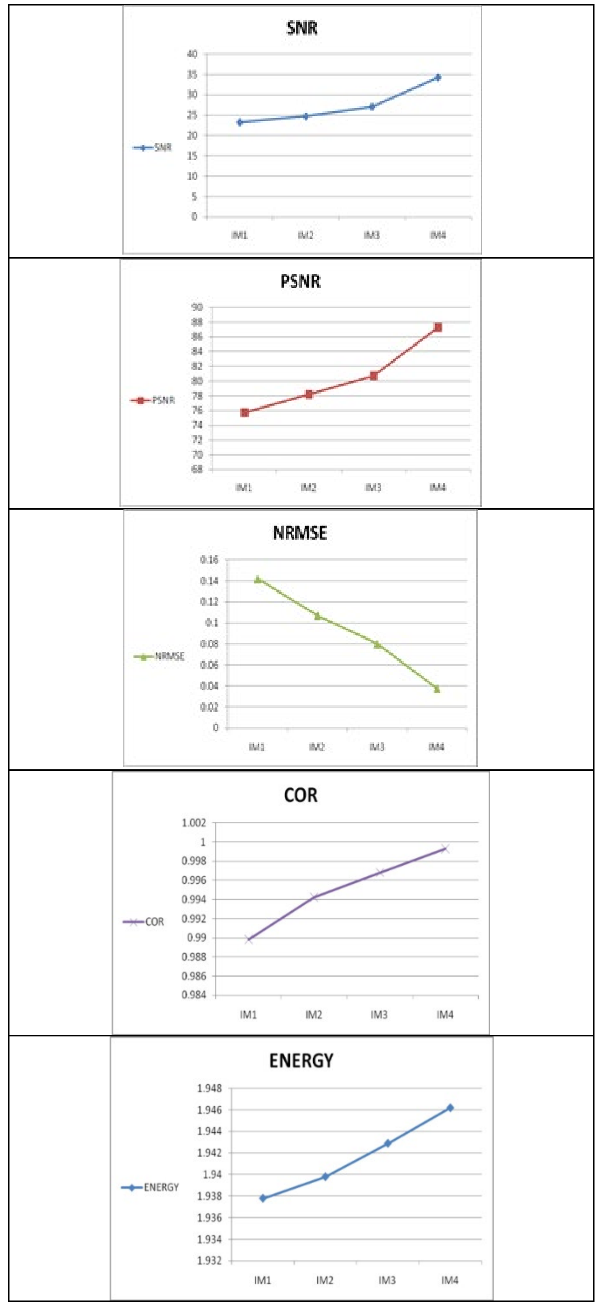



**Fig. 4.** Represented the graphical charts of the images (IM1, IM2, IM3, and IM4) when deconstructed to two levels.

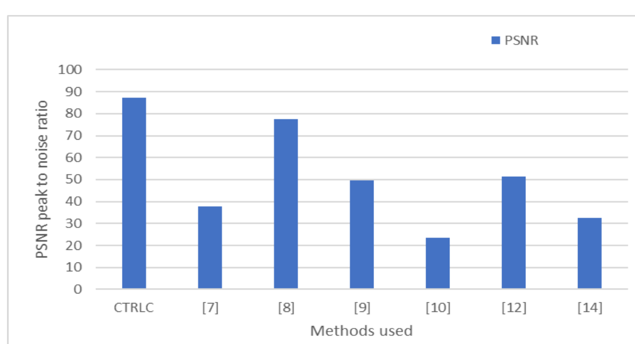



**Fig. 5.** Comparison with the methods used.

**Fig. 5** shows the comparison of PSNR values between various state-of-the-art methods along with the proposed CTRLC method for efficient image denoising. The results show that the PSNR achieved by the CTRLC approach was significantly higher compared to other methods. Specifically, the PSNR values of other methods ranged from 23.43 to 77.5, while the PSNR of our CTRLC method reached 87.2695, indicating the efficient performance of CTRLC in image restoration after noise removal.

## 6. Conclusions

Based on the findings of this research, it can be concluded that the proposed method, which involves applying the curvelet transform and selectively removing coefficients based on correlation, has shown promising results in image denoising. The analysis of the obtained results provides valuable insights into performing the method.

The comparison of various measures, including the correlation coefficient, signal-to-noise ratio (SNR), peak-to-noise ratio (PSNR), energy ratio, and mean square error (MSE), between the original images and the denoised images reveals interesting trends. As observed from **Fig. 4**, the values of the correlation coefficient, SNR, PSNR, and energy increase as the number of levels in the curvelet transform increases. This shows that the method effectively preserves and enhances the essential information in the images, leading to improved quality and clarity.

Furthermore, the decreasing trend in the MSE as the number of levels increases demonstrates the ability of the method to reduce noise and improve image fidelity. The affinity ratio of 99% between the recovered images and the original images underscores the high level of similarity achieved through the proposed method, indicating its efficiency in accurately restoring the underlying image structure.

These findings highlight the potential of the curvelet transform and the coefficient removal strategy in image denoising applications. However, it is important to note that further investigations and experiments are necessary to validate the generalizability and robustness of the method across different image types and noise levels.

In conclusion, the results got from this research provide valuable insights into the effectiveness of the proposed method, contributing to the field of image denoising and paving the way for future advancements in this area.

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