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Mathematical determination of the optimal control and maintenance scheme for industrial processes

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Abstract: Modern industrial projects face many challenges in order to sustain their productivity in a capital effective manner. Operational costs and production lines maintenance policy is on the top factors that play critical roles in that challenge. Linear programming is utilized in this paper to examine the possible minimization of the operational cost and determine an effective optimal maintenance policy for middle-sized furniture manufacturing plant in Baghdad city, taking under consideration all alternatives, non-sensitivity, and solidness. A mixture of Markov decision processes and linear programming analysis is implemented for the actual site and operational data to help decision-makers in planning for their project's mid and long term maintenance policy and performing Solidness and, as result, the tentative cost reduction scheme.

1. Introduction

The performance, productivity, and maintenance are considered among the main concerns of the manufacturers that may assure their success in satisfying the goods market. Markove decision processes MDP were widely considered by many decision-makers and researchers as a powerful tool in order to enhance these concerns for random processes that might be formulated as discrete of Markov series,[1-3]. The two main measures for evaluating the performance were the average cost per unit time and the average total discounted cost. The latter requires the determination of the specific value for the discount factor [4].

The problem of the current paper is to examine the hypothesis that the optimization test through the solution enhancement algorithm or the optimization policy in which all alternates are taken into consideration would result in a more effective method rather than MDP for the determination of optimal policies. The linear programming method by the reduced cost standard would be utilized in order to process that algorithm.

1.1. Linear programming and optimization policy

The main type of policy is called as the inevitable policy that was used by Markov decision processes to describe the decision $d_i(R)$ when the system is in state i , for all values of M , $i=0,1,\dots,M$ and so on. Hence the R-value was classified by the values $D_{ik}= 0$ or 1 in the matrix. The proper decision variable for the linear programming model is defined as follows for each $i=0,1,\dots,M$ and $k=1,2,\dots,k$ as a stable state; $X_{ik}=P$ (state= i and decision= K), and D_{ik} may be determined by the following:



$$D_{ik} = \frac{X_{ik}}{\sum_{k=1}^K X_{ik}} \tag{1}$$

So that

$$\sum_{k=1}^K X_{ik} = \sum_{i=0}^M \sum_{k=1}^K X_{ik} P_{ij}(k) \quad \text{for } j = 0,1,2,\dots,M \tag{2}$$

The expected long term average cost is determined by the following [5]:

$$E(c) = \sum_{i=0}^M \sum_{k=1}^K C_{ik} X_{ik} \tag{3}$$

Hence, the linear programming model becomes:

$$\text{Minimize} \quad Z = \sum_{i=0}^M \sum_{k=1}^K C_{ik} X_{ik} \tag{4}$$

And the final solution of which is as follows:

$$D_{ik} = \frac{X_{ik}}{\sum_{k=1}^K X_{ik}} \tag{5}$$

1.2. Optimal solution

The main benefit of the policy enhancement algorithm is to determine tackle optimization through relatively fewer trials, [6,7]. It utilizes α in order to measure the reduction factor where the value of α is between zero and one, [8]. The reduction factor might be explained as an equivalent to $1/(i+1)$ in which i represent the current interest average for each period, and hence, α represents the unit value for each of the future period's cost. The enhancement algorithm is:

1. The constraint value; for each policy R_n , C_{ik} and $D_{ij}(k)$ were used to solve a system $m+1$ as follows:

$$V_i(R_n) = C_{ik} + \alpha \sum_{i=0}^M P_{ij}(k) V_j(R_n), \quad \text{for } i = 0,1,\dots,M \tag{6}$$

2. The alternative policy R_{n+1} is determined as; for each state i ;

$$\text{Minimize}_{k=1,2,\dots,k} \quad \left[C_{ik} + \alpha \sum_{i=0}^M P_{ij}(k) V_j(R_n) \right]$$

1.3. Optimization test

The current policy R_{n+1} is optimal when it matches R_n , and hence the process halts, otherwise the repetition continues. The two properties of the algorithm are; the subsequent policy should be lower than the current one, and the algorithm ends with limited trials [9]. The following formula describes a model for linear programming to enhance the optimal solution;

$$\sum_{k=1}^M X_{ik} - \alpha \sum_{i=0}^M \sum_{k=0}^K X_{ik} P_{ij}(k) = \beta \quad \text{for } j=0,1,2,\dots,M \tag{7}$$

$$X_{ik} \geq 0 \quad \text{for } i=0,1,2,\dots,M ; k=1,2,\dots,K$$

where $\beta_0, \beta_1, \beta_2,$ and β_3 are arbitrarily chosen.

2. Case study

2.1. Problem definition

The management of Baghdad furniture factory was aiming at the set of optimal maintenance policy. The core production machines were degrading in performance due to the continuous heavy production load, hence, weekly maintenance processes were needed. The maintenance engineers have classified the status of the machine into four possible status types; 0 good as new, 1 simple practical failure, 2 major practical failure and 3 fail product (non-acceptable quality).

3. Modeling of the problem

The repetition matrix for each possible transition from each specific status to another one during a month period was made by the maintenance engineers through the collection of historical machine status records, as follows:

$$\begin{pmatrix} \text{State} & 0 & 1 & 2 & 3 \\ 0 & 0 & .78 & .11 & .11 \\ 1 & 0 & .66 & .17 & .17 \\ 2 & 0 & 0 & .48 & .52 \\ 3 & 0 & 0 & 0 & 1 \end{pmatrix}$$

The statistical analysis proved that the transition probability was not affected by the status of past months. The random variable X_t represents the machine status at the end of the month, hence the random process $(X_t, t=0,1,2,\dots)$ represents the discrete-time Markov series. After reaching status type 3, then the machine was no longer in service and represents the absorbing state, which leads to halt the production and the machine should be replaced and this, in turn, leads to the start from status 0. The replacement process lasts for about one week and this makes a loss of about \$2000 while the replacement itself costs about \$6000, which means a total of \$8000, not mentioning the cost of defect-ed products. The estimated weekly costs were listed in table 1.

Table 1. The estimated weekly cost for each status type.

State	Expected Cost Due to Defective Items, \$
0	0
1	2000
2	6000

The possible maintenance decisions after each testing process might be summarized as in table 2.

Table 2. The possible maintenance decisions.

Decision	Action	Relevant States
1	Do nothing	0,1,2
2	Overhaul	1,2,3
3	Replace	3

The relevant cost for each separate status is demonstrated in table 3 for the studied case. These data would be used as input for the suggested mathematical models.

Table 3. Cost as per each maintenance status.

Decision	Status	Expected Cost Due to Producing Defective Items,\$	Maintenance Cost,\$	Cost “Lost Profit” of Lost Production,\$	Total Cost per Week,\$
1.Do nothing	0	0	0	0	0
	1	2000	0	0	2000
	2	6000	0	0	6000
2.Overhaul	1	0	2000	4000	6000
	2	2000	4000	2000	8000
	3	6000	4000	2000	12000
3.Replace	3	0	8000	4000	12000

3.1. The mathematical model

The first and second columns in table 3 represent the status types and relevant decisions, hence the decision variables are X_{ik} . The far-right column represents the coefficients of these variables in the goal function. The application of equation (4) would result in the model of the problem of this study, as follows:

$$\text{Min } Z=2000X_{12}+6000X_{13}+6000X_{21}+8000X_{22}+12000X_{23}+12000X_{33} \text{ S.T}$$

$$X_{01}+X_{11}+X_{13}+X_{21}+X_{22}+X_{23}+X_{33} =1$$

$$X_{01} - (X_{13}+X_{23}+X_{33}) =0$$

$$X_{11}+X_{13}-(0.78X_{01}+0.66X_{11}+X_{22}) =0$$

$$X_{21}+X_{22}+X_{23}-(0.11X_{01}+0.17X_{11}+0.48X_{21}) =0$$

$$X_{33}-(0.11X_{01}+0.17X_{11}+0.52X_{21}) =0$$

and

$$\text{all } X_{ik} \geq 0$$

4. Solution and analysis

The solution of the suggested mathematical model was made via the application of the Quantitative System in Business QSB and the output of which were represented in figure 1.

Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
X01	0.1214	0	0	0	basic	-1,462.6870	M
X11	0.6357	2,000.0000	1,271.4290	0	basic	-M	3,606.5580
X13	0	12,000.0000	0	5,700.0000	at bound	6,300.0000	M
X21	0	6,000.0000	0	0	basic	-51,042,800,000.0000	M
X22	0.1214	8,000.0000	971.4286	0	basic	-M	9,342.4660
X23	0	12,000.0000	0	1,400.0000	at bound	10,600.0000	M
X33	0.1214	12,000.0000	1,457.1430	0	basic	-M	44,666.6800
Objective	Function	(Min.) =	3,700.0000				
Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
C1	1.0000	=	1.0000	0	3,700.0000	0	M
C2	0	=	0	0	-9,811.5380	-0.1694	0
C3	0.0000	=	0	0	-7,211.5390	-0.3778	0
C4	0	=	0	0	-2,911.5390	-0.1903	0
C5	0	=	0	0	-1,511.5390	-0.2040	0

Figure 1. Results of the application of the software QSB for the initial case.

The output of software QSB would in turn used in equation (5) in order to determine the optimal maintenance policies, as summarized in table 4. These results show that the lower possible maintenance cost policies were; do nothing, when the status was 0 or 1, repair the machine when the status was 2, and replace the machine when the status was 3.

Table 4. the optimal maintenance policies results.

Optimal maintenance policies	D ₀₁	D ₁₁	D ₁₃	D ₂₁	D ₂₂	D ₂₃	D ₃₃
values	1	1	0	0	1	0	1

4.1. Solution enhancement

The mathematical model that serves the enhancement of the optimal maintenance policy may be formulated via equation (7), with arbitrarily assuming $\beta = 0.25$ and $\alpha = 0.9$.

$$\text{Min } Z = 2000X_{11} + 1200X_{13} + 600X_{21} + 8000X_{22} + 1200X_{23} + 12000X_{33}$$

S.T

$$X_{01} - 0.9(X_{13} + X_{23} + X_{33}) = 0.25$$

$$X_{11} + X_{13} - 0.9(0.78X_{01} + 0.66X_{11} + X_{22}) = 0.25$$

$$X_{21} + X_{22} + X_{23} - 0.9(0.11X_{01} + 0.17X_{11} + 0.48X_{21}) = 0.25$$

$$X_{33} - 0.9(0.11X_{01} + 0.17X_{11} + 0.52X_{21}) = 0.25$$

and

$$\text{all } X_{ik} \geq 0$$

The solution of the enhanced model via the software QSB was summarized in figure 2.

Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
X01	1.4237	0	0	0	basic	-1,948.2740	M
X11	5.9682	2,000.0000	11,936.3700	0	basic	-M	4,123.9110
X13	0	12,000.0000	0	6,086.6680	at bound	5,913.3330	M
X21	0	6,000.0000	0	2,747.6890	at bound	3,252.3110	M
X22	1.3041	8,000.0000	10,432.6000	0	basic	-M	9,631.9840
X23	0	12,000.0000	0	1,690.5370	at bound	10,309.4600	M
X33	1.3041	12,000.0000	15,648.9000	0	basic	4,139.4800	52,897.8100
Objective	Function	(Min.) =	38,017.8600				
Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
C2	0.2500	=	0.2500	0	33,472.6100	-0.4688	M
C3	0.2500	=	0.2500	0	36,038.6800	-0.6375	M
C4	0.2500	=	0.2500	0	40,434.8100	-0.3967	M
C5	0.2500	=	0.2500	0	42,125.3500	-0.4084	M

Figure 2. The results of software QSB for the optimal solution for the enhanced case.

The results in table 5 represent the enhanced optimal policies D that can be gained via the application of equation (7). It is obvious that the results in table 6 seem much like these in table 4, hence the solution still represents the optimal policy and therefore the optimal maintenance policy is tightly confirmed. Many values were tested in order to determine the effect of indicators α and β on the model in equation (6), and the results of these tests were that these changes do not affect the final optimal solution for the essential variables of D, as demonstrated in table 6.

Table 5. The enhanced optimal maintenance policies.

The enhanced optimal maintenance policies		D ₀₁	D ₁₁	D ₁₃	D ₂₁	D ₂₂	D ₂₃	D ₃₃
Value		1	1	0	0	1	0	1

Table 6. The results of optimal policies for various values of α and β .

N	value		D ₀₁	D ₁₁	D ₁₃	D ₂₁	D ₂₂	D ₂₃	D ₃₃
	α	β							
1	0.25	0.9	1	1	0	0	1	0	1
2	0.4	0.9	1	1	0	0	1	0	1
3	0.5	0.9	1	1	0	0	1	0	1
4	0.25	0.8	1	1	0	0	1	0	1
5	0.4	0.8	1	1	0	0	1	0	1
6	0.5	0.8	1	1	0	0	1	0	1
7	0.4	0.7	1	1	0	0	1	0	1
8	0.6	0.75	1	1	0	0	1	0	1

5. Conclusions

The effectiveness of the linear programming process as a replacement of the MDP as studied and tested for the sake of building an optimal maintenance policy for Baghdad furniture factory, Iraq, depend-

ing on the collected maintenance history records. The major findings have proved the linear programming as a strong tool for setting optimal maintenance policy with the lower cost and also to enhance the optimal policy as compared to the random processes method that was commonly used via MDP and the iterative calculations that have long been used for such goals, as the linear programming method considers almost all available choices in addition to its precise and quick results, especially against the relatively large volume cases, which would benefit decision-makers to improve their administrative job.

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