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# Sensors structures and regional detectability of parabolic distributed systems

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**Abstract**

In this paper, we deal with the linear infinite dimensional systems in a Hilbert space where the dynamics of the system is governed by strongly continuous semi-groups. We study the concept of asymptotic (exponential) regional detectability in connection with the structures of sensors for a class of parabolic distributed parameter systems. For different sensors structures, we give the characterization of the asymptotic (exponential) regional detectability in order that asymptotic (exponential) regional observability be achieved. Furthermore, we apply these results to the regional observer for distributed parameter diffusion systems. © 2001 Elsevier Science B.V. All rights reserved.

*Keywords:* Regional detectability; Regional observers; Diffusion system; Sensors

## 1. Introduction

Many real problems in the control and asymptotic observation of distributed parameter systems can be reformulated as problems of infinite-dimensional systems, in a domain $\Omega$ [1–4]. An extension of detectability, is that of asymptotic (respectively exponential) regional $\omega$-detectability. This concept, was introduced recently by Al-Saphory and El Jai [5], and was focused on the state asymptotic (respectively exponential) detection in a given part $\omega$ of the domain $\Omega$. The purpose of this paper is to give some results related to the link between the regional detectability and sensors structures. We consider a class of distributed systems and we explore various results connected with different types of measurements, domains and boundary conditions. The main reason for introducing the concept of asymptotic (respectively exponential) regional $\omega$-detectability, is the possibility to construct an asymptotic (respectively exponential) regional observer for the considered system. Section 2 concerns the class of considered systems and the characterization of regional strategic sensors. Section 3 devotes to the introduction of regional detectability problem. We discuss this notion with regional observability and structures of sensors. In Section 4, we give an application to various situations of sensors locations and in the last section, we study the relation between the regional observer and regional detectability and we give an application of regional observer to a diffusion system.

## 2. The class of considered systems

Let the following assumptions be given:

- $\Omega$ be a regular bounded open set of $\mathbb{R}^n$ with boundary $\partial\Omega$.
- $[0, T]$, $T > 0$ a time measurement interval.
- $\omega$ be a non-empty given subregion of $\Omega$ with boundary $\Gamma_\omega$. We denote $Q = \Omega\times]0, T[$, $\mathcal{Q} = \Omega\times]0, \infty[$, $\Theta = \partial\Omega\times]0, \infty[$ and $\Sigma = \partial\Omega\times]0, T[$.
- $X, U, \mathcal{O}$ be separable Hilbert spaces where $X$ is the state space, $U$ the control space and $\mathcal{O}$ the observation space. We consider $X = L^2(\Omega)$, $U = L^2(0, \infty, \mathbb{R}^p)$ and $\mathcal{O} = L^2(0, \infty, \mathbb{R}^q)$, where $p$ and $q$ hold for the number of actuators and sensors. The considered system is described by the following parabolic distributed parameter system

$$\begin{cases} \dfrac{\partial x}{\partial t}(t) = Ax(\xi, t) + Bu(t), & \mathcal{Q} \\ x(\xi, 0) = x_0(\xi), & \Omega \\ x(\eta, t) = 0, & \Theta \end{cases} \tag{2.1}$$

augmented with the output function

$$z(\cdot, t) = Cx(\cdot, t) \tag{2.2}$$

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where $A$ is a second order linear differential operator which generates a strongly continuous semi-group $(S_A(t))_{t\geq 0}$ on the Hilbert space $X = L^2(\Omega)$ and is self-adjoint with compact resolvent. The operators $B \in \mathscr{L}(U, X)$ and $C \in \mathscr{L}(X, \mathcal{O})$ depend on the structure of actuators and sensors. Under the given assumptions, the system (2.1) has a unique solution given by

$$x(\xi, t) = S_A(t)x_0(\xi) + \int_0^t S_A(t-\tau)Bu(\tau)\,d\tau \tag{2.3}$$

The measurements can be obtained by the use of zone, pointwise or lines sensors which may be located in $\Omega$ (or $\partial\Omega$) [4].

- A sensor is defined by any couple $(D, f)$ where $D$, a nonempty closed subset of $\Omega$, is the spatial support of the sensor and $f \in L^2(D)$ defines the spatial distribution of the sensing measurements on $D$. According to the choice of the parameters $D$ and $f$, we have various types of sensors. In the case of a pointwise sensor, $D$ is reduced to the location $\{b\}$ with $b \in \Omega$ and $f = \delta(\cdot - b)$ where $\delta$ is the Dirac mass concentrated in $b$. In the case of $q$ sensors, we shall consider $(D_i, f_i)_{1\leq i\leq q}$ where $D_i \subset \Omega$ (or $D_i \subset \partial\Omega$) and $f_i \in L^2(D_i)$, with $D_i \bigcap D_j = \emptyset$ if $1 \leq i \neq j \leq q$. The output function (2.2) may be written in the form

$$z(\cdot, t) = \langle x(\cdot, t), f_i(\cdot)\rangle_{L^2(D_i)} \tag{2.4}$$

The sensors may be pointwise, denoted by $(b_i, \delta_{b_i})$ where $b_i \in \Omega$, the Eq. (2.2) then becomes

$$z(\cdot, t) = x(b_i, t) \tag{2.5}$$

Finally the sensors may be pointwise (or zone) with $D_i \subset \partial\Omega$. In the case of boundary pointwise sensors, the output function is similar to (2.5) with $b_i \in \partial\Omega$ and in the case with $D_i = \Gamma_i$ are zones subset of $\partial\Omega$. The output is given by

$$z(\cdot, t) = \langle x(\cdot, t), f_i(\cdot)\rangle_{L^2(\Gamma_i)} \tag{2.6}$$

- The sensors $(D_i, f_i)_{1\leq i\leq q}$ are said to be $\omega$-strategic if the system (2.1) together with the output function (2.2) is weakly regionally observable in $\omega$ [6].

## 3. Asymptotic (exponential) regional detectability

The detectability is in some sense a dual notion of stabilizability. In El Jai and Pritchard [4] this notion was considered in the whole domain. In this section, we shall extend these results to the regional case by considering $\omega$ as subregion of $\Omega$ (Fig. 1). Regional detectability characterization needs some assumptions which are related to the asymptotic behaviour of the system, i.e. stability.

### 3.1. Definitions and characterizations

We recall that the problem of stability is one of the most important in the analysis of control systems.

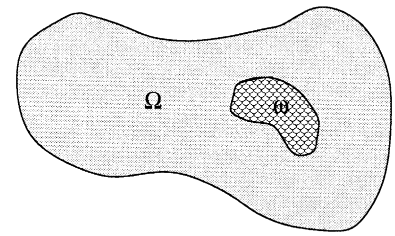


Fig. 1. Considered domain $\Omega$ and subregion $\omega$.

- The semi-group $(S_A(t))_{t\geq 0}$ is said to be asymptotically (respectively exponentially) stable if for every initial state $x_0(\cdot) \in L^2(\Omega)$ the corresponding solution of the autonomous system of (2.1) $x(\xi, t)$ converges asymptotically (respectively exponentially) to zero as $t$ tends to $\infty$. It is easy to see that the system (2.1) is exponentially stable if and only if for some positive constants $M$ and $\alpha$

$$||S_A(t)||_{L^2(\Omega)} \leq M e^{-\alpha t}, \quad \forall t \geq 0$$

If $(S_A(t))_{t\geq 0}$ is an asymptotically (respectively exponentially) stable semi-group, then for all $x_0(\cdot) \in L^2(\Omega)$ the solution of the associated autonomous system satisfies

$$\lim_{t\to\infty} ||x(\cdot, t)||_{L^2(\Omega)} = \lim_{t\to\infty} ||S_A(t)x_0(\cdot)||_{L^2(\Omega)} = 0 \tag{3.1}$$

- The system (2.1) is said to be asymptotically (respectively exponentially) stable, if the operator $A$ generates a semigroup which is asymptotically (respectively exponentially) stable. In the finite dimensional linear systems, the concept of exponential stability is equivalent to asymptotic stability. It is not the case if the state space $X$ is infinite dimensional.
- The system (2.1) together with the output (2.2) is said to be asymptotically (respectively exponentially) detectable if there exists an operator $H : \mathcal{O} \to X$ such that $A - HC$ generates a strongly continuous semi-group $(S_H(t))_{t\geq 0}$ which is asymptotically (exponentially) stable.
- If a system is asymptotically (respectively exponentially) detectable then it is possible to construct an asymptotic (respectively exponential) observer for the original system. If we consider the system

$$\begin{cases} \dfrac{\partial y}{\partial t}(\xi, t) = Ay(\xi, t) + Bu(t) + H(z(\cdot, t) - Cy(\xi, t)), & \mathcal{Q} \\ y(\xi, 0) = y_0(\xi), & \Omega \\ y(\eta, 0) = 0, & \Theta \end{cases} \tag{3.2}$$

then $y(\xi, t)$ estimates asymptotically (respectively exponentially) the state $x(\xi, t)$ because $e(\xi, t) = x(\xi, t) - y(\xi, t)$ satisfies $(\partial e/\partial t)(\xi, t) = (A - HC)e(\xi, t)$ with $e(\xi, 0) = x_0(\xi, t) - y_0(\xi, t)$ and if the system is asymptotically (respectively exponentially) detectable, it is possible to choose $H$ which realizes $\lim_{t\to\infty} ||e(\cdot, t)||_{L^2(\Omega)} = 0$.
- The system (2.1) together with the output function (2.2) is said to be asymptotically (respectively exponentially)

observable if there exists a dynamical system which is an asymptotic (respectively exponential) observer (estimator) for the systems (2.1)–(2.2).

**Remark 3.1.** In this paper we only need the relation (3.1) to be true on a given subdomain $\omega \subset \Omega$

$$\lim_{t\to\infty} ||x(\cdot,t)||_{L^2(\omega)} = 0 \tag{3.3}$$

We may refer to this as asymptotic (respectively exponential) regional $\omega$-stability which is equivalent for the considered class of systems to the asymptotically (respectively exponential) stability.

**Definition 3.2.** The system (2.1) is said to be asymptotically (respectively exponentially) regionally $\omega$-stable, if the operator $A$ generates a semi-group which is regionally asymptotically (respectively exponentially) $\omega$-stable.

**Definition 3.3.** The system (2.1) together with the output function (2.2) is said to be asymptotically (respectively exponentially) regionally $\omega$-detectable if there exists an operator

$$H_\omega : \mathcal{O} \to L^2(\omega)$$

such that $(A - H_\omega C)$ generates a strongly continuous semi-group $(S_{H_\omega}(t))_{t\geq 0}$ which is asymptotically (respectively exponentially) regionally $\omega$-stable.

It is clear that:

1. A system which is asymptotically (exponentially) detectable, is regionally asymptotically (exponentially) $\omega$-detectable,
2. A system which is exponentially regionally $\omega$-detectable, is asymptotically regionally $\omega$-detectable,
3. A system which is asymptotically (exponentially) regionally $\omega$-detectable, is asymptotically (exponentially) regionally $\omega_1$-detectable, for every subset $\omega_1$ of $\omega$.

### *3.2. Regional detectability and regional observability*

It has been shown that a system which is exactly observable is detectable [4]. This interesting result remains nonconstructive because it is related to the choice of the operator $H$ in (3.2). The autonomous system associated to (2.1) with output function (2.2) may be written by the form

$$\begin{cases} \dfrac{\partial x}{\partial t}(\xi,t) = Ax(\xi,t), & Q \\ x(\xi,0) = x_0(\xi), & \Omega \\ x(\eta,t) = 0, & \Sigma \\ z(\cdot,t) = Cx(\cdot,t), & Q \end{cases} \tag{3.4}$$

where $x(\xi,0)$ is supposed to be unknown. Define now the operator

$$K : x \in X \to Kx = CS_A(t)x \in \mathcal{O} \tag{3.5}$$

then $z(\cdot,t) = K(t)x_0(\cdot)$. We denote by $K^* : \mathcal{O} \to X$ the adjoint of $K$ given by

$$K^* z^*(\cdot,t) = \int_0^t S^*(s)C^* z^*(\cdot,s)\,\mathrm{d}s \tag{3.6}$$

Consider a subdomain $\omega$ of $\Omega$ and let $\chi_\omega$ be the function defined by

$$\begin{aligned} &\chi_\omega : L^2(\Omega) \to L^2(\omega) \\ &x \to \chi_\omega = x_{|_\omega} \end{aligned} \tag{3.7}$$

where $x_{|_\omega}$ is the restriction of the state $x$ to $\omega$.

- The system (3.4) is said to be exactly (respectively weakly) regionally observable in $\omega$ if

  $$\operatorname{Im} \chi_\omega K^* = L^2(\omega) \quad (\text{respectively} \quad \overline{\operatorname{Im} \chi_\omega K^*} = L^2(\omega))$$
- If the system (3.4) is weakly regionally observable in $\omega$ then $x(\xi,0)$ is given by

  $$x(\xi,0) = (K^*K)^{-1}K^*z(\cdot,t)$$

  where $(K^*K)^{-1}K^*$ is the pseudo-inverse of the operator $K$ [6,7]. These definitions have been extended to regional boundary case [8–10]. However, one can easily deduce the following important results.

**Corollary 3.4.** *A system which is exactly observable, then it is asymptotically (exponentially) observable.*

**Corollary 3.5.** *If the system* (2.1) *together with output function* (2.2) *is exactly regionally ω-observable, then it is regionally asymptotically (respectively exponentially) ω-detectable.*

From this result, we can easily deduce that there exists $\gamma > 0$ such that

$$||CS_A(t)x(\cdot,t)||_{L^2(0,T,\mathcal{O})} \geq \gamma ||\chi_\omega S_A(t)x(\cdot,t)||_{L^2(\omega)},$$
$$\forall x \in L^2(\omega)$$

Thus the notion of regional detectability is far less restrictive than that of exact regional observability in $\omega$.

### *3.3. Sensors and regional detectability*

As in El Jai and Pritchard [4], we shall develop a characterization result that links the regional detectability and sensors structures. For that purpose, let us consider the set $(\varphi_i)$ of functions of $L^2(\Omega)$ orthonormal in $L^2(\omega)$ associated with the eigenvalues $\lambda_i$ of multiplicity $m_i$ and suppose that the system (2.1) has $J$ unstable modes. Thus the sufficient condition of asymptotic (respectively exponential) regional $\omega$-detectability is given by the following theorem.

**Theorem 3.6.** *Suppose that there are q sensors* $(D_i,f_i)_{1\leq i\leq q}$ *and the spectrum of A contains J eigenvalues with non-negative real parts. The system* (2.1) *together with output function* (2.2) *is asymptotically (respectively exponentially)* *regionally ω-detectable if and only if*

1. $q \geq m$
2. *rank* $G_i = m_i, \quad \forall i,\ i = 1, \ldots, J$ *with*

$$G = (G_{ij}) = \begin{cases} \langle \varphi_j(\cdot), f_i(\cdot) \rangle_{L^2(D_i)}, & \text{for zone sensors} \\ \varphi_j(b_i), & \text{for pointwise sensors} \\ \langle \varphi_j(\cdot), f_i(\cdot) \rangle_{L^2(\Gamma_i)}, & \text{for boundary zone sensors} \end{cases}$$

*where* $\sup m_i = m < \infty$ *and* $j = 1, \ldots, \infty$.

**Proof.** The proof is limited to the case of zone sensors.

1. Under the assumptions of Section 2, the system (2.1) can be decomposed by the projections $P$ and $I - P$ on two parts, unstable and stable. The state vector may be given by $x(\xi, t) = [x_1(\xi, t)\, x_2(\xi, t)]^{\mathrm{tr}}$ where $x_1(\xi, t)$ is the state component of the unstable part of the system (2.1), may be written in the form

$$\begin{cases} \dfrac{\partial x_1}{\partial t}(\xi, t) = A_1 x_1(\xi, t) + PBu(t), & \mathcal{Q} \\ x_1(\xi, 0) = x_{1_0}(\xi), & \Omega \\ x_1(\eta, t) = 0, & \Theta \end{cases} \tag{3.8}$$

and $x_2(\xi, t)$ is the component state of the stable part of the system (2.1) given by

$$\begin{cases} \dfrac{\partial x_2}{\partial t}(\xi, t) = A_2 x_2(\xi, t) + (I - P)Bu(t), & \mathcal{Q} \\ x_2(\xi, 0) = x_{2_0}(\xi), & \Omega \\ x_2(\eta, t) = 0, & \Theta \end{cases} \tag{3.9}$$

The operator $A_1$ is represented by a matrix of order $(\sum_{i=1}^{J} m_i, \sum_{i=1}^{J} m_i)$ given by $A_1 = \mathrm{diag}[\lambda_1, \ldots, \lambda_1, \ldots, \lambda_J, \ldots, \lambda_J]$ and $PB = [G_1^{\mathrm{tr}}, G_2^{\mathrm{tr}}, \ldots, G_J^{\mathrm{tr}}]$. By using the condition (2) of this theorem, we deduce that the suite $(D_i, f_i)_{1 \leq i \leq q}$ of sensors is ω-strategic for the unstable part of the system (2.1), the subsystem (3.8) is weakly regionally observable in ω, and since it is finite dimensional, then it is exactly regionally observable in ω. Therefore it is asymptotically (respectively exponentially) regionally ω-detectable, and hence there exists an operator $H_\omega^1$ such that $A_1 - H_\omega^1 C$ which satisfies the following:

$$\exists M_\omega^1,\ \alpha_\omega^1 > 0 \text{ such that } \| \mathrm{e}^{(A_1 - H_\omega^1 C)t} \|_{L^2(\omega)} \leq M_\omega^1 \mathrm{e}^{-\alpha_\omega^1(t)}$$

and then we have

$$\| x_1(\cdot, t) \|_{L^2(\omega)} \leq M_\omega^1 \mathrm{e}^{-\alpha_\omega^1(t)} \| P x_0(\cdot) \|_{L^2(\omega)}$$

Since the semi-group generated by the operator $A_2$ is asymptotically (respectively exponentially) regionally ω-stable, then there exist $M_\omega^2,\ \alpha_\omega^2 > 0$ such that

$$\| x_2(\cdot, t) \|_{L^2(\omega)} \leq M_\omega^2 \mathrm{e}^{-\alpha_\omega^2(t)} \| (I - P) x_0(\cdot) \|_{L^2(\omega)} + \int_0^t M_\omega^2 \mathrm{e}^{-\alpha_\omega^2(t - \tau)} \| (I - P) x_0(\cdot) \|_{L^2(\omega)} \| u(\tau) \| \, \mathrm{d}\tau$$

and therefore $x(\xi, t) \to 0$ when $t \to \infty$. Finally, the systems (2.1)–(2.2) is asymptotically (respectively exponentially) regionally detectable in ω.

2. If the system (2.1) together with the output function (2.2) is asymptotically (respectively exponentially) regionally detectable in ω, there exists an operator $H_\omega \in \mathscr{L}(L^2(0, \infty, \mathbb{R}^q), L^2(\omega))$, such that $A - H_\omega C$ generates an asymptotically (respectively exponentially) regionally stable, strongly continuous semi-group $(S_{H_\omega}(t))_{t \geq 0}$ on the space $L^2(\omega)$ which satisfies the following

$$\exists M_\omega,\ \alpha_\omega > 0 \text{ such that } \| \chi_\omega S_{H_\omega}(t) \|_{L^2(\omega)} \leq M_\omega \mathrm{e}^{-\alpha_\omega(t)}$$

Thus the unstable subsystem (3.8) is asymptotically (respectively exponentially) regionally detectable in ω. Since this subsystem is of finite dimensional, then it is exactly regionally observable. Therefore (3.8) is weakly regionally observable and hence it is ω-strategic, i.e. $[K\chi_\omega^* x^*(\cdot, t) = 0 \Rightarrow x^*(\cdot, t) = 0]$ [6]. For $x^*(\cdot, t) \in L^2(\omega)$, we have

$$K\chi_\omega^* x^*(\cdot, t) = \left( \sum_{j=1}^{J} \mathrm{e}^{\lambda_j t} \langle \varphi_j(\cdot), x^*(\cdot, t) \rangle_{L^2(\omega)} \langle \varphi_j(\cdot), f_i(\cdot) \rangle_{L^2(\Omega)} \right)_{1 \leq i \leq q}$$

If the unstable system (3.8) is not ω-strategic, there exists $x^*(\cdot, t) \in L^2(\omega)$, such that $K\chi_\omega^* x^*(\cdot, t) = 0$, this leads

$$\sum_{j=1}^{J} \langle \varphi_j(\cdot), x^*(\cdot, t) \rangle_{L^2(\omega)} \langle \varphi_j(\cdot), f_i(\cdot) \rangle_{L^2(\Omega)} = 0$$

The state vectors $x_i$ may be given by

$$x_i(\cdot, t) = [\langle \varphi_1(\cdot), x^*(\cdot, t) \rangle_{L^2(\omega)} \langle \varphi_J(\cdot), x^*(\cdot, t) \rangle_{L^2(\omega)}]^{\mathrm{tr}} \neq 0$$

we then obtain $G_i x_i = 0$ for all $i,\ i = 1, \ldots, J$ and therefore rank $G_i \neq m_i$. •

## 4. Application to measurements structures

In this section we give the specific results related to some examples of sensors structures and we apply these results to different situations of the domain, which usually follow from symmetry considerations. We consider the two-dimensional system defined on $\Omega = ]0, 1[\times]0, 1[$ by the form

$$\begin{cases} \dfrac{\partial x}{\partial t}(\xi_1, \xi_2, t) = \dfrac{\partial^2 x}{\partial \xi_1^2}(\xi_1, \xi_2, t) + \dfrac{\partial^2 x}{\partial \xi_2^2}(\xi_1, \xi_2, t) + x(\xi_1, \xi_2, t) & \mathcal{Q} \\ x(\eta_1, \eta_2, t) = 0 & t > 0 \\ x(\xi_1, \xi_2, 0) = x_0(\xi_1, \xi_2) & \Omega \end{cases} \tag{4.1}$$

together with output function is given by (2.4), (2.5) or (2.6). Let $\omega = ]\alpha_1, \beta_1[\times]\alpha_2, \beta_2[$ be the considered region is subset of $]0,1[\times]0,1[$. In this case the eigenfunctions of the system (4.1) for Dirichlet boundary conditions are given by

$$\varphi_{ij}(\xi_1,\xi_2)=\left(\frac{4}{(\beta_1-\alpha_1)(\beta_2-\alpha_2)}\right)^{1/2}\times\sin i\pi\left(\frac{\xi_1-\alpha_1}{\beta_1-\alpha_1}\right)\sin j\pi\left(\frac{\xi_2-\alpha_2}{\beta_2-\alpha_2}\right)\quad(4.2)$$

associated with eigenvalues

$$\lambda_{ij}=-\left(\frac{i^2}{(\beta_1-\alpha_1)^2}+\frac{j^2}{(\beta_2-\alpha_2)^2}\right)\pi^2\quad(4.3)$$

In the case of Neumann or mixed boundary conditions, we have different functions. We illustrate some practical examples of the linear parabolic system (4.1).

### 4.1. Case of a zone sensor

Consider the system (4.1) together with output function (2.2) where the sensor supports $D$ are located in $\Omega$ (or $\partial\Omega$).

#### 4.1.1. Internal zone sensor

We discuss this case with different domains.

*Rectangular domain*: the output function (2.2) can be written by the form

$$z(t)=\int_D x(\xi_1,\xi_2,t)f(\xi_1,\xi_2)\,\mathrm{d}\xi_1\,\mathrm{d}\xi_2\quad(4.4)$$

where $D\subset\Omega$ is the location of the zone sensor and $f\in L^2(D)$. In the case of (Fig. 2), the eigenfunctions and the eigenvalues are given in the Eqs. (4.2) and (4.3). However, if we suppose that

$$\frac{(\beta_1-\alpha_1)^2}{(\beta_2-\alpha_1)^2}\notin\mathbb{Q}\quad(4.5)$$

then $m=1$ and one sensor may be sufficient for asymptotic (respectively exponential) regional $\omega$-detectability. The measurement supports is rectangular with $D=[\xi_{1_0}-l_1,\xi_{1_0}+l_1]\times[\xi_{2_0}-l_2,\xi_{2_0}+l_2]$. We then have the result.

**Corollary 4.1.** *Suppose that $f_1$ is symmetric about $\xi_1=\xi_{1_0}$ and $f_2$ is symmetric with respect to $\xi_2=\xi_{2_0}$. The system* (4.1) *together with the output function* (4.4) *is not asymptotically (respectively exponentially) regionally $\omega$-detectable if $i(\xi_{1_0}-\alpha_1)/(\beta_1-\alpha_1)$ and $i(\xi_{2_0}-\alpha_2)/(\beta_2-\alpha_2)\in\mathbb{N}$ for some $i$, $i=1,\dots,J$.*

*Circular domain*: consider the systems (4.1) and (2.4). So, this system may be given by the following form

$$\begin{cases}\dfrac{\partial x}{\partial t}(r,\theta,t)=\dfrac{\partial^2 x}{\partial r^2}(r,\theta,t)+\dfrac{\partial^2 x}{\partial\theta^2}(r,\theta,t)+x(r,\theta,t) & \mathcal{Q}\\ x(r,\theta,0)=x_0(r,\theta) & \Omega\\ x(1,\theta,t)=0 & \theta\in[0,2\pi],\ t>0\end{cases}\quad(4.6)$$

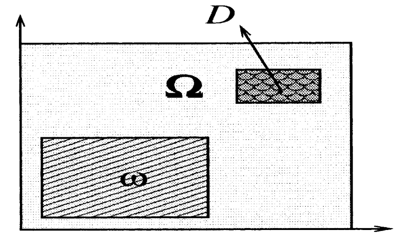


Fig. 2. Domain $\Omega$, subdomain $\omega$ and location $D$ of internal zone sensor.

and with output function

$$z_i(t)=\int_{D_i}x(r_i,\theta_i,t)f(r_i,\theta_i)\,\mathrm{d}r_i\,\mathrm{d}\theta_i,\quad 0\le\theta_i\le2\pi,\quad \frac{1}{2}r_{i_\omega}<r_i<\frac{1}{2},\quad 2\le i\le q\quad(4.7)$$

where $\Omega=D(0,1)$ is defined as in (Fig. 3). So, the eigenfunctions and eigenvalues concerning the region $\omega=D(0,r_\omega)\subset\Omega$, $\forall r_\omega\in]0,1[$ are defined by

$$\lambda_{ij}=-\beta_{ij}^2,\qquad i\ge0,\quad j\ge1\quad(4.8)$$

where $\beta_{ij}$ are the zeros of the Bessel functions $J_i$ and

$$\begin{array}{ll}\varphi_{0j}(r,\theta)=J_0(\beta_{0j}r), & j\ge1\\ \varphi_{ij_1}(r,\theta)=J_i(\beta_{ij_1}r)\cos(i\theta), & i,j_1\ge1\\ \varphi_{ij_2}(r,\theta)=J_i(\beta_{ij_2}r)\sin(i\theta), & i,j_2\ge1\end{array}\quad(4.9)$$

with multiplicity $m_i=2$ for all $i,j\neq0$ and $m_i=1$ for all $i,j=0$. In this case, the asymptotic (respectively exponential) regional detectability in $\omega$ is required at least two zone sensors $(D_i,f_i)_{2\le i\le q}$ where $D_i=(r_i,\theta_i)_{2\le i\le q}$ [4]. Thus we have the result.

**Corollary 4.2.** *Suppose that $f_i$ and $D_i$, are symmetric with respect to $\theta=\theta_i$, for all $i, 2\le i\le q$. Then the system* (4.6)–(4.7) *is not regionally asymptotically (respectively exponentially) $\omega$-detectable if $i_0(\theta_1-\theta_2)/\pi\in\mathbb{N}$ for some $i_0$, $i_0=1,\dots,J$.*

#### 4.1.2. Boundary zone sensor

We consider the system (4.1) with the Dirichlet boundary conditions and output function (2.6). We study this case with different geometrical domains.

*The domain* $]0,1[\times]0,1[$: now the output function (2.2) is given by

$$z(t)=\int_\Gamma\frac{\partial x}{\partial v}(\eta_1,\eta_2,t)f(\eta_1,\eta_2)\,\mathrm{d}\eta_1\,\mathrm{d}\eta_2\quad(4.10)$$

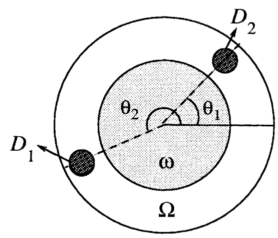


Fig. 3. Domain $\Omega$, subdomain $\omega$ and locations $D_1, D_2$ of internal zone sensors.

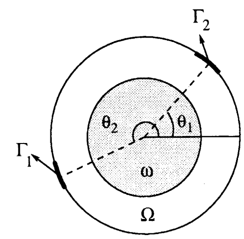


Fig. 5. Circular domain and locations $\Gamma_1, \Gamma_2$ of boundary zone sensors.

where $\Gamma \subset \partial\Omega$ is the support of the boundary sensor and $f \in L^2(\Gamma)$. The sensor $(D, f)$ may be located on the boundary in $\Gamma = [\eta_{1_0} - l, \eta_{1_0} - l] \times \{1\}$, then we have.

**Corollary 4.3.** *If the function f is symmetric with respect to $\eta_1 = \eta_{1_0}$, then the system* (4.1) *together with the output function* (4.10) *is not regionally asymptotically (respectively exponentially) $\omega$-detectable if $i(\eta_{1_0} - \alpha_1)/(\beta_1 - \alpha_1) \in \mathbb{N}$ for some $i$, $1 \le i \le J$.*

When the sensor is located in $\overline{\Gamma} = [\overline{\eta}_{1_0} - l_1, 1] \times \{0\} \bigcup \{1\} \times [0, \overline{\eta}_{2_0} + l_2] = \Gamma_1 \bigcup \Gamma_2$ where $\overline{\Gamma} \subset \partial\Omega$ (see Fig. 4). We obtain the following result.

**Corollary 4.4.** *Suppose that the function $f_{|_{\Gamma_1}}$ is symmetric with respect to $\eta_1 = \overline{\eta}_{1_0}$, and the function $f_{|_{\Gamma_2}}$ is symmetric about $\eta_2 = \overline{\eta}_{2_0}$. Then the system* (4.1) *together with the output function* (4.10) *is not regionally asymptotically (respectively exponentially) $\omega$-detectable if $(\overline{\eta}_{1_0} - \alpha_1)/(\beta_1 - \alpha_1)$ and $(\overline{\eta}_{2_0} - \alpha_2)/(\beta_2 - \alpha_2) \in \mathbb{N}$ for some $i$, $1 \le i \le J$.*

*The domain $D(0,1)$*: here, we consider the system (4.6) with output function

$$z_i(t) = \int_{\Gamma_i} \frac{\partial x}{\partial v}(1, \theta_i, t) f(1, \theta_i)\, \mathrm{d}\theta_i, \quad 0 \le \theta_i \le 2\pi, \quad t > 0 \tag{4.11}$$

In this case, it is necessary to have at least two boundary zone sensors $(\Gamma_i, f_i)_{2 \le i \le q}$ with $\Gamma_i = (1, \theta_i)_{2 \le i \le q}$ and if the function $f_{|_{\Gamma_i}}$ is symmetric with respect to $\theta = (\theta i)_{2 \le i \le q}$ as in Fig. 5. So, we have.

**Corollary 4.5.** *The system* (4.6) *together with the output function* (4.11) *is not regionally asymptotically (respectively exponentially) $\omega$-detectable if $i(\theta_1 - \theta_2)/\pi \in \mathbb{N}$ for some $i$, $1 \le i \le J$.*

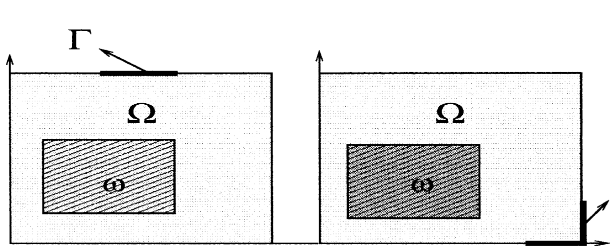


Fig. 4. Rectangular domain and locations $\Gamma, \overline{\Gamma}$ of boundary zone sensors.

### 4.2. Case of a pointwise sensor

Let us consider the case of pointwise sensor located inside of $\Omega$ or on the boundary of $\partial\Omega$.

#### 4.2.1. Internal pointwise sensor

We can discuss the following.

*Case of Fig. 6*: the system (4.1) is augmented with the following output

$$z(t) = \int_{\Omega} x(\xi_1, \xi_2, t)\delta(\xi_1 - b_1, \xi_2 - b_2)\, \mathrm{d}\xi_1\, \mathrm{d}\xi_2 \tag{4.12}$$

where $b = (b_1, b_2)$ is the location of pointwise sensor in $\Omega$ is defined as in Fig. 6. If $(\beta_1 - \alpha_1)/(\beta_2 - \alpha_1) \notin \mathbb{Q}$ then $m = 1$ and one sensor $(b, \delta_b)$ may be sufficient for asymptotic (respectively exponential) regional $\omega$-detectability. Then we obtain.

**Corollary 4.6.** *The systems* (4.1)–(4.12) *is regionally asymptotically (respectively exponentially) $\omega$-detectable if $i(b_1 - \alpha_1)/(\beta_1 - \alpha_1)$ and $i(b_2 - \alpha_2)/(\beta_2 - \alpha_2) \notin \mathbb{N}$, for every $i$, $i = 1, \ldots, J$.*

*Case of Fig. 7*: consider the system (4.6) with the function defined by form

$$z_i(t) = \int_{\Omega} x(r_i, \theta_i, t) f_i(r_i, \theta_i)\, \mathrm{d}r_i\, \mathrm{d}\theta_i \tag{4.13}$$

where
$2 \le i \le q$, $0 \le \theta_i \le 2\pi$, $(1/2)r_{i_\omega} < r_i < (1/2)$, $t > 0$. The sensors may be located in $p_1 = (r_1, \theta_1)$ and $p_2 = (r_2, \theta_2) \in \Omega$ (see Fig. 7).

**Corollary 4.7.**

1. *The system* (4.6)–(4.13) *is regionally asymptotically (respectively exponentially) $\omega$-detectable if $i(\theta_1 - \theta_2)/\pi \notin \mathbb{N}$ for all $i = 1, \ldots, J$.*

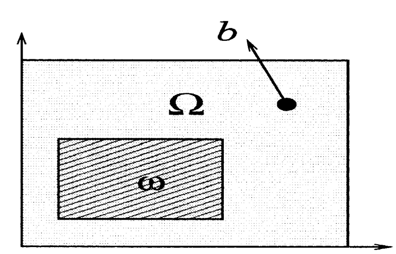


Fig. 6. Rectangular domain and location $b$ of internal pointwise sensor.

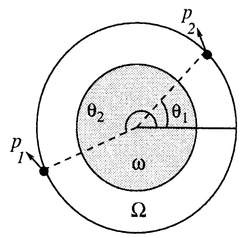


Fig. 7. Circular domain and locations $p_1, p_2$ of internal pointwise sensors.

2. *If* $r_1 = r_2$, *the system* (4.6)–(4.13) *is regionally asymptotically (respectively exponentially) ω-detectable if* $i(\theta_1 - \theta_2)/\pi \notin \mathbb{N}$ *for all* $i = 1, \ldots, J$.

*4.2.2. Filament sensors*

Consider the case where $\Omega = ]0,1[\times]0,1[$ and $\omega = ]\alpha_1, \beta_1[\times]\alpha_2, \beta_2[\subset \Omega$. Suppose that the observation on the curve $\sigma = \mathrm{Im}(\gamma)$ with $\gamma \in C^1(0,1)$ (Fig. 8), then we have.

**Corollary 4.8.** *If the observation recovered by the filament sensor* $(\sigma, \delta_\sigma)$ *such that* $\sigma$ *is symmetric with respect to the line* $\xi = \xi_0$. *The system* (4.1)–(4.12) *is regionally asymptotically (respectively exponentially) ω-detectable if* $i(\xi_{1_0} - \alpha_1)/(\beta_1 - \alpha_1)$ *and* $i(\xi_{2_0} - \alpha_2)/(\beta_2 - \alpha_2) \notin \mathbb{N}$, *for every* $i$, $i = 1, \ldots, J$.

*4.2.3. Boundary pointwise sensor*

Let us consider the system (4.1) with Dirichlet boundary condition, so, we can study the following.

*Case of Fig. 9*: in this case the sensor $(b, \delta_b)$ is located on $\partial\Omega$ (Fig. 9). The output function is given by

$$z(t) = \int_{\partial\Omega} \frac{\partial x}{\partial v}(\eta_1, \eta_2, t)\delta(\eta_1 - b_1, \eta_2 - b_2)\, \mathrm{d}\eta_1\, \mathrm{d}\eta_2 \quad (4.14)$$

Then we can obtain.

**Corollary 4.9.** *The system* (4.1)–(4.14) *is regionally asymptotically (respectively exponentially) ω-detectable if* $i(b_1 - \alpha_1)/(\beta_1 - \alpha_1)$ *and* $i(b_2 - \alpha_2)/(\beta_2 - \alpha_2) \notin \mathbb{N}$, *for every* $i$, $i = 1, \ldots, J$.

The case of Fig. 10: in this case we consider the system (4.6) with

$$z_i(t) = \int_{\partial\Omega} \frac{\partial x}{\partial v}(1, \theta_i, t) f(1, \theta_i) \mathrm{d}\theta_i, \quad \theta_i \in [0, 2\pi],\ t > 0 \quad (4.15)$$

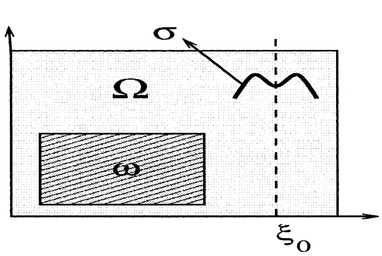


Fig. 8. Rectangular domain and location $\sigma$ of internal filament sensors.

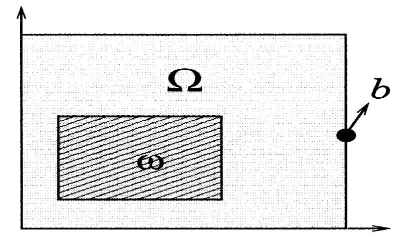


Fig. 9. Rectangular domain $\Omega$ and location $b$ of boundary pointwise sensor.

When the pointwise sensors at the polar coordinates $p_i = (1, \theta_i)$ where $\theta_i \in [0, 2\pi]$ and $2 \le i \le q$. We have the following result.

**Corollary 4.10.** *Then the system* (4.6)–(4.15) *is regionally asymptotically (respectively exponentially) ω-detectable if* $i(\theta_1 - \theta_2)/\pi \notin \mathbb{N}$ *for every* $i$, $1 \le i \le J$.

These results can be extended with boundary Neumann conditions.

## 5. Regional observer and regional detectability

In this section, we give an approach which allows to determine a regional asymptotic (exponential) estimator of $Tx(\xi, t)$ in $\omega$, based on the regional asymptotic (exponential) $\omega$-detectability. This approach derives from Luenberger observer type as introduced by Gressang and Lamont [11]. For that purpose, we recall that some definitions concerns the regional observer and the regional detectability.

*5.1. Definitions and characterizations*

**Definition 5.1.** Suppose that there exists a dynamical system with state $y(\xi, t) \in Y$ (a Hilbert space) given by

$$\begin{cases} \dfrac{\partial y}{\partial t}(\xi, t) = F_\omega y(\xi, t) + G_\omega u(t) + H_\omega z(\cdot, t), & \mathcal{Q} \\ y(\xi, 0) = y_0(\xi), & \Omega \\ y(\eta, t) = 0, & \Theta \end{cases} \quad (5.1)$$

where $F_\omega$ generates a strongly continuous semi-group which is asymptotically (respectively exponentially) stable on the space $Y$, $G_\omega \in \mathscr{L}(U, Y)$ and $H_\omega \in \mathscr{L}(\mathcal{O}, Y)$. The system

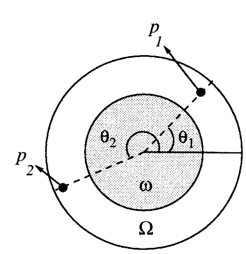


Fig. 10. Circular domain and locations $p_1, p_2$ of boundary pointwise sensors.

(5.1) defines a regional asymptotic (respectively exponential) estimator for $\chi_\omega Tx(\xi,t)$ if

1. $\lim_{t\to\infty}[\chi_\omega Tx(\xi,t) - y(\xi,t)] = 0, \quad \xi\in\omega$,
2. $\chi_\omega T$ maps $D(A)$ into $D(F_\omega)$ where $x(\xi,t)$ and $y(\xi,t)$ are the solutions of (2.1)–(2.2) and (5.1).

**Definition 5.2.** The system (5.1) specifies an asymptotical (respectively exponential) observer for the system (2.1)–(2.2) if it satisfies the following conditions:

1. There exists $M_\omega\in\mathscr{L}(\mathcal{O},L^2(\omega))$ and $N_\omega\in\mathscr{L}(L^2(\omega))$ such that $M_\omega C + N_\omega\chi_\omega T = I_\omega$.
2. $\chi_\omega TA - F_\omega\chi_\omega T = G_\omega C$ and $H_\omega = \chi_\omega TB$.
3. The system (5.1) determines a regional asymptotic (respectively exponential) estimator for $\chi_\omega Tx(\xi,t)$.

**Definition 5.3.** The system (5.1) is said to be an identity asymptotic (respectively exponential) regional observer for the system (2.1)–(2.2) if $\chi_\omega T = I_\omega$ and $X = Y$.

**Definition 5.4.** The system (5.1) is said to be an asymptotic (respectively exponential) regional reduced-order observer for the system (2.1)–(2.2) if $X = \mathcal{O}\oplus Y$.

**Definition 5.5.** The system (2.1)–(2.2) is asymptotically (respectively exponentially) regionally observable in $\omega$ if there exists a dynamical system which is asymptotic (exponential) regional observer for the original system.

**Proposition 5.6.** *Suppose that the system* (2.1) *together with output function* (2.2) *is an asymptotically (respectively exponentially) regionally ω-detectable then, the dynamical system described by the following parabolic equation*

$$\begin{cases} \dfrac{\partial y}{\partial t}(\xi,t) = Ay(\xi,t) + Bu(t) - H_\omega(Cy(\xi,t) - z(\cdot,t)), & \mathcal{Q}\\ y(\xi,0) = 0, & \Omega\\ y(\eta,t) = 0, & \Theta \end{cases} \tag{5.2}$$

*is an asymptotically (respectively exponentially) regional observer of the system* (2.1)–(2.2), *if*

$$\lim_{t\to\infty}[x(\xi,t) - y(\xi,t)] = 0, \quad \xi\in\omega.$$

**Proof.** Let $e(\xi,t) = x(\xi,t) - y(\xi,t)$ where $y(\xi,t)$ is the solution of the system (5.2). Deriving the above equation and inserting the Eqs. (2.1) and (5.2), we obtain

$$\begin{aligned}\frac{\partial e}{\partial t}(\xi,t) &= \frac{\partial x}{\partial t}(\xi,t) - \frac{\partial y}{\partial t}(\xi,t)\\ &= Ax(\xi,t) + Bu(t) - Ay(\xi,t) - Bu(t)\\ &\quad + H_\omega C(y(\xi,t) - x(\cdot,t)) = (A - H_\omega C)e(\xi,t).\end{aligned}$$

Since the system (2.1)–(2.2) is asymptotically (respectively exponentially) regionally $\omega$-detectable, there exists an operator $H_\omega\in\mathscr{L}(L^2(0,\infty,\mathbb{R}^q),L^2(\omega))$, such that the operator $A - H_\omega C$ generates asymptotically (respectively exponentially) regionally stable, strongly continuous semi-group $(S_{H_\omega}(t))_{t\geq 0}$ on $L^2(\omega)$ which is satisfied the following relations.

$\exists M_\omega,\ \alpha_\omega > 0$ such that $||\chi_\omega S_{H_\omega}(t)||_{L^2(\omega)} \leq M_\omega \mathrm{e}^{-\alpha_\omega(t)}$

Finally, we have

$$||e(\cdot,t)||_{L^2(\omega)} \leq ||\chi_\omega S_{H_\omega}(t)||_{L^2(\omega)}||e_0(\cdot)|| \leq M_\omega \mathrm{e}^{-\alpha_\omega(t)}||e_0(\cdot)||$$

with $e_0(\cdot) = x_0(\cdot) - y_0(\cdot)$ and therefore $e(\xi,t)$ converges to zero as $t\to\infty$. •

Thus the dynamical system (5.2) may be considered as an (identity) asymptotically (respectively exponentially) regionally observer for the system (2.1)–(2.2) without needing the stability of the system (2.1).

**Remark 5.7.** It is easily to show the following:

1. A system which is exactly regionally observable in $\omega$, is asymptotically (exponentially) regionally observable in $\omega$.
2. A system which is asymptotically (exponentially) observable, is asymptotically (exponentially) regionally observable in $\omega$.
3. A system which is asymptotically (exponentially) regionally observable in $\omega$, is asymptotically (exponentially) regionally observable in every subset $\omega_1$ of $\omega$. These results can be extended to (general case and reduced-order) of regional observer as in [5].

### 5.2. Application to a regional observer for distributed parameter diffusion systems

Consider the case of two dimensional distributed parameter diffusion system described by the parabolic equations

$$\begin{cases} \dfrac{\partial x}{\partial t}(\xi_1,\xi_2,t) = \dfrac{\partial^2 x}{\partial\xi_1^2}(\xi_1,\xi_2,t) + \dfrac{\partial^2 x}{\partial\xi_2^2}(\xi_1,\xi_2,t) + \beta x(\xi_1,\xi_2,t), & \mathcal{Q}\\ x(\eta_1,\eta_2,t) = 0, & t>0\\ x(\xi_1,\xi_2,0) = x_0(\xi_1,\xi_2), & \Omega \end{cases} \tag{5.3}$$

where the above-stated equations in discussing heat-conduction problems [12]. Let $\omega$ be a subregion of $\Omega = ]0,1[\times]0,1[$ and suppose there exists a single sensor $(D_1,f)$ with $D_1 = [d_1 - l_1, d_1 + l_1]\times[d_2 - l_2, d_2 + l_2]\subset]0,1[\times]0,1[$. Thus the augmented output function is given by

$$z(t) = \int_{D_1} x(\xi_1,\xi_2,t)f(\xi_1,\xi_2)\,\mathrm{d}\xi_1\,\mathrm{d}\xi_2 \tag{5.4}$$

The eigenfunctions of the operator $((\partial^2/\partial\xi_1^2) + (\partial^2/\partial\xi_2^2) + \beta)$ for the Dirichlet boundary conditions are defined by

$$\varphi_{ij}(\xi_1,\xi_2) = 2\sin i\pi(d_1)\sin j\pi(d_2)$$

associated with the eigenvalues

$\lambda_{ij} = \beta - (i^2 + j^2)\pi^2$

The system

$$\begin{cases} \dfrac{\partial y}{\partial t}(\xi_1, \xi_2, t) = \dfrac{\partial^2 y}{\partial \xi_1^2}(\xi_1, \xi_2, t) + \dfrac{\partial^2 y}{\partial \xi_2^2}(\xi_1, \xi_2, t) + \beta y(\xi_1, \xi_2, t) + Bu(\xi_1, \xi_2, t) - H_\omega(Cy(\xi_1, \xi_2, t) - z(t)), & \mathcal{Q} \\ y(\eta_1, \eta_2, t) = 0, & \Theta \\ y(\xi_1, \xi_2, 0) = y_0(\xi_1, \xi_2), & \Omega \end{cases} \quad (5.5)$$

together with the system (5.3)–(5.4) are equivalent to the system (2.1), (2.2)–(5.2) with the operator $A = ((\partial^2/\partial\xi_1^2) + (\partial^2/\partial\xi_2^2) + \beta)$. Using the Proposition 5.6 and Corollary 4.1, the system (5.5) is a regional observer for the system (5.3)–(5.4) if

$$\int_{D_1} x(\xi_1, \xi_2, t) f(\xi_1, \xi_2)\, \mathrm{d}\xi_1\, \mathrm{d}\xi_2 \neq 0 \quad (5.6)$$

Equivalently, if $i(d_1 - \alpha_1)/(\beta_1 - \alpha_1)$ and $i(d_2 - \alpha_2)/(\beta_2 - \alpha_2) \notin \mathbb{N}$. Under this condition, there exists an operator $H_\omega \in \mathscr{L}(L^2(0, \infty, \mathbb{R}^q), L^2(\omega))$ such that $A - H_\omega C$ generates a regionally exponentially stable, strongly continuous semi-group $(S_{H_\omega}(t))_{t \geq 0}$ on $L^2(\omega)$ and weobtain

$$\lim_{t \to \infty} [x(\xi, t) - y(\xi, t)] = 0, \quad \xi \in \omega$$

We can easily extend this application to the case of pointwise sensor and to the case of boundary zone (or pointwise) sensor with different boundary conditions.

## 6. Conclusion

In this paper we have presented the existence of the sufficient condition of regional asymptotic (respectively exponential) $\omega$-detectability and we have discussed the regional asymptotic (respectively exponential) detection problem for a class of distributed parameter systems with regional observability and regional observer. Many interesting results concerning the choice of sensors structures are explored and illustrated in specific situations. We have shown that it is possible to construct a regional observer for distributed parameter diffusion system by using the concept of regional asymptotic (respectively exponential) $\omega$-detectability. An extension of these results to the problem of regional stabilizability in connection the actuator structures is under consideration.

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