

Calculations of Ground Band in even-even $^{170-180}\text{W}$ nuclei by
Interacting boson model (IBM-1)

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Mushtaq Abed Dawood Al-Jubbori* Hussein Ali Hassan Al-Saffar**

* Department of Physics, College of Education for Pure Science, Mosul University, Iraq

**Ministry of Education , General Directorate for Nineva Education, Iraq

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Abstract

In this work, the properties of $^{170-180}\text{W}$ isotopes are studied by the E-GOS curves and the relation between energy levels and E_2 showed that $^{170-180}\text{W}$ lie in SU(3)-O(6) transition region were investigated to calculate the energy levels of ground state band according to the Interacting Boson Model (IBM-1). The entire calculations and drawing of the figures implemented by one program written by matlab language. The results are compared with experimental data and showed good agreement.

Keywords: energy levels, IBM(1), SU(3)-O(6), E-GOS, W isotopes.

حساب مستويات الطاقة للحزمة الأرضية لنظائر $^{170-180}\text{W}$ الزوجية – الزوجية باستخدام
أنموذج البوزونات المتفاعلة (IBM-1)

مشتاق عبد داود الجبوري* حسين علي حسن الصفار**

*قسم الفيزياء، كلية التربية للعلوم الصرفة، جامعة الموصل، العراق
**وزارة التربية، المديرية العامة لتربية نينوى، العراق

الخلاصة

في هذا البحث تم دراسة خصائص نظائر التنكستن $^{170-180}\text{W}$ بواسطة منحنيات كما مقسوماً على الزخم E-GOS وتبين من العلاقة بين مستويات الطاقة ومستوي E_2 ان هذه النظائر تقع بين التحديدين SU(3)-O(6) أي في المنطقة الانتقالية بين النوى الدورانية والنوى ذات خصائص كما الناعمة. حسبت مستويات الطاقة للنظائر المدروسة باستخدام انموذج البوزونات المتفاعلة IBM-1 من خلال برنامج حاسوبي كتب بلغة matlab يفي بهذا الغرض من اجراء الحسابات ورسم الأشكال المطلوبة. قورنت الحسابات الحالية مع النتائج العملية وتبين انها تتفق بشكل جيد.

الكلمات المفتاحية: نظائر التنكستن، منحنيات E-GOS، SU(3)-O(6)، أنموذج البوزونات المتفاعلة IBM-1، مستويات الطاقة.

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Introduction

Several phenomenological and geometrical models [1] have been proposed to investigate the nuclear structure by the prediction of the ground states and the description of electromagnetic transition rates [2]. The quadrupole correlations and electromagnetic transition in nuclei depends mainly on the neutron-proton interaction. However, the excitation energies of collective quadrupole excitations in nuclei near a closed shell are strongly dependent on the number of nucleons outside the closed shell [3,4]. The tungsten isotopes received considerable attention both theoretically and experimentally in recent years. Abdul Ameer and Al-Shimmary [5] calculated the energy levels, $B(E2)$ transition probabilities and electric quadrupole moment of the even-even $^{180-190}\text{W}$ isotopes in the transition region $SU(3)-O(6)$. The energy levels, electric quadrupole moments, $B(E2)$ values of $^{182-186}\text{W}$ isotopes have been calculated by, Salem *et al* [6] within the framework of the interacting boson model IBM-2. Due to the increased interest in this subject recent years, the energy levels and E-Gos of even-even $^{170-180}\text{W}$ isotopes in $SU(3)-O(6)$ transition region within framework are studied by the Interacting Boson Model (IBM-1).

Theory
Energy Gamma Over Spin (E-GOS)

Many nuclei exhibit a decay sequence consistent with quasi-vibrational excitations at lower spins, the perfect harmonic vibrator of gamma-ray decay energies are given by :

$$E_{\gamma}(J \rightarrow J - 2) = \hbar w \quad (1)$$

where J is spin state, \hbar is plank's constant and w is angular frequency. While, for an axially symmetric rotor,

$$E_{\gamma}(J \rightarrow J - 2) = \frac{\hbar^2}{2\mathcal{I}}(4J - 2) \quad (2)$$

where \mathcal{I} is moment of inertia of the nucleus.

The gamma-soft nucleus can be written as

$$E_{\gamma}(J \rightarrow J - 2) = \frac{E2_1^+}{4}(J + 2) \quad (3)$$

The ratio $R = \frac{E_{\gamma}(J \rightarrow J - 2)}{J}$ provides an effective way of distinguishing

axially symmetric rotational, γ -unstable and harmonic vibrational mode [7].

at $J \rightarrow 0$

$$\text{for vibrator } R = \frac{\hbar w}{J} \rightarrow 0 \quad (4)$$

$$\text{Rotor } R = \frac{\hbar^2}{2\mathcal{I}}(4 - \frac{2}{J}) \rightarrow 4 \frac{\hbar^2}{2\mathcal{I}} \quad (5)$$

$$\gamma\text{-unstable } R = \frac{E2_1^+}{4}(1 + \frac{2}{J}) \rightarrow \frac{E2_1^+}{4} \quad (6)$$

Figure (1) shows these theoretical limits plotted for three schematic nuclei: (i) a vibrator in which the first 2^+ excited state lies at an energy of 500 keV, (ii) a rotor where this energy is

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100 keV and (iii) a γ -unstable of energy 300 keV (These values were taken to represent typical nuclear vibrator, rotor and γ -unstable energies, respectively.)

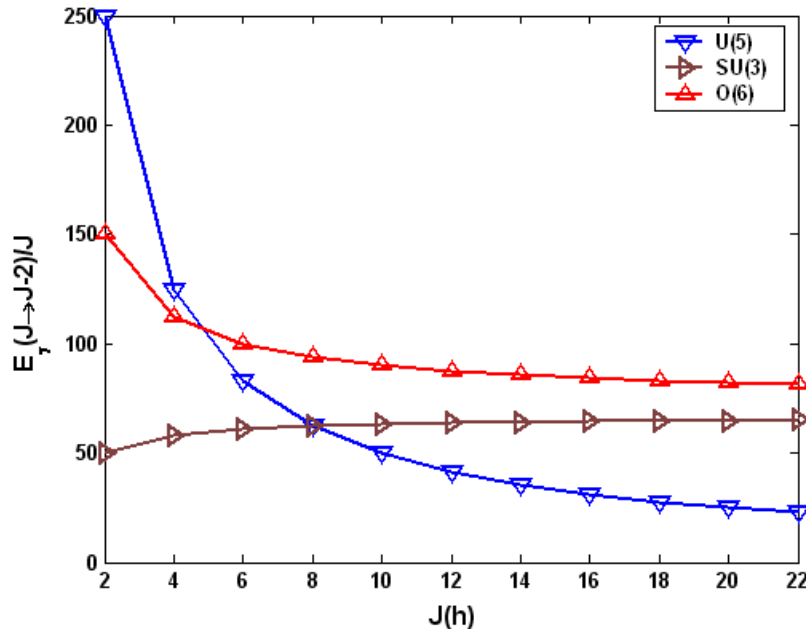


Figure (1): Standard curve of E-GOS plot for vibrational U(5), gamma unstable O(6) and rotational bands SU(3)

Interacting Boson Model (IBM-1)

In the present work, the IBM-1 states of the low lying collective state of even-even nuclei can be described by the interaction of s and d-bosons, carrying angular momentum L = 0 and L= 2, respectively. The IBM-1 Hamiltonian is written

$$H = \epsilon \hat{n}_d + a_0 \hat{P}^+ \hat{P} + a_1 \hat{L} \cdot \hat{L} + a_2 \hat{Q} \cdot \hat{Q} + a_3 \hat{T}_3 \cdot \hat{T}_3 + a_4 \hat{T}_4 \cdot \hat{T}_4 \quad (7)$$

Where a_0 , a_1 , a_2 , a_3 and a_4 are strength of pairing, angular momentum and multipole terms. The Hamiltonian tends to reduce into three limits, the vibration U(5), γ -soft O(6) and the rotational SU(3) nuclei [8]. In U(5) limit, the effective parameter is ϵ , in the γ -soft limit O(6) the effective parameter is the pairing a_0 and in the SU(3) limit the effective parameter is the quadrupole a_2 .

The eigenvalues for the SU(3)-O(6) limit is given by [9]

$$E(\lambda, \mu, J) = K_3[N(N+4) - \sigma(\sigma+4)] + K_2(\lambda^2 + \mu^2 + 3(\lambda + \mu) + \lambda\mu) + K_5J(J+1) \quad (8)$$

for low-lying $N=\sigma$, $\mu=0$ and $\lambda=2N$ therefore

$$E(\lambda, J) = K_2(\lambda^2 + 3\lambda) + K_5J(J+1) \quad (9)$$

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Discussion and Conclusions

The parameters of Eq.(9) were calculated for ¹⁷⁰⁻¹⁸⁰W nucleus and listed in table (1), it was found that K₂ decreases and K₅ increases with mass number.

Table(1): The Parameters K₂, K₅ and the ratio R=E₄/E₂ for ¹⁷⁰⁻¹⁸⁰W isotopes

Nucleus	K ₂ (keV)	K ₅ (keV)	R=E ₄ /E ₂
¹⁷⁰ W	25.7672	4.1226	2.95
¹⁷² W	18.4796	5.2798	3.0609
¹⁷⁴ W	13.9465	7.7379	3.154
¹⁷⁶ W	12.6059	8.4789	3.2151
¹⁷⁸ W	10.7882	9.5505	3.2364
¹⁸⁰ W	10.4675	9.6598	3.26

Figure (2) shows that the ratio E_J/E₂ of ¹⁷⁰⁻¹⁸⁰W versus spin (J) in which the limit lies between SU(3)-O(6). Eq.(9) used to calculate the energy levels of ground state band.

Figure (3) shows the E-GOS curves of the ground states band of isotopes. Comparing these curves with the ideal limits of vibration, rotational and γ -soft show the evolution in the property of these isotopes, were the slow reduce of all the curves from the first to the last excited states confirms that the ¹⁷⁰⁻¹⁸⁰W isotopes lies between SU(3)-O(6) limit.

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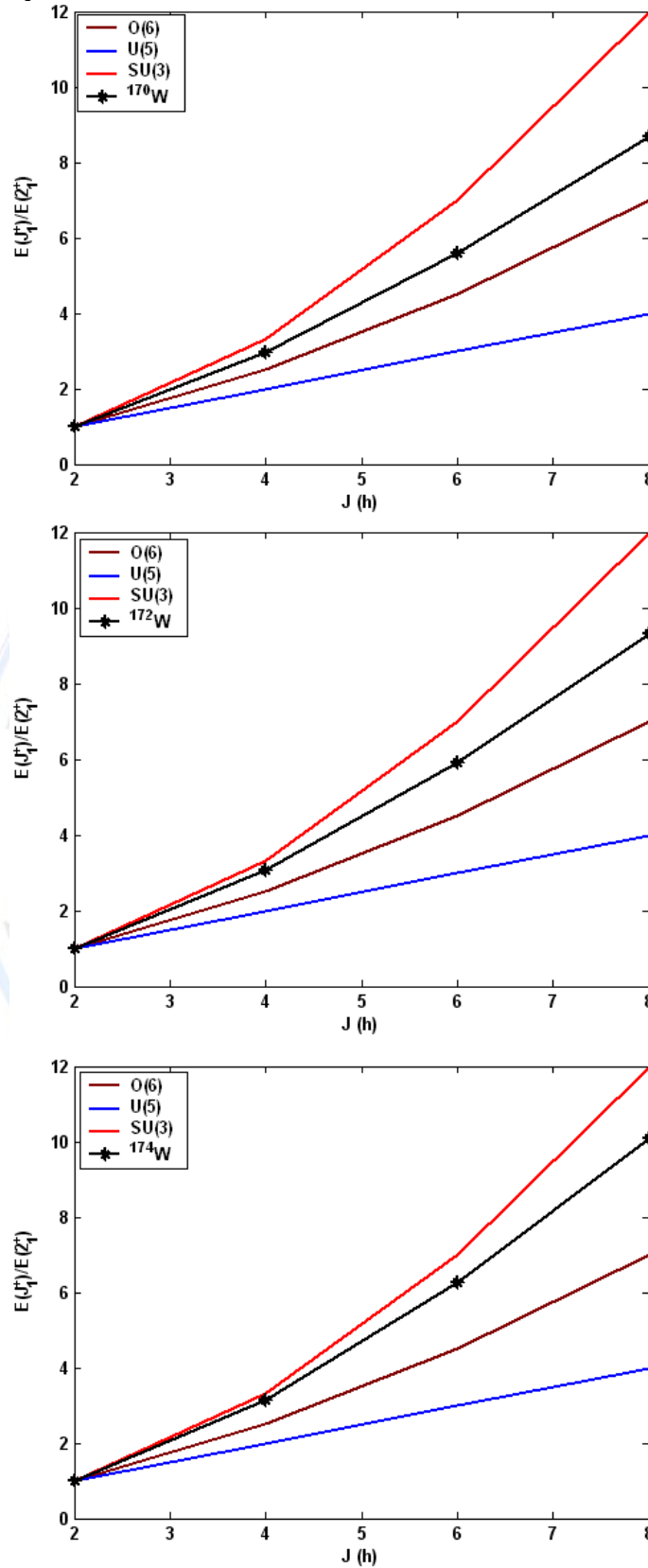


Figure (2): $E(J_1)/E(2_1)$ versus spin J

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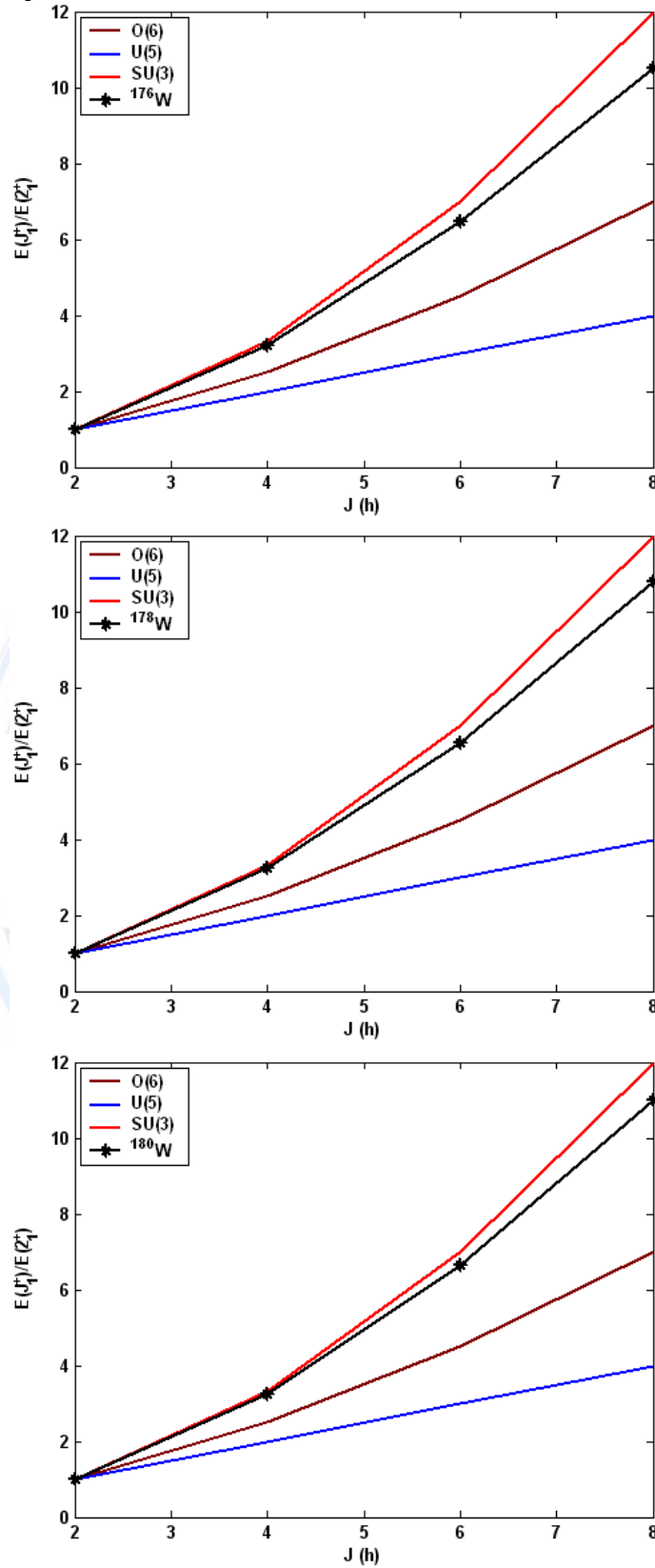


Fig. 2. (continued)

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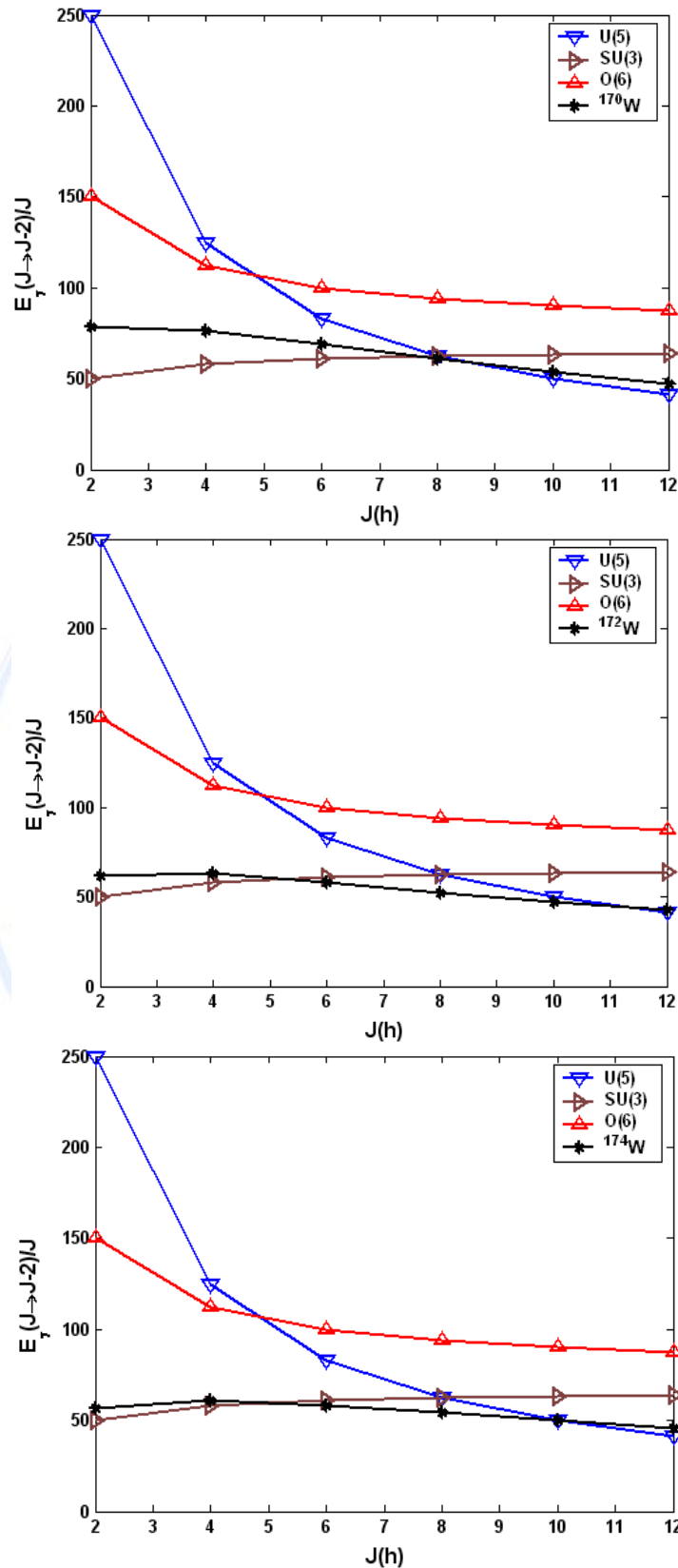


Figure (3): The E-GOS of ¹⁷⁰⁻¹⁸⁰W isotopes.

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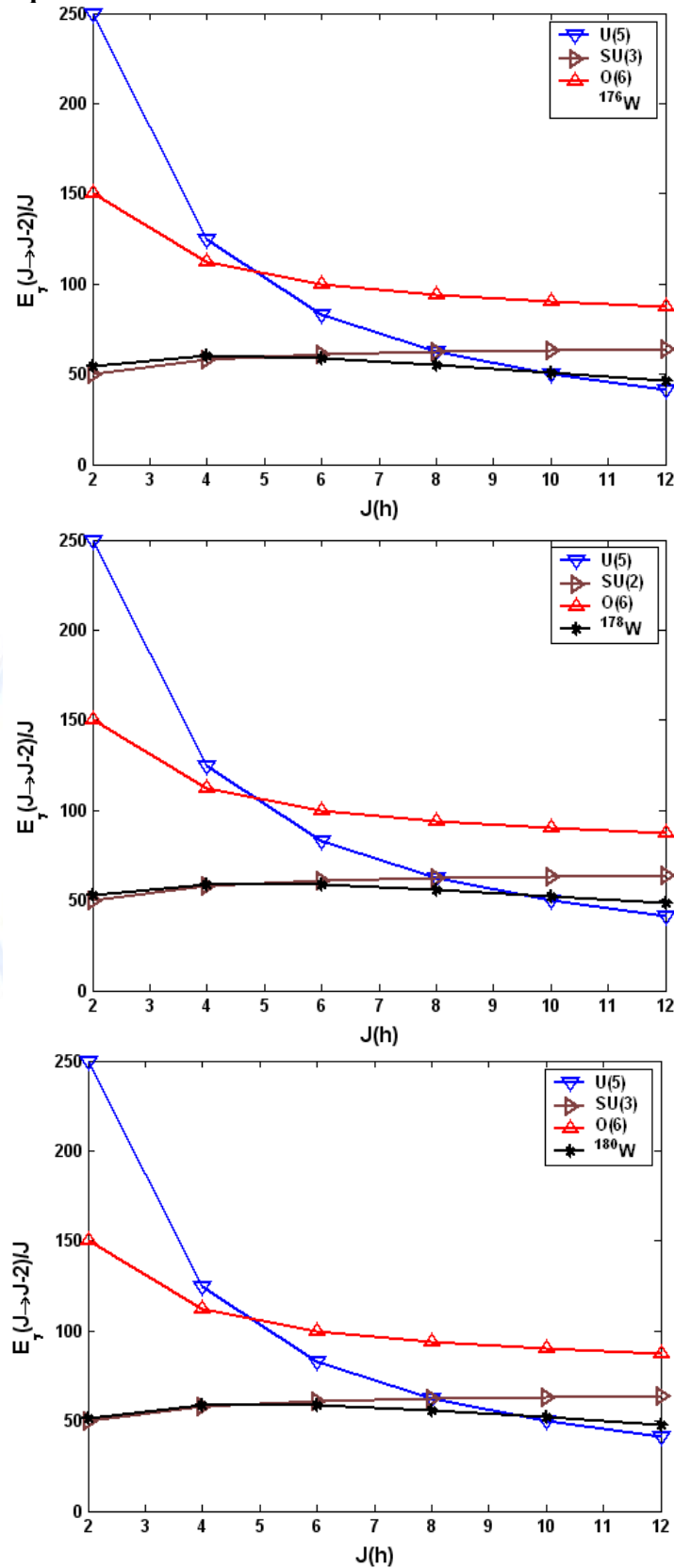


Fig. 3. (continued)

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Figure (4) shows the present calculations of the energy levels were found to be in a reasonable agreement with measured values.

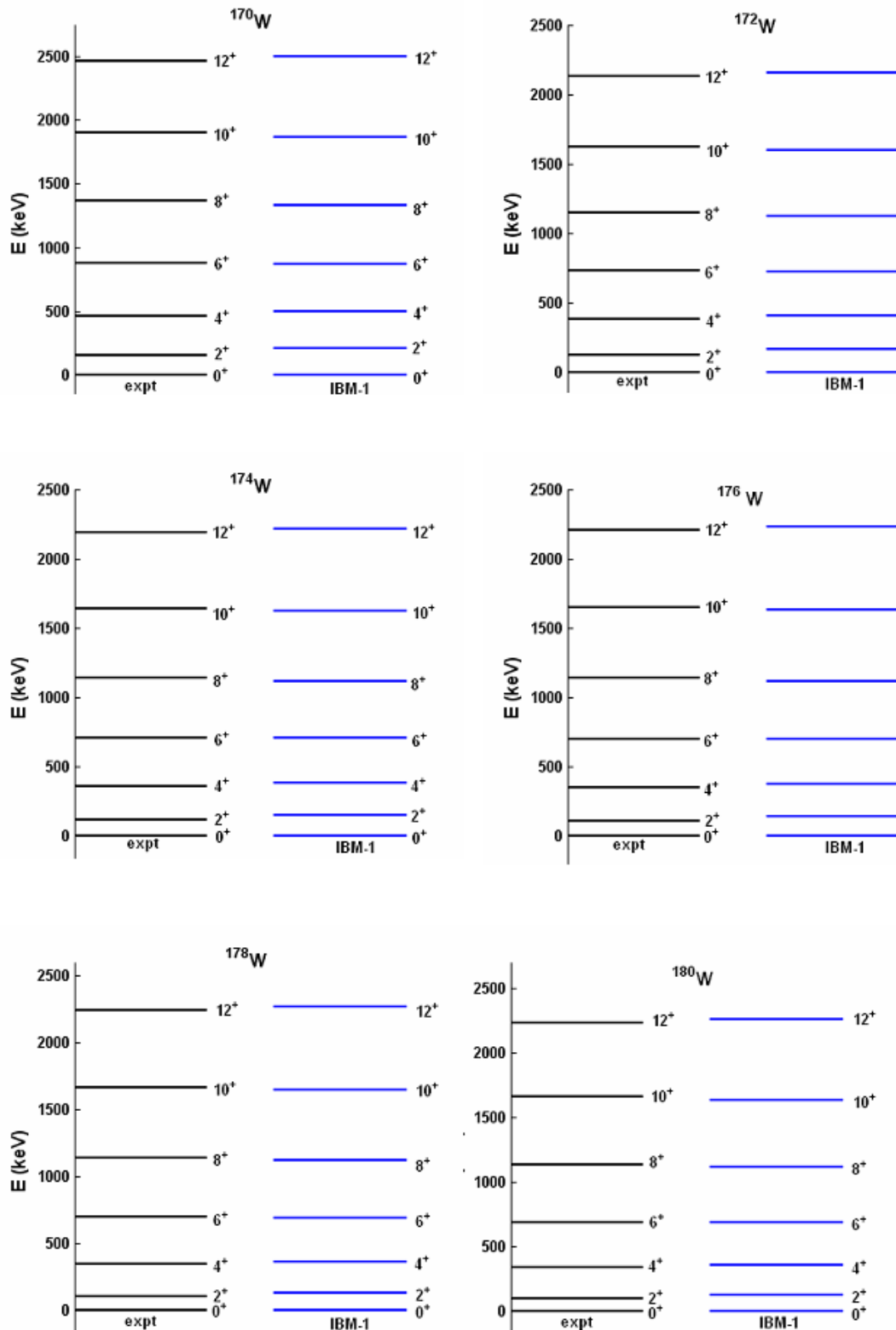


Figure (4): Comparison between experimental energy levels and calculated data

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