

# Design A Non-Newtonian Porosity Mathematical Model For The Effect Of Flexibility On Knee Joint Performance

Enas Yahya Abdullah<sup>1</sup> and Hala Khedhi<sup>2</sup>

<sup>1</sup>University of Kufa / College Of basic Education

[inasy.abdullah@uokufa.edu.iq](mailto:inasy.abdullah@uokufa.edu.iq)

<sup>2</sup>University of Kufa/ College Of Education for girl

[hala.khdhir@gmail.com](mailto:hala.khdhir@gmail.com)

**Abstract :** *In this paper ,the introduce flexibility dynamic of synovial human knee joint in stance phase and swing phase Relationship between flexibility and squeeze film characteristic are introduced with using mathematical model and analysis of the results with appropriate boundary condition for types of lubrication .It is observed that flexibility considerable influence the plecte number and curvature during gait cycle .It effect of flexibility is to increase the pressure and load carrying and decreased in time approach .*

**Keywords:** Non-Newtonian, Porosity, Mathematical Model, Performance

## Introduction

Flexibility refers to the extensibility of joint tissues to allow normal or physiological motion. Flexibility plays a prominent role in the functional ability of a joint to move through its full range of motion with in stance phase and swing phase without incurring pain or a limit to performance. The main movement of the knee is flexion – extension , existence flexibility joint perform movements with ease [1]. Flexibility different between male and female explain that stride length for male very high this lead to increasing flexibility dynamic while in female inverse relationship .[2]. Classification of flexibility to many types : static flexibility range of motion about a joint with no emphasis on speed. Ballistic flexibility usually associated with bobbing or bouncing motion. dynamic (functional) flexibility ability to use range of motion in the performance of a physical activity. The dynamics and deformations of immersed flexible are at the heart of important industrial and biological processes, induce peculiar mechanical and transport properties in the fluids that contain them, and are the basis for novel methods of flow control in porous medium [3]. The study considers a mathematical model for the influence of varying film thickness of articular cartilage and pecllet number on squeeze film characteristics for non-Newtonian fluid with variable flexibility through porous medium . The study uses Mathematica 8 the to solve the problem. The results of the physical parameter problem are discussed by using the graphs.[4]

### 1.1 Assumptions of Hydrodynamic Lubrication: [5]

2. Newtonian fluid
3. Ios-viscous fluid
4. Incompressible fluid
5. Body force and a inertia force in the equation of motion are negligible.
6. The velocity component across the film is negligible compared with the other velocity component .
7. The pressure gradient across the film could be neglected with respect to the pressure gradient along film .

### 1.2 Basic Equation:

The main equations that describe the flexibility of the synovial human knee joint during different movement in the case stand phase and swing phase are introduced. Pore size of surface of articular cartilage with variable load and peak load will be considered and discussed taking into account the effect of roughness and plecte number on the dynamic joint performance.

### 1.3 Mathematical Formulation and Solution of The Flexibility Problem:

The squeeze film mechanism as represents a rigid sphere of radius (R) approaching an infinite plate with a velocity

$V = \frac{\partial h}{\partial t}$  with vary flexibility ( $f$ ) synovial human knee joint. The lubricant is taken to be a Stockes couple stress fluid. The

geometry and coordinates of the flow domain in the present problem are shown in figure (3.1). Using momentum equations and continuity equation expresses the synovial fluid flow.

$$\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} \right) = X - \mu \frac{\partial^2 u}{\partial z^2} - \frac{\partial p}{\partial r} \left( 1 - \frac{P_e}{R_a} \right) - \gamma \frac{\partial^4 u}{\partial z^4} \quad (1.1)$$

$$\frac{\partial p}{\partial z} = 0 \quad (1.2)$$

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial(r\rho u)}{\partial r} + \frac{\partial(\rho w)}{\partial z} = 0 \quad (1.3)$$

Where  $\rho$  density,  $X$  body force,  $(u, w)$  are the velocity components of the lubricant in  $r$  and  $z$  directions respectively,  $p$  is pressure,  $\mu$  dynamic viscosity,  $\gamma$  is a material constant accounting for couple stresses due to polar additives in the lubricant,  $R_a$  surface roughness and  $P_e$  plecte number. Under the assumptions hydrodynamic lubrication theory the fluid film is thin, the fluid inertia is small and body forces are absent, then the momentum equations and continuity equation governing the flow of lubricant in polar coordinates reduce to the form:

$$\frac{\partial p}{\partial r} \left( \beta - \frac{\beta * P_e}{R_a} \right) = \mu \frac{\partial^2 u}{\partial z^2} - \gamma \frac{\partial^4 u}{\partial z^4} \quad (1.4)$$

$$\frac{\partial p}{\partial z} = 0 \quad (1.5)$$

$$\frac{1}{r} \frac{\partial(ru)}{\partial r} + \frac{\partial w}{\partial z} = 0 \quad (1.6)$$

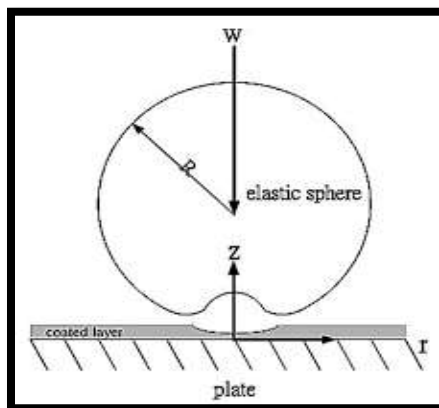


Figure (1.1): Squeeze film action between a sphere and a flat plate.

The ratio  $\left(\frac{\gamma}{\mu}\right)$  is about of dimensional square length and hence characterizes the chain length of the polymer additives.

$$l = \sqrt{\frac{\gamma}{\mu}} \quad (1.7)$$

The Boundary conditions for the velocity component at the surfaces of the plate and spheres are:

$$u(r,0) = \frac{\partial^2 u(r,0)}{\partial z^2} = 0 \quad , \quad w(r,0) = 0 \quad (1.8)$$

$$u(r,h) = \frac{\partial^2 u(r,h)}{\partial z^2} = 0 \quad , \quad w(r,h) = \frac{\partial h}{\partial t} \quad (1.9)$$

Integrating equation (1.4) with the boundary conditions yields the expression of  $u(r, z)$

$$u(r, z) = \frac{1}{2\mu} \frac{\partial p}{\partial r} \left(1 - \frac{P_e}{Ra}\right) \left[ z^2 - hz + 2l^2 - 2l^2 \left( \frac{\cosh \left( \frac{z(z-h)}{2l} \right)}{\cosh \left( \frac{h}{2l} \right)} \right) \right] \quad (1.10)$$

If the ratio of hyaluronic acid lion is high in synovial and cartilaginous cells between the spherical and plane,  $h_m$  is minimum film thickness and  $R$  is a curvature radius, then the film thickness  $h$  can be written as follows :

$$h = h_m + \frac{r^2}{2R} \quad (1.11)$$

Integrating equation (1.6) with respect to  $z$  with the boundary conditions of  $w(r, z)$ , we can achieve the porosity non-Newtonian

$$w = -\frac{\partial}{\partial r} \cdot \frac{1}{2\mu} \cdot \frac{\partial p}{\partial r} \left(1 - \frac{P_e}{Ra}\right) \left[ \frac{z^3}{3} - h \frac{z^2}{2} + 2zl^2 - \frac{2l^3}{\cosh \left( \frac{h}{2l} \right)} \left( \sinh \left( \frac{2z-h}{2l} \right) - 2l^3 \tanh \left( \frac{h}{2l} \right) \right) \right] \quad (1.12)$$

The flow of hyaluronic acid fluid in a porous matrix (articular cartilage) is specific by the modified Darcy law that show up the simple proportional relationship between the immediate discharge rate through a porous medium, the viscosity of the fluid and the direction pressure in the gap between articular cartilage with different type of lubrication expression of law is given by:

$$Q = -\frac{f}{\mu} \nabla p \quad (1.13)$$

$Q$  is the total discharge, and  $Q = (u^*, w^*)$  where  $u^*, w^*$  represent modified Darcy velocity components in  $r, z$  directions, respectively,  $u^*, w^*$  have expression following :

$$u^* = -\frac{f}{\mu} \frac{\partial p^*}{\partial r} \quad w^* = -\frac{f}{\mu} \frac{\partial p^*}{\partial z}, \mu \neq 0 \quad (1.12)$$

Where  $p^*$  is porous region,  $f$  is the flexibility of synovial human knee joint that represents the ratio of the stride length and viscosity to time and weight. If flexibility of knee joint is high and velocity of synovial fluid so then the microstructure additives present in the lubricant block the pores in the porous layer and thus reduce the Darcy flow through the porous matrix. When the microstructure size of the particle is very small compared to the pore size, it should be noted negative sign because fluid flows from high pressure to low pressure.  $p^*$  in the porous region, due to continuity, satisfies the Laplace equation:

$$\nabla^2 p^* = \frac{\partial^2 p^*}{\partial r^2} + \frac{\partial^2 p^*}{\partial z^2} = 0 \quad (1.13)$$

Integrating equation (1.13) with respect to  $(z)$  and using the boundary conditions of solid bearing ( $z = -H_0$  at  $z = 0$ ) where  $H_0$  is the porous layer thickness so noted negative sign because fluid flows less, therefore film thickness decreased.

$$\frac{\partial p^*}{\partial z} = -\int_{-H_0}^0 \left( \frac{\partial^2 p^*}{\partial r^2} \right) dz \quad (1.14)$$

Assuming that the porous layer thickness  $H_0$  is very small using the pressure continuity condition pressure generated in film region ( $p$ ) equal to the pressure generated in the porous region at the interface  $z = 0$  of porous matrix and fluid film in healthy synovial knee joint, integral equation (1.14) become:

$$\frac{\partial p^*}{\partial z} = -H_0 \frac{\partial^2 p^*}{\partial r^2} \quad (1.15)$$

We will focus our attention on the squeeze action  $w^*$  that change with the movement where the person's weight presses downward and the reaction of the ground upwards with different type of flexibility, That will ignore the speed slide  $u^*$  and become equation (1.15) form:

$$w^* = \frac{f}{\mu} H_0 \frac{\partial^2 p}{\partial r^2} \tag{1.16}$$

The relevant boundary conditions for the velocity components are:

(i) at the static flexibility  $z = 0$

$$u(r,0) = \frac{\partial^2 u(r,0)}{\partial z^2} = 0 \quad , \quad w(r,0) = 0 \tag{1.17}$$

(ii) at the dynamic (functional) flexibility  $z = h$

$$u(r,h) = \frac{\partial^2 u(r,h)}{\partial z^2} = 0 \quad , \quad w(r,h) = \frac{\partial h}{\partial t} - w^* \tag{1.18}$$

The squeeze action was obtained by integrate the continuity equation (1.6) with respect to  $z$  and applying boundary condition squeeze action on the upper and lower surface of the synovial human knee joint. From equation(3.31) we obtain:

$$w(r, h) = - \frac{\partial}{\partial r} \cdot \frac{1}{12\mu} \cdot \frac{\partial p}{\partial r} \left(1 - \frac{P_e}{Ra}\right) [f(h, l)] \tag{1.19}$$

Where:

$$f(h, l) = h^3 - 12l^2 h + 24l^3 \tanh\left[\frac{h}{2l}\right] \tag{1.20}$$

Then the modified Reynolds equation governing the film pressure can be written as:

$$12\mu \frac{\partial h}{\partial t} = \frac{\partial^2 p}{\partial r^2} \left[12 f H_0 + \left(1 - \frac{P_e}{Ra}\right) f(h, l)\right] \tag{1.21}$$

#### 1.4 Squeeze Film Pressure:

Introducing the non-dimensional parameters in the governing equations for the pressure is of importance for both theoretical and computational purposes. It is also of importance to present the various parameters in the lubrication system, in non-dimensional form.

$$\begin{aligned} p^* &= - \frac{p h_0^2}{\mu R \frac{\partial h}{\partial t}} & l^* &= \frac{l}{h_0} & h^* &= \frac{h}{h_0} & S^* &= \frac{S}{D} \\ r^* &= \frac{r}{h_0} & \beta &= \frac{R}{h} & f &= \frac{S^2 \mu}{t W} \end{aligned} \tag{1.22}$$

Where S,W,D are the stride length . body weight of human and distance. Apply equation (1.22) into equation (1.21) it was obtained the final form of dimensionless modified Reynolds equation as:-

$$\frac{12 R^2}{\beta} = \frac{\partial p^*}{\partial r^*} \left[12 \frac{f H}{h_0} + \left[1 - \frac{P_e}{Ra}\right] f(h^*, l^*)\right] \frac{\partial}{\partial r^*} \tag{1.23}$$

Boundary condition for the fluid film pressure and radial of chain polymer as follows :

$$\left. \begin{aligned} p^* &= 0 \text{ at } r^* = 2 \\ \frac{\partial p^*}{\partial r^*} &= 0 \text{ at } r^* = 0 \end{aligned} \right\} \tag{1.24}$$

After integration equation (1.23) one with respect to  $r^*$  the squeeze film pressure is given b

$$p^* = \frac{6R^2(4-r^{*2})}{\beta[12fH^* + [\beta - \frac{\beta P_e}{Ra}]h^3 - 12l^2h + 24l^3 \tanh[\frac{h}{2l}]]} \quad (1.25)$$

With the film pressure known, the squeeze film characteristics can now be calculated.

### 1.5 Load Carrying Capacity For Synovial Knee Joint:

The load carrying capacity during with different activities performed by human and varying flexibility level .The load carrying capacity of the porous flat plate ( $W$ ) can be determined:

$$w = 2\pi \int p r dr \quad (1.26)$$

Introduce the dimensionless load carrying capacity in consideration to flexibility knee joint.

$$w^* = -\frac{Wh_0^2}{\mu R^2 \frac{\partial h}{\partial t}} \quad (1.27)$$

Substituted quantity (1.27) into equation (1.26) yield

$$w^* = 2\pi \int_0^2 p^* r^* dr^* \quad (1.28)$$

We integrate dimensionless pressure with respect to the dimensionless radial and thus we obtain the general form:

$$w^* = \frac{48\pi R^2}{\beta[12fH^* + [\beta - \frac{\beta P_e}{Ra}]h^{*3} - 12l^{*2}h^* + 24l^{*3} \tanh[\frac{h^*}{2l^*}]]} = g(h^*, P_e, l^*) \quad (1.29)$$

### 1.6 Squeeze Time-Film For Flexibility Knee Joint: [1]

Film thickness between two articular cartilage different with cycle time where squeeze projected on knee joint transfer film thickness to minimal film. Time film thickness was depended on load carrying capacity and flexibility knee joint from the equation (1.29) we obtain the time of approach as follows:

$$\frac{dh_m^*}{dt^*} = \frac{1}{12\pi \int_0^2 g(h^*, P_e, l^*) dr^*} \quad (1.30)$$

where  $t^* = \frac{Wh_0^2}{\mu R^2 \phi}$  the dimensionless time of approach .

$$t^* = -12\pi \int_{h_m^*}^1 g(h^*, \beta, l^*, \phi^*) dh_m^* \quad (1.31)$$

If the dimensionless time approach tends to zero , then the minimum film thickness is high. Equation (1.32) is a highly non-linear differential with initial condition,  $h_m^* = 2$  at  $t^* = 0$ .

$$t^* = \frac{226.06(R)^4}{\{\beta * 12 * F * 4 + (\beta - \frac{\beta P_e}{Ra})((h)^3 - 12 * (l)^2 + 24 * (l)^3 * \text{Tanh}[\frac{h}{2 * (l)}])\}} - \frac{226.06 * (R)^4 * h_m}{\{\beta * 12 * F * 4 + (\beta - \frac{\beta P_e}{Ra})((h)^3 - 12 * (l)^2 + 24 * (l)^3 * \text{Tanh}[\frac{h}{2 * (l)}])\}} \quad (1.32)$$

## 1.7 Result and Discussion:

On the basis of momentum equations and continuity equation, this chapter discusses effective of plecte number and flexibility of the articular cartilage on squeeze film characteristics in synovial human knee joint in daily active, and determine type of flexibility in different lubrication.

### 1.7.1 Squeeze Film Pressure:

The variation of the dimensionless squeeze film pressure ( $p^*$ ) generated by the squeeze film action as a function of dimensionless radial coordinate ( $r^*$ ) for different values of peclt number parameters ( $P_e$ ) is shown in figure (2) with using equation (3.48). It is observed that the effect of the peclt number fluid ( $P_e \neq 0$ ) is to increase film pressure, especially in the nearby areas of the position coordinate ( $r^* = 0$ ) than those of the Newtonian case ( $P_e = 0.7$ ), see table (3.2), this result appears important peclt number relationship with the fluid flow about increased pressure distribution. The effect of couple stress

parameter ( $l$ ) on the variation of ( $p^*$ ) with coordinate ( $r^*$ ) is shown in figure (3). It is observed that the pressure film ( $p^*$ ) increases with increasing values of ( $l$ ). The effect of flexibility parameter ( $F$ ) on the variation of ( $p^*$ ) with coordinate ( $r^*$ ) is shown in figure (4). It is observed that the pressure film ( $p^*$ ) increases with decreasing values of ( $F$ ) when have joint. high flexibility then the pressure distribution is lower. The effect of a film thickness of gab between two articular parameter ( $h$ ) on the variation of ( $p^*$ ) with coordinate ( $r^*$ ) is shown in figure (5). It is observed that the pressure film ( $p^*$ ) increases with decreasing values of ( $h$ ) in different type lubrication (hydrodynamic, squeeze and elastohydrodynamic). The effects of the surface roughness parameter ( $R_a$ ), and efective radius of curvature parameter ( $R$ ) on the variations of ( $p^*$ ) with coordinate ( $r^*$ ) is shown in figures (6) and (7). It is observed that the pressure film ( $p^*$ ) increases with increases values of ( $R_a$ ) and ( $R$ ).

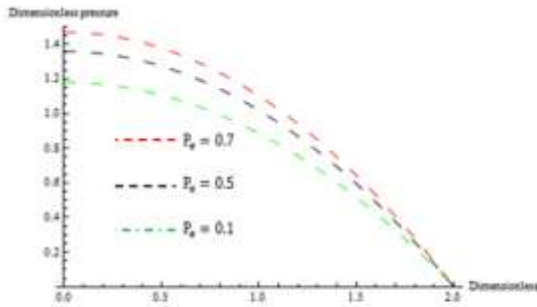


Figure (2): shows the variation of dimensionless pressure ( $p^*$ ) with dimensionless radial coordinate ( $r^*$ ) for different peclet number parameters ( $P_e$ )

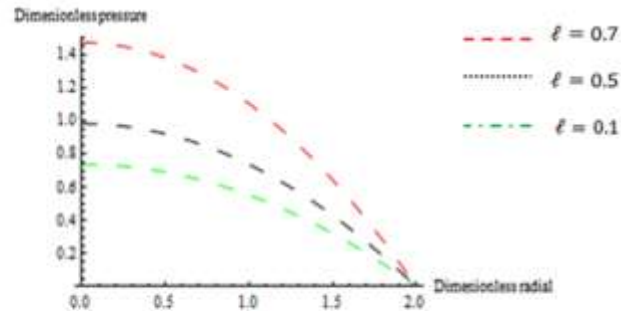


Figure (3): shows the variation of dimensionless pressure ( $p^*$ ) with dimensionless radial coordinate ( $r^*$ ) for different couple stress length parameter ( $l$ )

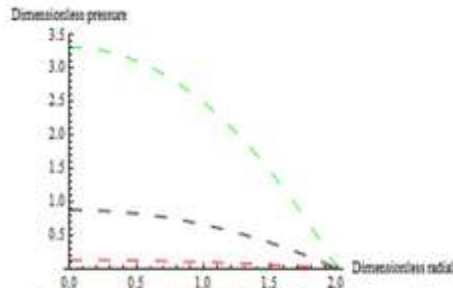


Figure (4): shows the variation of dimensionless pressure ( $p^*$ ) with dimensionless radial coordinate ( $r^*$ ) for different flexibility parameter ( $F$ ) ( $\beta = 0.02, P_e = 0.7$  and  $h = 16$ )

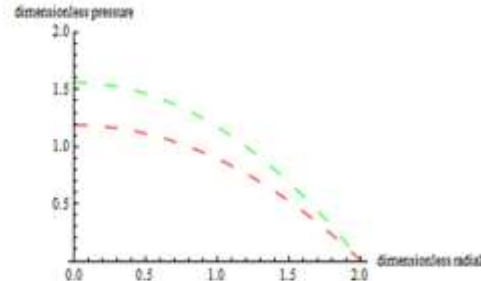


Figure (5): shows the variation of dimensionless pressure ( $p^*$ ) with dimensionless radial coordinate ( $r^*$ ) for the different film thickness of gab between two articular parameters.

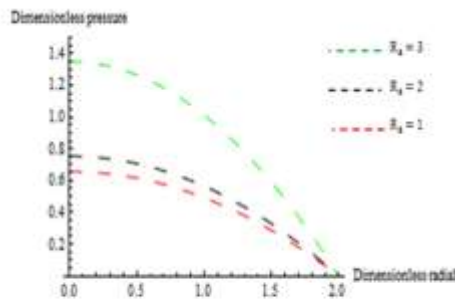


Figure (6): shows the variation of dimensionless pressure ( $p^*$ ) with dimensionless radial coordinate ( $r^*$ ) for different the surface roughness parameter ( $R_a$ ) ( $\beta = 0.04, h = 10$  and  $F = 3$ )

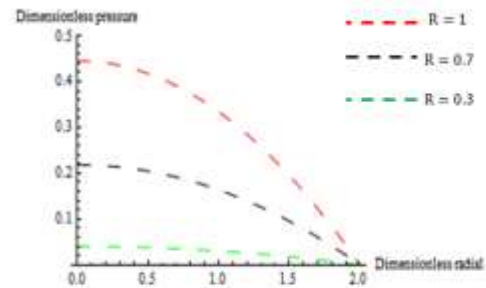


Figure (7): shows the variation of dimensionless pressure ( $p^*$ ) with dimensionless radial coordinate ( $r^*$ ) for the different effective radius of curvature parameter ( $R$ ) ( $\beta = 0.04, h = 11$  and  $F = 4$ ).

### 1.7.2 Load Carrying Capacity:

The dimensionless load carrying capacity ( $w^*$ ) as a function of dimensionless flexibility ( $F$ ) for different values of pecllet number parameters ( $P_e$ ) is shown in figure (8). After applying equation (1.29) in the computer program. It is observed that the effect of the pecllet number when film thickness ( $h^* = 10$ ) is increase load carrying capacity, especially in the nearby areas of the position ( $F = 0$ ) because plecte number effects give a higher film pressure so in case SL. where film thickness ( $h^* = 7$ ). Load carrying capacity different with time cycle this appear clearly in EHL. The dimensionless load carrying capacity ( $w^*$ ) as a function of dimensionless flexibility ( $F$ ) for different dimensionless couple stress is shown in figure (9). It is observed that the effect of the couple stress fluid ( $\Gamma^* \neq 0$ ) is to increase load carrying capacity especially in the vicinity of the position ( $h^* = 4$ ) and it found couple stress different with type of flexibility The dimensionless load carrying capacity ( $w^*$ ) as a function of the effective radius of curvature ( $R$ ) for different values of flexibility parameters ( $F$ ) is shown in figure (10). It is observed that increase the flexibility ( $F$ ) of knee joint when film thickness ( $h = 10$ ) lead to increase load carrying capacity, since they expand the tissue and thus increase the flow of fluid responsible for generating pressure. So EHL. when film thickness ( $h = 4$ ) increased flexibility leads to increased load carrying capacity of joint. The effect of a film thickness of gab between two articular parameter ( $h$ ) on the variation of ( $w^*$ ) with ( $F$ ) is shown in figure (11). It is observed that the load carrying capacity increases with decreasing values of ( $h$ ). The effect of the surface roughness parameter ( $R_a$ ) on the variations of ( $w^*$ ) with ( $F$ ) is shown in figure (12). It is observed that the load carrying capacity increases with decreasing values of ( $R_a$ ), since the roughness of the knee indicates a decrease in joint endurance and with age, the joint loses its ability to bear weight. The effect of the effective radius of curvature parameter ( $R$ ) on the variation of ( $w^*$ ) with ( $F$ ) is shown in figure (13). It is observed that the load carrying capacity increases with increasing values of ( $R$ ).

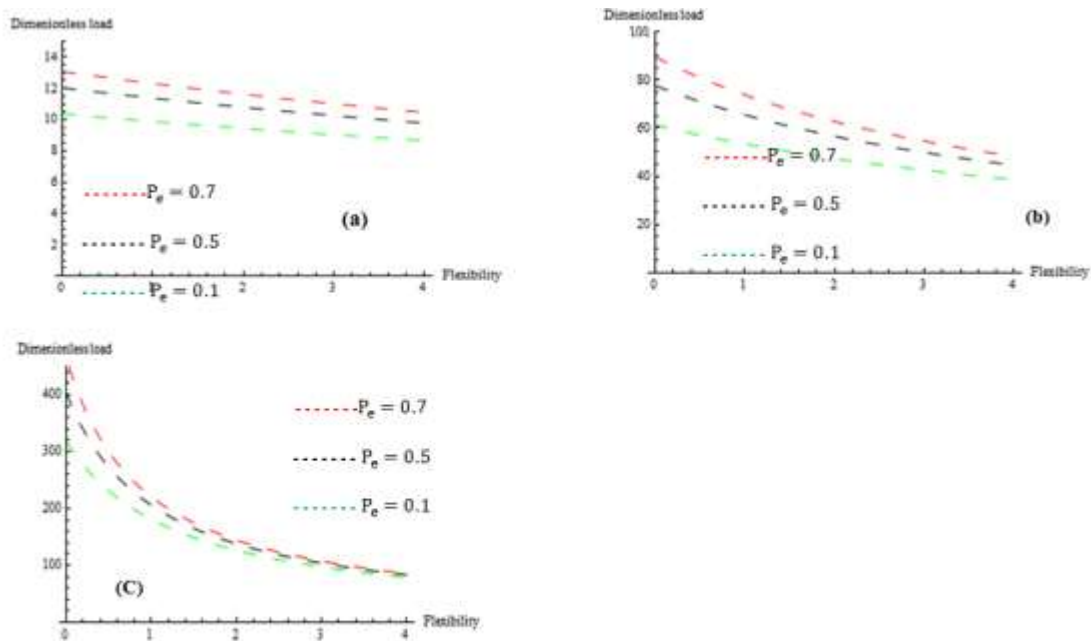


Figure (8): shows the variation of dimensionless load carrying capacity ( $w^*$ ) of with flexibility ( $F$ ) for different pecllet number parameters ( $P_e$ ) ( $\beta = 0.02, R_a = 3, R = 1$ ). (a) HL, (b) SQL and (c) EHL.

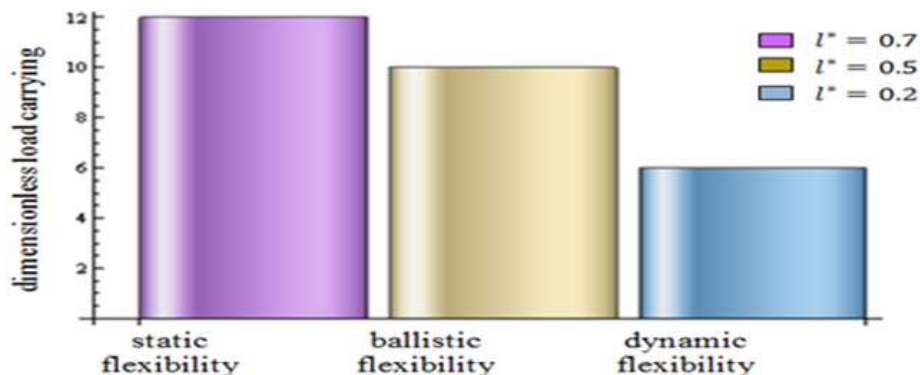


Figure (9): shows the variation of dimensionless load carrying capacity ( $w^*$ ) with dimensionless flexibility ( $F$ ) for different couple stress length parameter ( $l$ ).

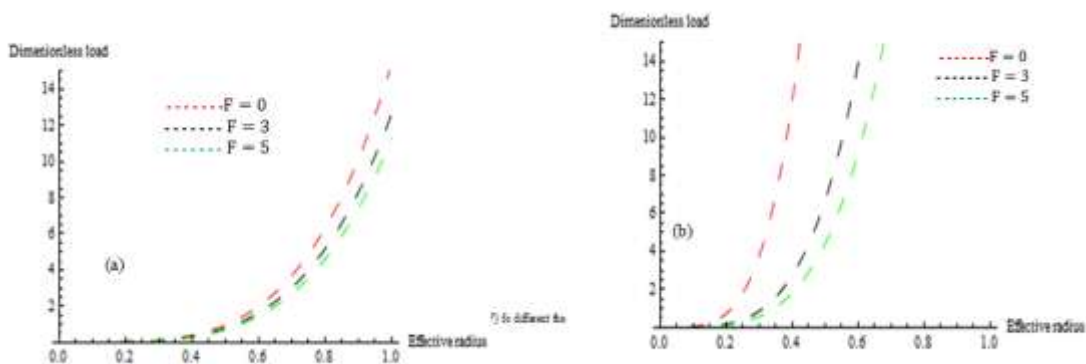


Figure (10): shows the variation of dimensionless load carrying capacity ( $w^*$ ) of with a dimensionless effective radius of

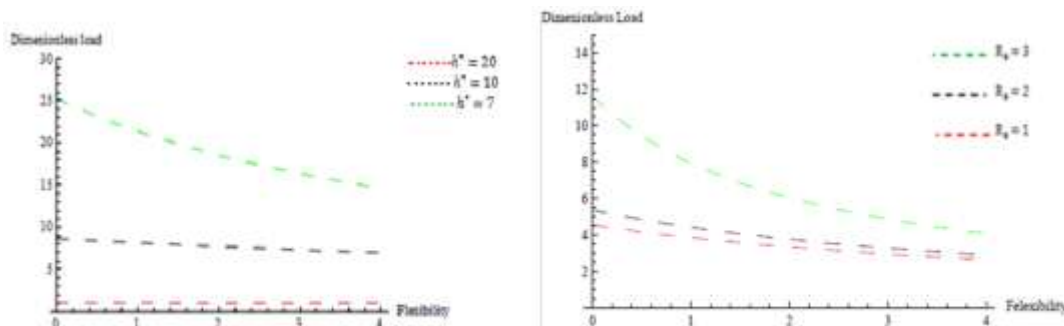


Figure (11): shows the variation of dimensionless load carrying capacity ( $w^*$ ) with a dimensionless flexibility ( $f$ ) for different film thickness

Figure (12): shows the variation of dimensionless load carrying capacity ( $w^*$ ) with a dimensionless flexibility ( $f$ ) for different surface roughness



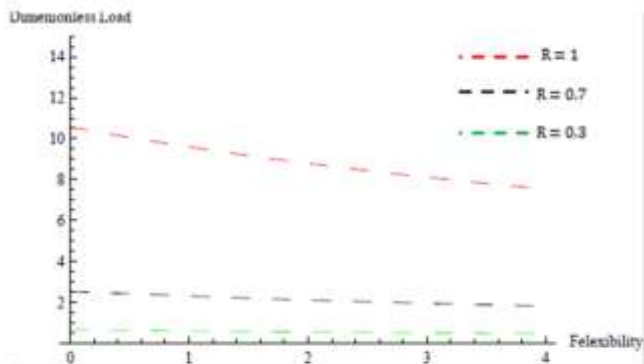


Figure (13): shows the variation of dimensionless load carrying capacity ( $w^*$ ) with a dimensionless flexibility ( $F$ ) for different curvature

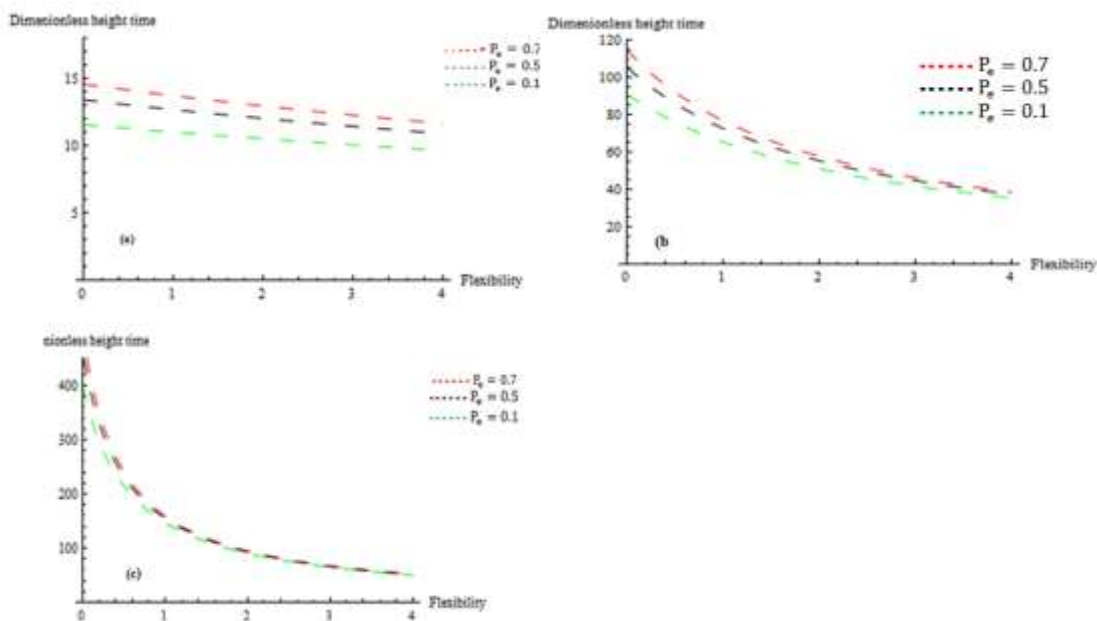


Figure (14): shows the variation of the dimensionless time ( $t^*$ ) of with flexibility ( $F$ ) for different peclot number parameters, (a)HL (b)SQL and (c) EHL.

### Squeeze Time-Film Thickness

The height time of the film thickness is an important factor in describing squeeze film characteristics. this is the time elapsed to reduce the film thickness of different type lubrication to the minimum permissible squeeze film height. the variation of the flexibility ( $F$ ) with the dimensionless height time ( $t^*$ ) for the different values of peclot number parameters ( $P_e$ ) is shown in figure (14) by solving equation (1.32) in the computer program. In non- Newtonian lubricant the film thickness turn into minimum film thickness for different peclot number This process needs longer time compared to the Newtonian lubricant . The effect of couple stress length parameter ( $l$ ) on the variation of ( $t^*$ )with ( $F$ ) is shown in figure (15). It is observed that the time increases with increases values of ( $l$ ). The presence of hyaluronic molecules makes the hydrodynamic pressure very high . The effect of flexibility ( $F$ ) on the variations of ( $t^*$ ) with ( $R$ ) is shown in figure (16). It is observed that the squeeze time increases with decreasing values of ( $F$ ) where appear clearly in EHL more than HL since the type of movements performed by human make the time of transformation film thickness to minimum film thickness more. The effect of a film thickness of gab between two articular parameter ( $h$ ) on the variation of ( $t^*$ ) with ( $F$ ) is shown in figure (17). It is observed that the time ( $t^*$ ) increases with decreasing

values of ( $h$ ) in different stage lubrication (hydrodynamic, squeeze and elastohydrodynamic). The figure (18) explain the surface roughness parameter ( $R_a$ ) on the dimensionless response time ( $t^*$ ) with a value of ( $F$ ), The response time of the squeeze film ( $t^*$ ) decreasing with increasing value the surface roughness parameter ( $R_a$ ). The figure (19) explain the effective radius of curvature parameter ( $R$ ) on the dimensionless response time ( $t^*$ ) with a value of ( $F$ ), The response time of the squeeze film ( $t^*$ ) increasing with increasing value the effective radius of curvature parameter ( $R$ ).

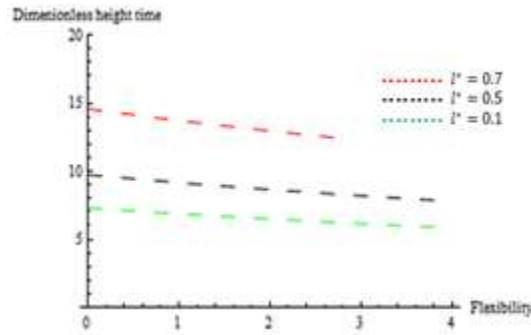


Figure (15): The variation of the dimensionless time ( $t^*$ )e of with a dimensionless flexibility with different couple stress radius of curvature( $R$ ) for different flexibility parameters)

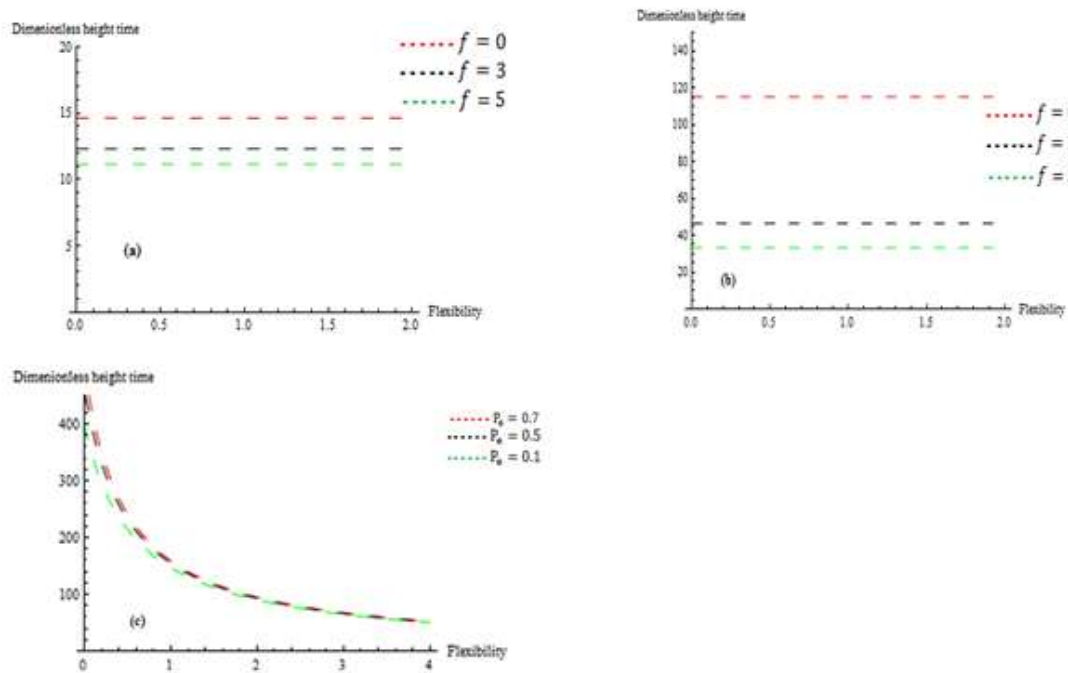


Figure (16): The variation of the dimensionless time ( $t^*$ ) of with a dimensionless effective radius of curvature( $R$ ) for different flexibility parameters( $f$ ) ( $h_m^* = 0.005, \beta = 0.02, P_e = 0.7, h^* = 10, R_a = 3$ ). (a) HL (b) SQL.

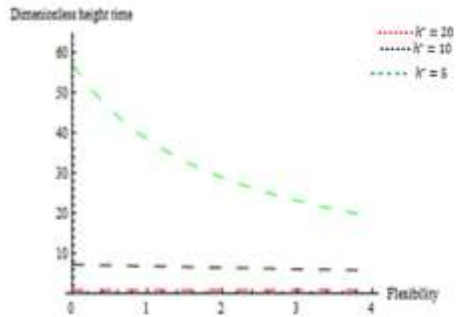


Figure (17): The variation of the dimensionless time ( $t^*$ ) with dimensionless flexibility ( $f$ ) for the different film thickness of gap between two articular parameters ( $h^*$ ) ( $h_m^* = 0.007, \beta = 0.04, P_a = 0.7, R_a = 3, R = 1$ )

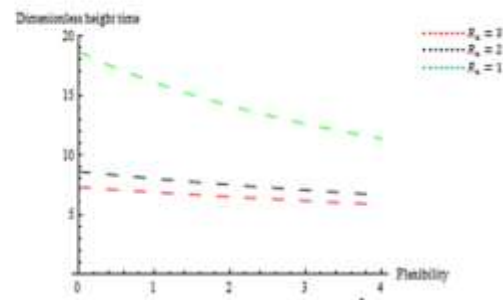


Figure (18): The variation of the dimensionless time ( $t^*$ ) with dimensionless flexibility ( $f$ ) for different the surface roughness parameter ( $R_a$ ) ( $h_m^* = 0.007, \beta = 0.04, P_a = 0.7, h^* = 10, R = 1$ )

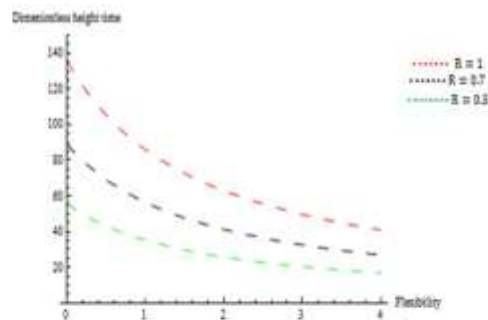


Figure (19): The variation of the dimensionless time ( $t^*$ ) with dimensionless flexibility ( $f$ ) for the different effective radius of curvature parameter ( $R$ ) ( $h_m^* = 0.009, \beta = 0.04, P_a = 0.7, h^* = 10, R_a = 2$ )

## 1.8 Conclusions

The effects of couple stresses on the pressure film are presented in the synovial knee joint on the basis of momentum equations and continuity equation. The modified Reynolds equation, which governs the squeeze film, numerically solved using "(Wolfram Mathematic 8)". We obtained the following results:

1. The effect of couple stress is increase film pressure, load carrying capacity and time in one side and decrease friction coefficient on the other side significantly as compared to Newtonian case.
2. The effect of film thickness parameters is to increases in film pressure , load carrying capacity and time .
3. The effect decrease flexibility parameters is to increases in film pressure, time, and increases of flexibility parameters is to increases load carrying capacity.
4. Roughness when increasing that mean pressure and load so increasing while film thickness decreasing that lead to time approach drop .

## 1.9 Reference

- [1] O. Roure, A. Lindner (2019) "Dynamics of Flexible Fibers in Viscous Flows and Fluids" Annu. Rev. Fluid Mech..51:pp. 539-572.
- [2] C.Wang and Y. H. J. Au (2011)" Study of design parameters for squeeze film air journal bearing – excitation frequency and amplitude, Vol .2:p.147-155.
- [3] E.Yahya (2019) " Squeeze film characteristics in synovial hip joint" IOP Conference Series: Materials Science and Engineering.pp.1-14

[4].E. Yahya, N. Edan "Study surface roughness and friction force of synovial human knee joint with using mathematical model" scientific international conference,pp.109-118,2018.

[5] Heather K. and Susan S." Hyaluronic Acid (HA) Viscosupplementation on Synovial Fluid Inflammation in Knee Osteoarthritis: A Pilot Study " the open Orthopaedics Journal,Vol.7,pp.378-384 ,2013