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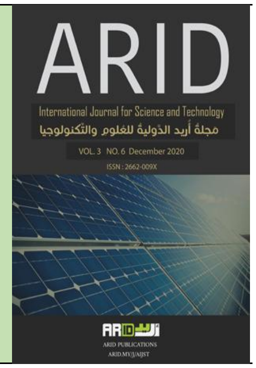

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RETRACTING 1-SIMPLICIAL CHAOTIC GRAPHS WITH DENSITY VARIATION

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تقلـص الكثافة للمخططات البسيـطة الفوضـويـة (المشـوّشـة)

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ABSTRACT

In this paper we will discuss retraction transformation on chaotic graphs for different cases of density variation, Two types of retraction will be discussed, geometric retraction and chaotic edges retraction, We shall study and discuss the effects of retraction on the shape and density degree of chaotic graphs shown on figures, the adjacent and incidence matrices with be presented. The density character may present many applications in life such as degree of green color in plants, net perturbation resonance, signals in the nerve system and so many, we focus our study on plants, how retraction effects of the degree of green color of plants leaves and shape of the leave its self.

The variation of the density character shows the variation of the degree of green color of plant (i.e. Chlorophyll), so we divided in three cases, the first case when the leave of plant is unit and constant everywhere, or varied from level to level or varied even in the same level line of a plant leave, each case will be discussed for two types of retraction and deduce results for both types.

Keywords: adjacent matrix, chaotic graph, density, geometric graph, incidence matrix, retraction.

الملـخــص

في هذا البحث سوف يتم مناقشة نوع من أنواع التطبيقات الهندسية التي تجري على المخططات الفوضوية (المشوّشة) التي تحمل خاصية الكثافة، وهذا البحث يدرس تأثير التقلص على أوراق النباتات بمعنى أنه كيف يمكن للتقلص أن يؤثر على نسبة مستوى كثافة وشكل هذه المخططات. سوف يتم مناقشة نوعين من التقلص (التقلص الهندسي وتقلص المخططات المشوشة) وسوف يتم استنتاج المصفوفة الملاصقة ومصفوفة السقوط لمعظم هذه المخططات.

مصطلح الكثافة في المخططات المشوّشة يمثل العديد من التطبيقات في الحياة فقد يمثل تنوع الكثافة في المخططات المشوشة تغاير اللون الأخضر في أوراق النبات (نسبة لمادة الكلوروفيل) وقد تمثل الإشارات العصبية في الجهاز العصبي أو من الممكن أن تمثل إشارات إرسال شبكات الإنترنت .

سيتم تمثيل أوراق النبات بمخطط مشوش وإجراء تطبيق التقلص بنوعيه عليه واكتشاف تأثيراته على هذا المخطط وإن هذا البحث سوف يدرس هذا التطبيق من ناحية التبولوجي الهندسي.

الكلمـات المفتاحيـة: المصفوفة الملاصقة، المخطط الفوضوي (المشوش)، الكثافة، المخطط الهندسي، مصفوفة السقوط، تقلص.

1. INTRODUCTION

There are many physical systems whose performance depends not only on the characteristics of the components but also on the relative locations of the elements. An obvious example is an electrical network. If we change a resistor to a capacitor, some of the properties (such as an input impedance of the network) also change. This indicates that the performance of a system depends on the characteristics of the components. If, on the other hand, we change the location of one resistor, the input impedance again may change, which shows that the topology of the system is influencing the system's performance [1].

One simple way of displaying a structure of a system is to draw a diagram consisting of points called "vertices" and line segments called "edges" which connect these vertices so that such vertices and edges indicate components and relationships between these components such a diagram is called a "Linear graph" whose name depends on the kind of physical system we deal with, this means that it may be called a network, a net, a circuit, a graph, a diagram, a structure, and so on [1]

The generalization of this graph is the “fuzzy graph” and the most generalization of these graphs is the “chaotic graph”, which applied in many uncertain circuits, resonance, perturbation theory and many other medical applications. More advanced applications using the more complicated graphs are the chaotic graphs [1 ,2 ,3, 4]

1.1.Definitions and backgrounds

- Adjacency and incidence: Let v and w be vertices of a graph, if v and w are joined by an edge e, then v and w are said to be adjacent. Moreover, v and w are said to be incident with e, and e is said to be incident with v and w [5, 6].

- The adjacency matrix: Let G be a graph without loops, with n-vertices labeled $1,2,3,...,n$ the "adjacency matrix" $A(G)$ is the $n \times n$ matrix in which the entry in row i and column j is the number of edges joining the vertices i and j [5, 6].

- The incidence matrix: Let G be a graph without loops, with n-vertices labeled $1,2,3,...,n$ and m- edges labeled $1,2,3,...,m$.The" incidence matrix" $I(G)$ is the $n \times m$ matrix in which the entry in row i and column j is 1 if vertex i is incident with edge j and 0 otherwise [5, 6].

Example:

Consider the graph G in Figure (1):

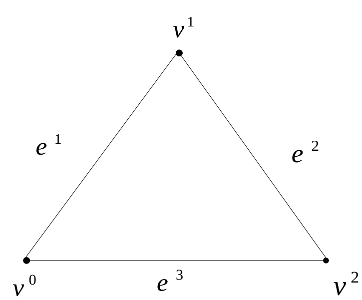

Figure (1): Simple graph

The adjacency matrix $A(G)$ is: $A(G) = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

And its incidence matrix $I(G)$ is: $I(G) = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

- Loop: A loop is an edge which starts and ends on the same vertex [3].

- Multiple edges: Tow or ore edges joining the same pair of vertices are called "multiple edges" [3].
- Simple graph $G(V,E)$: is a graph with no loops or multiple edges [4].
- Null graph: is a graph consists of a set of vertices and no edges [5].
- Chaotic graph $G_h(V_h,E_h)$: is a geometric graph that carries many physical characters, these geometric graphs might have similar properties or different [5, 7, 8].
- Density (d): is a physical property of matter, as each element and compound has a unique density associated with it [1].
- 1-simplicial chaotic graph: is a graph consists of (v_{0h}^0, v_{0h}^1) vertices the geometric edge e_{0h}^1 and smooth chaotic edges e_{ih}^1 , $i=1,2,3,...,\infty$ with no loops or multiple edges (Figure (2)) [1, 6, 7, 8, 9].

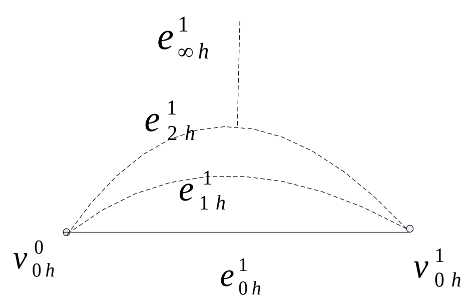

Figure (2): 1-simplicial chaotic graph

This paper the physical character is presented by density, the density may represent the degree of green color of a leave of a plant (Chlorophyll degree), so the density might be constant everywhere or vary from one place into another place. There are three cases, the first case is when all chaotic edges have the same physical characters (i.e. fixed density) such that all e_{ih}^1 , $i=1,2,3,...,\infty$, has fixed density; for example the color of a plant leaves is a perfect green (constant and unit), and the second case is when chaotic edges and the geometric edge have various densities such that e^1_{0h} represent degree 1 of green color, e^1_{1h} represent degree 2 of green color,…and so on; the third case when even each chaotic level has various densities, so each area of the plant leave has a different degree of green [1]. The adjacent and incidence matrix for each case can be obtained easily.

Generally; consider 1-simplicial geometric graph G without loops, with 2-vertices (v^0, v^1) and one geometric edge (e), it's adjacent and incidence matrices are respectively:

$$A(G)=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}, I(G)=\begin{bmatrix}1\\ 1\end{bmatrix}.$$

Now, considers the chaotic graph $G_h(v^0_{0h}v^1_{0h})$, this graph consists of the geometric edge e^1_{0h} and smooth chaotic edges $e^1_{ih}, i=1,2,3,...,\infty$.The adjacent and incidence matrices of this chaotic graph whatever the variation of the density are:

$$A(G_h)=\begin{bmatrix}0_{(012..\infty)h} & 1_{(012..\infty)h}\\ 1_{(012..\infty)h} & 0_{(012..\infty)h}\end{bmatrix}, I(G_h)=\begin{bmatrix}1_{(012..\infty)h}\\ 1_{(012..\infty)h}\end{bmatrix}.$$

So the adjacent and incidence matrices of 1-simplicial geometric graph are different to the adjacent and incidence matrices of 1-simplicial chaotic graph.

There are several transformations run on chaotic graph such as folding, unfolding and retraction. In previous study [1] the case of folding transformation impact on chaotic graph with density variation is discussed and indicated that the topological folding increases the density of the graph and reduces the length of the graph. The limit of successive folding a vertex into another vertex is a geometric vertex overlapped on by different chaotic loops and each loop has its own density characters, while folding chaotic edges induces a geometric graph with their basic edges, and we deduce that density increases

Similarly, we shall do the same steps on retraction transformation; we shall discuss the impact of retraction on the density character and the shape of chaotic graphs.

2. Retraction transformation on chaotic graphs with density character

Basically, the idea of retraction (r) is simply is a subset A of a topological space X is called "retract "of X if there exists a continuous map $r : X \to A$ (called a retraction), such that $\{r(a) = a, \forall a \in A\}$, where A is closed and X is open. In other words, a retraction is a continuous map of a space into a subspace leaving each point of the subspace fixed [10].

The application of retraction can be applied on a leave of a plant, so the retracted cells of the leave presents the subspace A, while the whole leave presents the topological space X [8, 9].

In our research two types of retractions will be applied on chaotic graph with density variation which is geometric retraction and physical chaotic character retraction (i.e. chaotic edges retraction).

2.1 Geometric retraction

Basically, the idea of geometric retraction is simply defined by this relation:

$$r_i \left(G_h - \{ v_{0h}^i \} = \{ \begin{matrix} v_{0h}^0 & , i=1 \\ v_{0h}^1 & i=0 \end{matrix} \} \right) \to G_h^1$$

Since 1-simplicial chaotic graph $G_h^1 = \{v^0 v^1\} = \{\lambda v^0 + (1-\lambda) v^1 : \lambda \geq 0\}$ is open, and then $G_h^1 - \left\{ v^i : i = 0,1 \right\}$ is open and its limited final retraction results a chaotic vertex at v_{0h}^0 or v_{0h}^1, which is the null graph of chaotic graph (i.e. one chaotic vertex).This leads to a change of the shape of the graph and a change in its adjacent and incidence matrices (Figure (3)) .

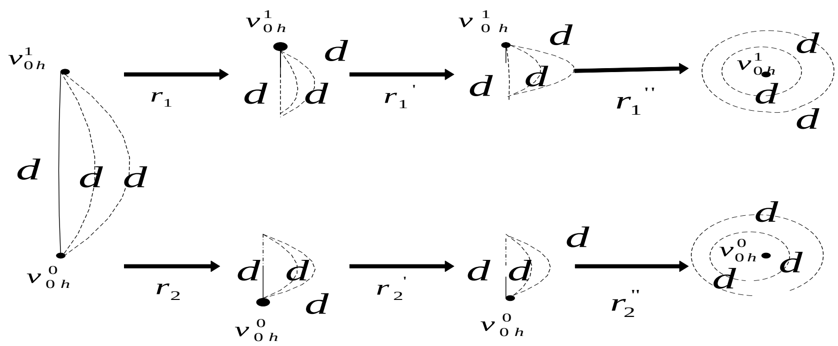

Figure (3): Geometric retraction of 1-simplicial chaotic graph with density variation

The adjacent and incidence matrices of 1-simplicial chaotic graph before applying retractions are:

$$A(G_h)=\begin{bmatrix}0_{(012..\infty)h} & 1_{(012..\infty)h}\\ 1_{(012..\infty)h} & 0_{(012..\infty)h}\end{bmatrix}, I(G_h)=\begin{bmatrix}1_{(012..\infty)h}\\ 1_{(012..\infty)h}\end{bmatrix}$$

And the adjacent and incidence matrices for the resulted graph are respectively:

$$A(G_h^r)=\left[0_{(012..\infty)h}\right], I(G_h^r)=\phi$$

So they are different to each other, since the resulted graph is the null chaotic graph.

The affection of geometric retraction on 1-simplicial chaotic graph on the density characterization can understand it via examples.

1- Consider a 1-simplicial chaotic graph with constant density, where the geometric edge and the chaotic levels have a constant density $(d=1/2)$.

The geometric retraction changes the shape of the graph from 1-simplicial chaotic graph of constant density into chaotic vertex, the chaotic edges keeps the same density as before retraction, while the resulted graph loses its geometric edge with its density character, so the total density of the resulted graph will be reduced as the chaotic graph loses its geometric edge (Figure (4)).

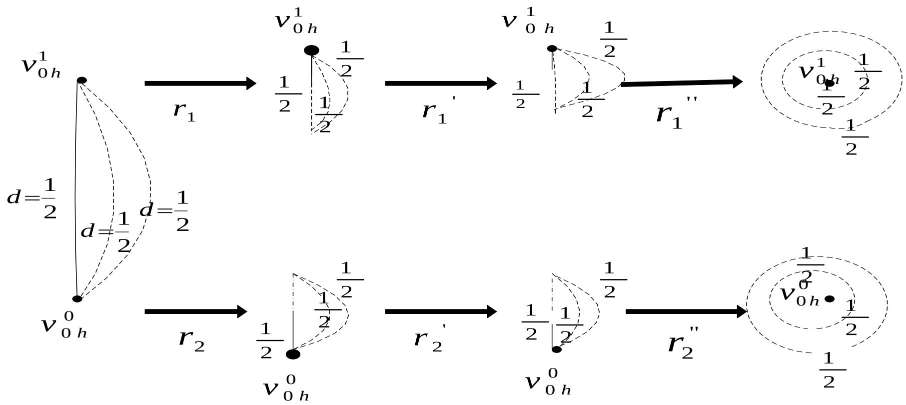

Figure (4): Geometric retraction of 1-simplicial chaotic graph with constant density

2- Suppose we have a 1-simplicialchaotic graph where the geometric edge and chaotic edges have a constant density and they differ to each other, for example $(d = 1/2, 1/4, 1/5)$.

The same result deduced as in the previous example (Figure (5)).

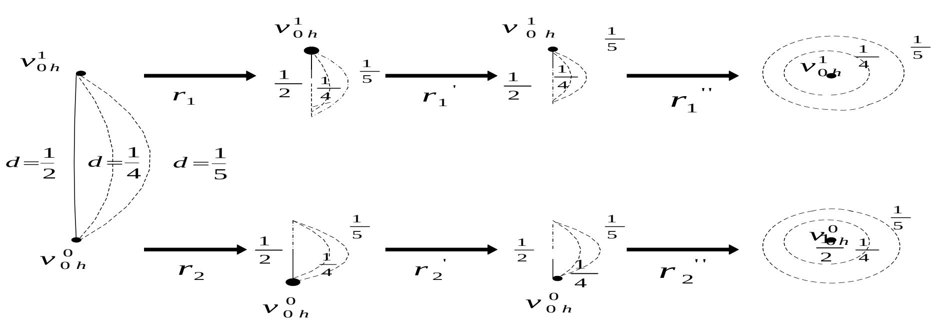

Figure (5): Geometric retraction of 1-simplicial chaotic graph with different density value to the geometric edge and chaotic edges

3- Consider a chaotic graph which has a different density for each area in each chaotic graphs $(d = 1/2, 1/4)$, chaotic level one is divided between two densities (1/2) and (1/4) and similarly for the second chaotic level (Figure (6)).

The same result deduced as before.

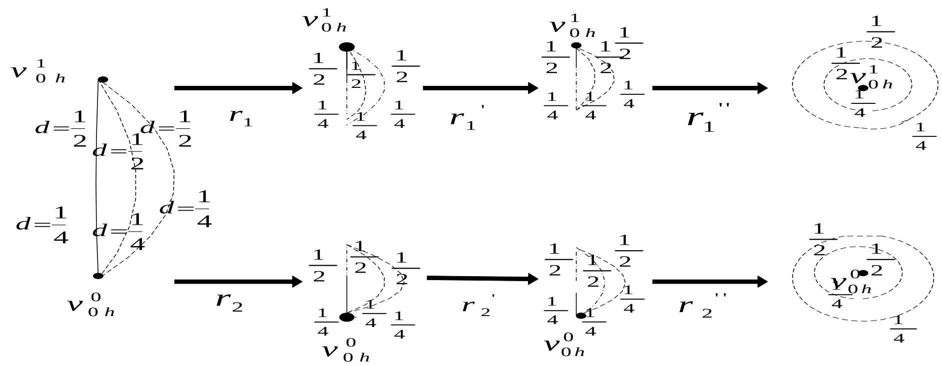

Figure (6): Geometric retraction of 1-simplicial chaotic graph with density variation

2.2 Retraction of chaotic edges

- Making a retraction for some chaotic edges $(e^1_{ih}, i = 1,2,...,\infty h)$ of chaotic graph G_h with density variation reduces number of chaotic edges by removing some of its chaotic edges which leads to vanish the chaotic edge we are retracting, so the resultant graph has $(e^1_{(i-1)h}, i = 1,2,...\infty)$, the other significant result is a reduction in the density degree of the total graph density as well, as the resulted graph loses some of its chaotic edges. (Figure(7))

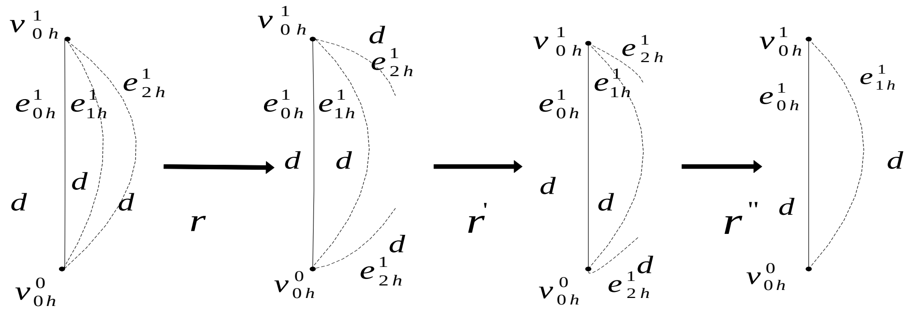

Figure (7): Chaotic retraction of 1-simplicial chaotic graph with constant density

The adjacent and incidence matrices of 1-simplicial chaotic graph before applying a retraction to some of its chaotic edges are respectively:

$$A(G_h)=\begin{bmatrix}0_{(012..\infty)h} & 1_{(012..\infty)h}\\ 1_{(012..\infty)h} & 0_{(012..\infty)h}\end{bmatrix}, I(G_h)=\begin{bmatrix}1_{(012..\infty)h}\\ 1_{(012..\infty)h}\end{bmatrix}$$

While the adjacent and incidence matrices after applying a retraction to some of its chaotic edges are respectively:

$$A(G_h^r)=\begin{bmatrix}0_{(012..m)h} & 1_{(012..m)h}\\ 1_{(012..m)h} & 0_{(012..m)h}\end{bmatrix}, I(G_h^r)=\begin{bmatrix}1_{(012..m)h}\\ 1_{(012..m)h}\end{bmatrix}$$

Where m denotes number of the chaotic edges after applying retraction, so we can see they are different to each other, since the resulted graph loses some of its chaotic edges.

Working that to the three different cases of density variation (examples studied in previous section), the results are the same, the resulted graph loses some of its chaotic edges and keeps the rest of the graph as before applying retraction (no change of density degree to the geometric edge) except the total density of the chaotic graph will be reduced as the graph loses some of its chaotic edge. The adjacent and incidence matrices of the resulted graph are different after applying retraction. (Figures (8, 9, 10))

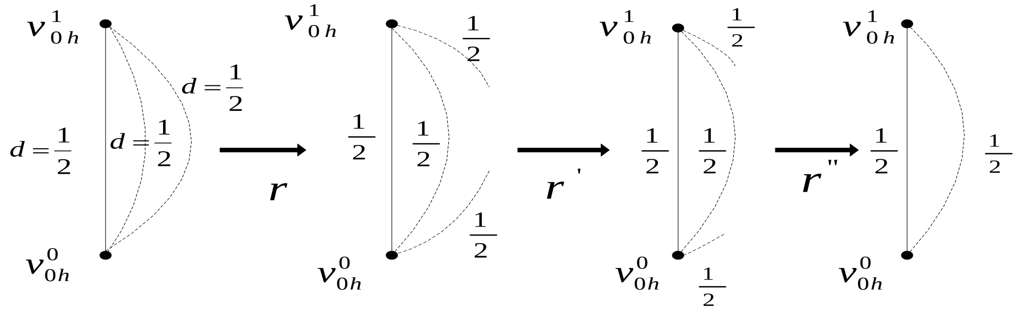

Figure (8): some chaotic retraction of 1-simplicial chaotic graph with constant density

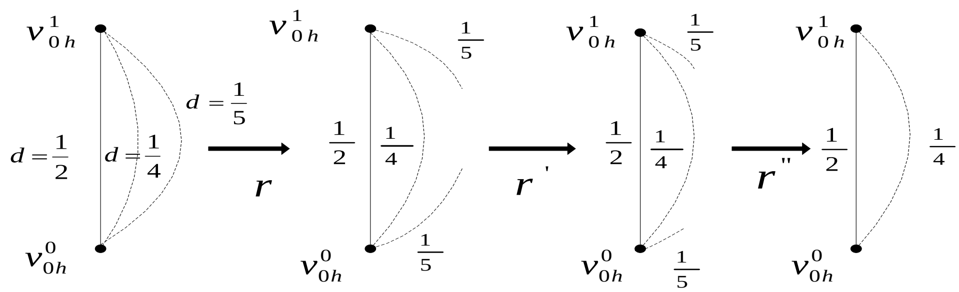

Figure (9): some chaotic retraction of 1-simplicial chaotic graphs with different density value to the geometric edge and chaotic edges on each level.

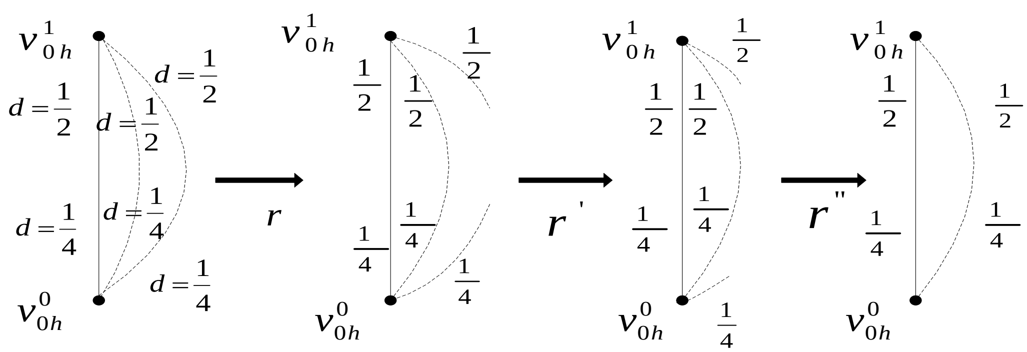

Figure (10): Some chaotic retraction of 1-simplicial chaotic graph with density variation

- Making a retraction for all chaotic edges $(e_{ih}^1, i = 1,2,3,...\infty)$ of 1-simplicial chaotic graph with density variation G_h, the result graph is 1-simplicail geometric graph losing all its chaotic edges and it does not effect on the density values of the geometric edge. (Figure (11))

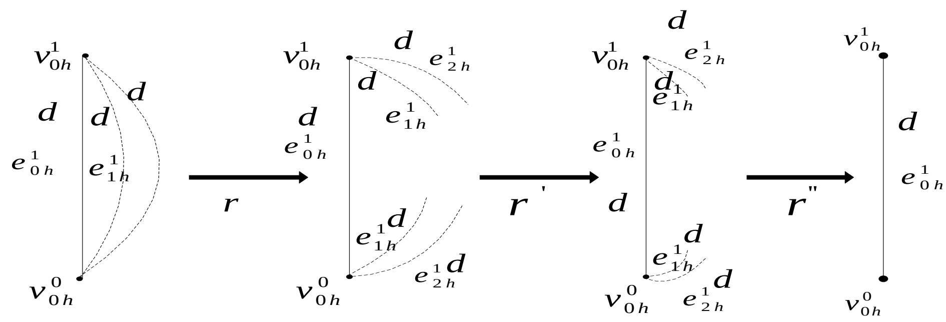

Figure (11): Chaotic retraction to all chaotic edges of 1-simplicial chaotic graph

The adjacent and incidence matrices of 1-simplicial chaotic graph before applying a retraction to all chaotic edges as mentioned before are respectively:

$$A(G_h)=\begin{bmatrix}0_{(012..\infty)h} & 1_{(012..\infty)h}\\ 1_{(012..\infty)h} & 0_{(012..\infty)h}\end{bmatrix}, I(G_h)=\begin{bmatrix}1_{(012..\infty)h}\\ 1_{(012..\infty)h}\end{bmatrix}$$

While the adjacent and incidence matrices of the resulted graph after applying a retraction to all chaotic edges are respectively:

$$A(G_h^r)=\begin{bmatrix}0_{0h} & 1_{0h}\\ 1_{0h} & 0_{0h}\end{bmatrix}, I(G_h^r)=\begin{bmatrix}1_{0h}\\ 1_{0h}\end{bmatrix}$$

So, it is obvious that they are different to each other, since the graph loses all chaotic edges. Working that to the three different cases of density variation give the same result, it results a geometric graph with lower density degree (Figures (12, 13, 14)).

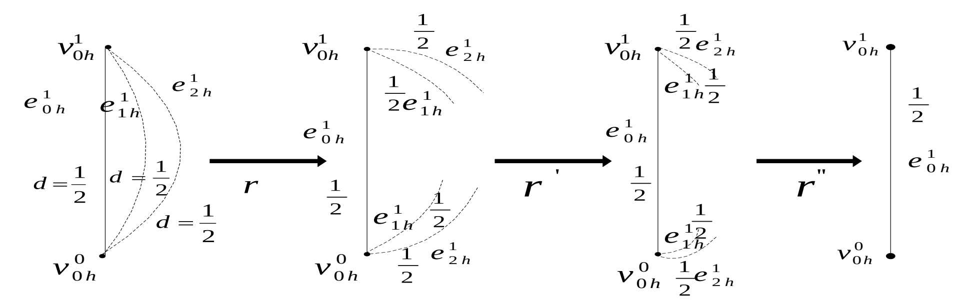

Figure (12): Chaotic retraction to all chaotic edges of 1-simplicial chaotic graph with constant density

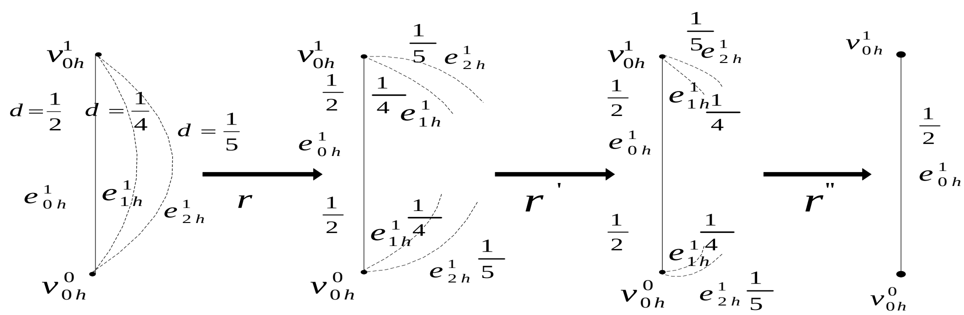

Figure (13): Chaotic retraction to all chaotic edges of 1-simplicial chaotic graph with different density value to the geometric edge and chaotic edges

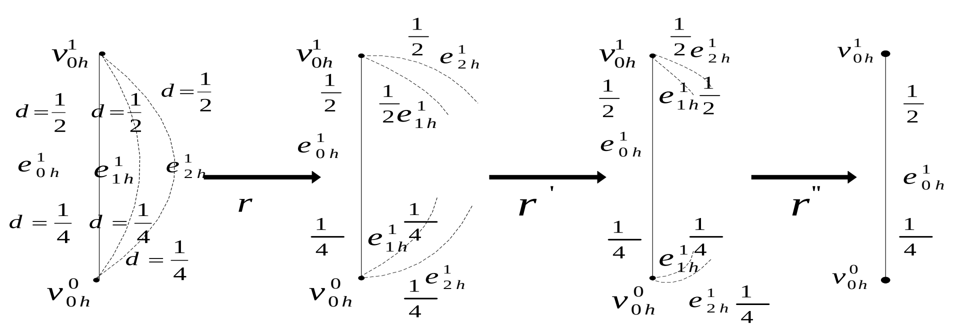

Figure (14): Chaotic retraction to all chaotic edges of 1-simplicial chaotic graph with density variation

3 Conclusion

- This paper discussed the transformation of retraction on 1-simplicial chaotic graph with density variation, two types of retractions are applied, the geometric retraction and physical chaotic retraction.
- The limit geometric retraction of 1-simplicial chaotic graph results a null graph, a graph with one vertex that carries all its chaotic edges losing its geometric edge, so the total density of the graph is reduced as it loses its geometric edge with its density character. The adjacent and incidence matrices of the resulted graph are different after applying retraction as the resulted retracted graph is the null graph.
- Applying a retraction for some chaotic edges removes some of its chaotic edges of the graph which leads to vanish some of its chaotic edges resulting a reduction of the total density of the graph. The adjacent and incidence matrices of the resulted graph are different after applying retraction.
- Applying a retraction of all chaotic edges produces a1-simplicial geometric graph reducing its total density and the adjacent and incidence matrices of the resulted graph are different after applying retraction as the retracted graph changes from 1-simplicial chaotic graph into 1-simplicial geometric graph losing all chaotic edges.

List of Abbreviations:

d	Density
G	Graph
G_h	Chaotic graph
$A_h(G)$	Adjacency matrix of chaotic graph
$I_h(G)$	Incidence matrix of chaotic graph
r	Retraction
h	An index represents chaotic graph
G_h^r	Resulted chaotic graph after applying retraction

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