Results for Some of the Projective Special Linear Groups

Rana Noori Majeed [1], Rasha Ibrahim Khalaf [2], Niran Sabah Jasim [3]

[1,2, 3] Department of Mathematics, College of Education for Pure Science/ Ibn Al-Haitham, University of Baghdad

Abstract: ***In this labor we compute the cyclic decomposition for the projective special linear groups $PSL(2,s^v)$ where v = 2 and s = 3, 5 and 7.***

Keywords: General linear group, special linear group, projective special linear group, cyclic decomposition

1. Introduction

The projective special linear group denoted by PSL(n,F) get it by factor out the special linear group SL(n,F) by its center. This group consists two cases the first case where F ≡ +1 (mod 4) while the other case F ≡ – 1 (mod 4).

In this labor we consider the case where F = s^2 and s = 3,5, and 7, so we count for the case F ≡ +1 (mod 4).

This labor consists two sections, in the first section some basic concept presented in it, while the cyclic decomposition calculate for the groups PSL(2,9), PSL(2,25) and PSL(2,49) in the next section.

2. Preliminaries

This section offers some notions needed it.

Theorem 2.1: [1]
(i) The group $PSL(2,s^v)$ is simple for $s^v > 3$.

(ii)

$$\left|PSL(2,s^v)\right| = \begin{cases} (s^v+1)\,s^v\,(s^v-1) & if \quad s=2 \\ \frac{1}{2}(s^v+1)\,s^v\,(s^v-1) & if \quad s \text{ is a prime } \ s \neq 2. \end{cases}$$

Lemma 2.2: [1]
$PSL(2,s^v)$ has exactly $(2s^v+10)/4$ conjugacy classes $C_{<z>g}$ for $<z>g \in PSL(2,s^v)$.
For $s^v \equiv +1 \pmod 4$:

<z>g	C_g	$\lvert C_g \rvert$	$\lvert C_G(g) \rvert$
<z>	$C_{<z>}$	1	$s^v(s^{2v}-1)/2$
<z> c	$C_{<z>\,c}$	$(s^{2v}-1)/2$	s^v
<z> d	$C_{<z>\,d}$	$(s^{2v}-1)/2$	s^v
<z> a^η	$C_{<z>}\,a^\eta$	$s^v(s^v+1)$	$(s^v-1)/2$
$<z>a^{(s^v-1)/4}$	$C<z>a^{(s^v-1)/4}$	$s^v(s^v+1)/2$	(s^v-1)
<z> b^ϖ	$C_{<z>\,b^\varpi}$	$s^v(s^v-1)$	$(s^v+1)/2$

where $1 \le \eta \le (s^v-5)/4$ and $1 \le \varpi \le (s^v-1)/4$.

Theorem 2.3: [2]
Let $\rho \in \mathbb{C}$ be a (s^v-1)-th root of oneness and $\sigma \in \mathbb{C}$ be a (s^v+1)-th root of oneness, where i = 2, 4, 6, …, $(s^v-5)/2$, j = 2, 4, 6, …, $(s^v-1)/2$, $1 \le \eta \le (s^v-5)/4$ and $1 \le \varpi \le (s^v-1)/4$. Then for $s^v \equiv +1 \pmod 4$ the ordinary character table of $PSL(2,s^v)$, is:

	<z>	<z> c	<z> d	<z> a^η	$<z>a^{\frac{s^v-1}{4}}$	<z> b^ϖ
1_G	1	1	1	1	1	1
ψ	s^v	0	0	1	1	−1
χ_i	s^v+1	1	1	$\rho^{i\eta}+\rho^{-i\eta}$	$\rho^{i\frac{s^v-1}{4}}+\rho^{-i\frac{s^v-1}{4}}$	0
θ_j	s^v-1	−1	-1	0	0	$-(\sigma^{j\varpi}+\sigma^{-j\varpi})$
ξ_1	$\frac{s^v+1}{2}$	$\frac{1+\sqrt{s^v}}{2}$	$\frac{1-\sqrt{s^v}}{2}$	$(-1)^\eta$	$(-1)^{\frac{s^v-1}{4}}$	0
ξ_2	$\frac{s^v+1}{2}$	$\frac{1-\sqrt{s^v}}{2}$	$\frac{1+\sqrt{s^v}}{2}$	$(-1)^\eta$	$(-1)^{\frac{s^v-1}{4}}$	0

Theorem 2.4: [3]
Let G be a cyclic p–group. Then

$$K(G) = Z_p.$$

Theorem 2.5: [3]
Let G be a cyclic group of order p^n. Then

$$K(G) = \bigoplus_{i=1}^{n} Z_{p^i}$$

3. The Cyclic Decomposition for $K(PSL(2,s^2))$ where $s = 3, 5$ and 7

As in [4] if the diagonalization of the matrix for the rational valued character table presume as

$$\begin{pmatrix} v_1 & 0 & 0 & 0 & 0 \\ 0 & v_2 & 0 & 0 & 0 \\ 0 & 0 & v_3 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & v_n \end{pmatrix}$$

Then the cyclic decomposition for the group $K(SL(2,s^2))$ is:

$$K(PSL(2,s^2)) = Z_{v_1} \oplus Z_{v_2} \oplus Z_{v_3} \oplus \ldots \oplus Z_{v_n} \qquad \ldots(*)$$

3.1 The Cyclic Decomposition for K(PSL(2,9))

$|PSL(2,9)| = 360$

i = 2, j = 2,4, $\eta = 1$, $\varpi = 1,2$, ρ is the 8-th root of oneness and σ is the 10-th root of oneness, so the character table of PSL(2,9)

	$\langle z\rangle$	$\langle z\rangle\ c$	$\langle z\rangle\ d$	$\langle z\rangle\ a$	$\langle z\rangle\ a^2$	$\langle z\rangle\ b$	$\langle z\rangle\ b^2$
$\lvert C_g \rvert$	1	40	40	90	45	72	72
$\lvert C_G(g) \rvert$	360	9	9	4	8	5	5
1_G	1	1	1	1	1	1	1
ψ	9	0	0	1	1	−1	−1
χ_2	10	1	1	0	−2	0	0
θ_2	8	−1	−1	0	0	− 0.618	1.618
θ_4	8	−1	−1	0	0	1.618	− 0.618
ξ_1	13	3	−2	−1	1	0	0
ξ_2	13	−2	3	−1	1	0	0

Compile θ_2 with θ_4, we take out

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 9 & 0 & 0 & 1 & 1 & -1 & -1 \\ 10 & 1 & 1 & 0 & -2 & 0 & 0 \\ 16 & -2 & -2 & 0 & 0 & 1 & 1 \\ 5 & 2 & -1 & -1 & 1 & 0 & 0 \\ 5 & -1 & 2 & -1 & 1 & 0 & 0 \end{pmatrix}$$

Removing one of the frequent columns we take out

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 9 & 0 & 0 & 1 & 1 & -1 \\ 10 & 1 & 1 & 0 & -2 & 0 \\ 16 & -2 & -2 & 0 & 0 & 1 \\ 5 & 2 & -1 & -1 & 1 & 0 \\ 5 & -1 & 2 & -1 & 1 & 0 \end{pmatrix}$$

The diagonalization of this matrix is

$$\begin{pmatrix} 360 & 0 & 0 & 0 & 0 & 0 \\ 0 & -6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Thus by $(*)$, we take out

$K(PSL(2,9)) = Z_{360} \oplus Z_6 \oplus Z_1 \oplus Z_1 \oplus Z_1 \oplus Z_1$

3.2 The Cyclic Decomposition for K(PSL(2,25))

$|PSL(2,25)| = 7800$

i = 2,4,6,8,10, j = 2,4,6,8,10,12, $1 \le \eta \le 5$,, $1 \le \varpi \le 6$, ρ is the 24-th root of oneness and σ is the 26-th root of oneness, so the character table of PSL(2,25)

	$\langle z\rangle$	$\langle z\rangle\ c$	$\langle z\rangle\ d$	$\langle z\rangle\ a$	$\langle z\rangle\ a^2$	$\langle z\rangle\ a^3$	$\langle z\rangle\ a^4$	$\langle z\rangle\ a^5$	$\langle z\rangle\ a^6$	$\langle z\rangle\ b$	$\langle z\rangle\ b^2$	$\langle z\rangle\ b^3$	$\langle z\rangle\ b^4$	$\langle z\rangle\ b^5$	$\langle z\rangle\ b^6$
$\lvert C_g \rvert$	1	312	312	650	650	650	650	650	325	600	600	600	600	600	600
$\lvert C_G(g) \rvert$	7800	25	25	12	12	12	12	12	24	13	13	13	13	13	13
1_G	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
ψ	25	0	0	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1
χ_2	26	1	1	1.732050806	1	0	−1	− 1.732050806	−2	0	0	0	0	0	0
χ_4	26	1	1	1	−1	−2	−1	1	2	0	0	0	0	0	0
χ_6	26	1	1	0	−2	0	2	0	−2	0	0	0	0	0	0
χ_8	26	1	1	−1	−1	2	−1	−1	2	0	0	0	0	0	0
χ_{10}	26	1	1	− 1.732050806	1	0	−1	1.732050806	−2	0	0	0	0	0	0
θ_2	24	−1	−1	0	0	0	0	0	0	− 1.77091205	− 1.13612948	− 0.24107336	0.709209774	1.497021496	1.941883634
θ_4	24	−1	−1	0	0	0	0	0	0	− 1.13612948	0.709209774	1.941883634	1.497021496	− 0.24107336	− 1.77091205
θ_6	24	−1	−1	0	0	0	0	0	0	− 0.24107336	1.941883634	0.709209774	− 1.77091205	− 1.13612948	1.497021496
θ_8	24	−1	−1	0	0	0	0	0	0	0.709209774	1.497021496	− 1.77091205	− 0.24107336	1.941883634	− 1.13612948
θ_{10}	24	−1	−1	0	0	0	0	0	0	1.497021496	− 0.24107336	− 1.13612948	1.941883634	− 1.77091205	− 1.13612948
θ_{12}	24	−1	−1	0	0	0	0	0	0	1.941883634	− 1.77091205	1.497021496	− 1.13612948	0.709209774	− 0.24107336
ξ_1	13	3	−2	−1	1	−1	1	−1	1	0	0	0	0	0	0
ξ_2	13	−2	3	−1	1	−1	1	−1	1	0	0	0	0	0	0

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Compile χ_2 with χ_{10} and θ_2 with θ_4, θ_6, θ_8, θ_{10}, θ_{12}, we take out

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 25 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 52 & 2 & 2 & 0 & 2 & 0 & -2 & 0 & -4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 26 & 1 & 1 & 1 & -1 & -2 & -1 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 26 & 1 & 1 & 0 & -2 & 0 & 2 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 26 & 1 & 1 & -1 & -1 & 2 & -1 & -1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 144 & -6 & -6 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 13 & 3 & -2 & -1 & 1 & -1 & 1 & -1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 13 & -2 & 3 & -1 & 1 & -1 & 1 & -1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Removing six of the frequent columns we take out

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 25 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & -1 \\ 52 & 2 & 2 & 0 & 2 & 0 & -2 & -4 & 0 \\ 26 & 1 & 1 & 1 & -1 & -2 & -1 & 2 & 0 \\ 26 & 1 & 1 & 0 & -2 & 0 & 2 & -2 & 0 \\ 26 & 1 & 1 & -1 & -1 & 2 & -1 & 2 & 0 \\ 144 & -6 & -6 & 0 & 0 & 0 & 0 & 0 & 1 \\ 13 & 3 & -2 & -1 & 1 & -1 & 1 & 1 & 0 \\ 13 & -2 & 3 & -1 & 1 & -1 & 1 & 1 & 0 \end{pmatrix}$$

The diagonalization of this matrix is

$$\begin{pmatrix} 7800 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Thus by (∗), we take out

$K(PSL(2,25)) = Z_{7800} \oplus Z_1\ Z_1 \oplus Z_3 \oplus Z_2 \oplus Z_4 \oplus Z_1 \oplus Z_1 \oplus Z_1$

3.3 The Cyclic Decomposition for K(PSL(2,49))

$|PSL(2,49)| = 58800$

$i = 2,4,6,\ldots,22$, $j = 2,4,6,\ldots,24$, $1 \le \eta \le 11$,, $1 \le \varpi \le 12$, ρ is the 48-th root of oneness and σ is the 50-th root of oneness, so the character table of PSL(2,49)

	$\langle 1\rangle$	$\langle 1\rangle c$	$\langle 1\rangle d$	$\langle 1\rangle a$	$\langle 1\rangle a^2$	$\langle 1\rangle a^3$	$\langle 1\rangle a^4$	$\langle 1\rangle a^5$	$\langle 1\rangle a^6$	$\langle 1\rangle a^7$	$\langle 1\rangle a^8$	$\langle 1\rangle a^9$	$\langle 1\rangle a^{10}$	$\langle 1\rangle a^{11}$	$\langle 1\rangle a^{12}$
$\lvert C_g\rvert$	1	1200	1200	2450	2450	2450	2450	2450	2450	2450	2450	2450	2450	2450	1225
$\lvert C_G(g)\rvert$	58800	49	49	24	24	24	24	24	24	24	24	24	24	24	48
1_G	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
ψ	49	0	0	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	50	1	1	1.931851652	1.732050806	1.414213562	1	0.51763809	0	−0.51763809	−1	−1.414213562	−1.732050806	−1.931851652	−2
χ_4	50	1	1	1.732050806	1	0	−1	−1.732050806	−2	−1.732050806	−1	0	1	1.732050806	2
χ_6	50	1	1	1.414213562	0	−1.414213562	−2	−1.414213562	0	1.414213562	2	1.414213562	0	−1.414213562	−2
χ_8	50	1	1	1	−1	−2	−1	1	2	1	−1	−2	0	1	2
χ_{10}	50	1	1	0.51763809	−1.732050806	−1.414213562	1	1.931851652	0	−1.931851652	−1	1.414213562	1.732050806	−0.51763809	−2
χ_{12}	50	1	1	0	−2	0	2	0	−2	0	2	0	−2	0	2
χ_{14}	50	1	1	−0.51763809	−1.732050806	1.414213562	1	−1.931851652	0	1.931851652	−1	−1.414213562	1.732050806	0.51763809	−2
χ_{16}	50	1	1	−1	−1	2	−1	−1	2	−1	−1	2	−1	−1	2
χ_{18}	50	1	1	−1.414213562	0	1.414213562	−2	1.414213562	0	−1.414213562	2	−1.414213562	0	1.414213562	−2
χ_{20}	50	1	1	−1.732050806	1	0	−1	1.732050806	−2	1.732050806	−1	0	1	−1.732050806	2
χ_{22}	50	1	1	−1.931851652	1.732050806	−1.414213562	1	−0.51763809	0	0.51763809	−1	1.414213562	−1.732050806	1.931851652	−2
θ_2	48	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0
θ_4	48	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0
θ_6	48	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0
θ_8	48	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0
θ_{10}	48	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0
θ_{12}	48	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0
θ_{14}	48	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0
θ_{16}	48	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0
θ_{18}	48	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0
θ_{20}	48	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0
θ_{22}	48	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0
θ_{24}	48	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0
ξ_1	25	4	−3	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1
ξ_2	25	−3	4	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1

$\langle g \rangle\ b$	$\langle g \rangle\ b^2$	$\langle g \rangle\ b^3$	$\langle g \rangle\ b^4$	$\langle g \rangle\ b^5$	$\langle g \rangle\ b^6$	$\langle g \rangle\ b^7$	$\langle g \rangle\ b^8$	$\langle g \rangle\ b^9$	$\langle g \rangle\ b^{10}$	$\langle g \rangle\ b^{11}$	$\langle g \rangle\ b^{12}$
2352	2352	2352	2352	2352	2352	2352	2352	2352	2352	2352	2352
25	25	25	25	25	25	25	25	25	25	25	25
1	1	1	1	1	1	1	1	1	1	1	1
−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
−1.937166322	−1.75261336	−1.457937254	−1.07165359	−0.618033988	−0.125581038	0.374762628	0.851558582	1.274847978	1.61833988	1.85955297	1.984229402
−1.75261336	−1.07165359	−0.125581038	0.851558582	1.61833988	1.984229402	1.85955297	1.274847978	0.374762628	−0.618033988	−1.457937254	−1.937166322
−1.457937254	−0.125581038	1.274847978	1.984229402	1.61833988	0.374762628	−1.07165359	−1.937166322	−1.75261336	−0.618033988	0.851558582	1.85955297
−1.07165359	0.851558582	1.984229402	1.274847978	−0.618033988	−1.937166322	−1.937166322	0.374762628	1.85955297	1.61833988	−0.125581038	−1.75261336
−0.618033988	1.61833988	1.61833988	−0.618033988	−2	−0.618033988	1.61833988	1.61833988	−0.618033988	−2	−0.618033988	1.61833988
−0.125581038	1.984229402	0.374762628	−1.937166322	−0.618033988	1.85955297	0.851558582	−1.75261336	−1.07165359	1.61833988	1.274847978	−1.457937254
0.374762628	1.85955297	−1.75261336	−1.457937254	1.61833988	0.851558582	−1.937166322	−0.125581038	1.984229402	−0.618033988	−1.75261336	1.274847978
0.851558582	1.274847978	−1.937166322	0.374762628	1.61833988	−1.75261336	−0.125581038	1.85955297	−1.457937254	−0.618033988	1.984229402	−1.07165359
1.274847978	0.374762628	−1.75261336	−0.125581038	−0.618033988	−1.07165359	1.984229402	−1.457937254	−0.125581038	1.61833988	−1.937166322	0.851558582
1.61833988	−0.618033988	−0.618033988	1.61833988	−2	1.61833988	−0.618033988	−0.618033988	1.61833988	−2	1.61833988	−0.618033988
1.85955297	−1.457937254	0.851558582	−0.125581038	−0.618033988	1.274847978	−1.75261336	1.984229402	−1.937166322	1.61833988	−1.07165359	0.374762628
1.984229402	−1.937166322	1.85955297	−1.75261336	1.61833988	−1.457937254	1.274847978	−1.07165359	0.851558582	−0.618033988	0.374762628	−0.125581038
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0

Compile χ_2 with χ_4, χ_6, χ_{10}, χ_{14}, χ_{18}, χ_{20}, χ_{22} and θ_2 with θ_4, θ_6, θ_8, θ_{10}, θ_{12}, θ_{14}, θ_{16}, θ_{18}, θ_{20}, θ_{22} and θ_{24} we take out

$$\begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
49 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
400 & 8 & 8 & 0 & 2 & 0 & -2 & 0 & -4 & 0 & -2 & 0 & -2 & 0 & -8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
50 & 1 & 1 & 1 & -1 & -2 & -1 & 1 & 2 & 1 & -1 & -2 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
50 & 1 & 1 & 0 & -2 & 0 & -2 & 0 & -2 & 0 & -2 & 0 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
50 & 1 & 1 & -1 & -1 & 2 & -1 & -1 & 2 & -1 & -1 & 2 & -1 & -1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
576 & -12 & -12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
25 & 4 & -3 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
25 & -3 & 4 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Removing the frequent columns we take out

$$\begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
49 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & -1 \\
400 & 8 & 8 & 2 & 0 & -2 & -4 & -8 & 0 \\
50 & 1 & 1 & -1 & -2 & -1 & 2 & 2 & 0 \\
50 & 1 & 1 & -2 & 0 & -2 & -2 & 2 & 0 \\
50 & 1 & 1 & -1 & 2 & -1 & 2 & 2 & 0 \\
576 & -12 & -12 & 0 & 0 & 0 & 0 & 0 & 0 \\
25 & 4 & -3 & 1 & -1 & 1 & 1 & 1 & 0 \\
25 & -3 & 4 & 1 & -1 & 1 & 1 & 1 & 0
\end{pmatrix}$$

The diagonalization of this matrix is

$$\begin{pmatrix}
58800 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

Thus by (∗), we take out

$K(PSL(2,49)) = Z_{58800} \oplus Z_3\ \ Z_1\ \ Z_1 \oplus Z_6 \oplus Z_1 \oplus Z_2 \oplus Z_1 \oplus Z_1$

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