

Comparison of Laser Range Finder Mathematical Models under Different Weather Conditions

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Abstract

A theoretical study carried out to compare a three laser range finder models at different weather conditions depends on the power detector as a function of range R . two widely known conventional range equations are exclusively used when the targets reflects partially the transmitted beam eq. (12), and the targets reflects totally the transmitted beam eq. (13). A new Gaussian beam equation eq. (19) can be applied for all cases partially and totally reflected beam by compare with the two range equation mention above. The Gaussian beam range equation provides correct results and unified the two conventional range equations mention above. The three range equation applied a different atmospheric absorption coefficient, it was found that the three range equation can be used for rain and fog weather case because it gives an approach values, on the other hand for clear sky and very clear sky weather case the Gaussian range equation was the best because it gives a correct results for all values of range R compare with partially and totally range equation.

((مقارنة موديلات رياضية لمقدرة مدى ليزرية تحت ظروف مناخية مختلفة))

الخلاصة

اجريت هذه الدراسة النظرية لمقارنة ثلاث موديلات رياضية لمقدرة مدى ليزرية في ظروف مناخية مختلفة بالاعتماد على القدرة الليزرية المكتشفة كدالة الى المدى R . استخدمت معادلتين للمدى الاولى عندما يكون الهدف عاكس جزئي للحزمة المرسله المتمثلة بالمعادلة (12), والثانية عندما يكون الهدف عاكس كلي للحزمة المرسله المعادلة (13). اما المعادلة الكاوسية الجديدة معادلة (19) فقد شملت الحالتين للانعكاس الكلي والجزئي للحزمة وتم مقارنة النتائج مع نتائج المعادلتين السابقتين. وقد لوحظ ان معادلة المدى الكاوسية نتائجها ادق وهي توحد المعادلتين المذكورة اعلاه. وقد تم تطبيق الحسابات لقيم مختلفة من معامل الامتصاص الجوي (α) ووجد بأن المعادلات الثلاثة يمكن استخدامها للطقس الماطر والضبابي لانها تعطي نتائج متقاربة جدا, بينما عندما تكون السماء صافية والصافية جدا كانت معادلة المدى الكاوسية الافضل كونها تعطي نتائج ادق لجميع قيم المدى R مقارنة مع المعادلتين ذات الانعكاس الكلي والجزئي.

1. Introduction

Laser range finder (LRF) was one of the first and successful applications of laser technology so far [1]. There are many methods to measure distance using laser light, which are based on: (1) interferometry, (2) beam modulation telemetry, and (3) time of flight [2]. They provide different ranges and accuracies. The first method is based on an interferometry in which the frequency of a cw laser itself is used. With this fringe counting technique, displacement in distance can be measured in the order of wavelength of the used laser light. This technique is generally used for relative displacement measurements in laboratory environment. The second one utilizes an amplitude modulated cw laser. In this method, the laser beam is directed at a target. When the light signal is reflected back its phase is shifted proportionally by the range being measured. Since the phase measurement is unambiguous only up to the range corresponding to 2π radians, a method of removing range ambiguities using several frequencies must be used. This technique has an accuracy of a few millimeter orders at ranges of several kilometers. In the third method, a narrow pulse (usually less than 50 ns) is transmitted to the target. Its time of flight which is proportional to the range is measured. The time of flight method is the most representative principle of LRFs [1]. This method has definitely moved from laboratory to field use and developed into full production. The principle of time-of-flight LRF is similar to the principle of radar. Instead of using radio waves, the range finder uses light waves. Both techniques are based on the transmission of a short pulse of electromagnetic radiation and the reception of back scattered signals from a target.

In general, the accuracy of Laser range measurement is affected by a number of factors, including ambient temperature, ambient light, the material type, colors, gray levels, surface orientation, mixed pixels, signal strength, noises in the electronics, sampling rates, and the reflectivity of targets. By identifying and quantifying these factors, highly accurate range data of objects can be acquired. Characterization is such a process in which the factors that affect the quality of

the range data are determined. This can be performed by analyzing the physical process with which the range measurement is obtained theoretically and identifying sources of error [3].

2. Range Equations

If the transmitter is a kind of point source, and the power is spread in a forward hemisphere, the power arriving at a unit area on the hemisphere is given by:-

$$P_{Hemisphere} = \frac{P_{Transmitter}}{2\pi R^2} \quad (1)$$

Where $P_{Transmitter}$ is the power at the transmitter, and R is the radius of the hemisphere. The denominator corresponds to the area of a hemisphere (half of a sphere). The power arriving at the target is given by

$$P_{Target} = A_{Target} * \frac{P_{Transmitter}}{2\pi R^2} \quad (2)$$

Where A_{Target} is the area of the target and R is the distance between the transmitter and the target. We will refer the distance between the target and transmitter/detector as “range” or R , and the device for measuring the distance is called “range finder”. The power of reflected light, which is detected by the detector, will be [4]

$$P_{Detector} = A_{Detector} * \frac{P_{Target}}{2\pi R^2} \quad (3)$$

Where $A_{Detector}$ is the area of the detector. Substitution of eq. (3) into eq. (2) yields.

$$P_{Detector} = \frac{A_{Detector} \cdot A_{Target}}{(2\pi R^2)^2} \cdot P_{Transmitter} \quad (4)$$

It is apparent that the detected power is inversely proportional to R^4 of the transmitted power. It is of course difficult to detect the reflected light, since its power is attenuated by $1/R^4$ when it arrives at the detector. The range finder could

not be possibly built until the invention of laser. Accordingly, a range finder using a laser is called a “laser range finder” or LRF.

2.1. Geometrical Range Equation

The reason for using laser for range finder is that a laser can emit high power light, which is almost collimated with a small beam divergence, typically in the order of milliradian. Therefore, with respect to the range to be measured, beam size of the transmitted laser may be either totally or partially reflected by the target [5].

Fig. (1) shows a schematic diagram of the range measurement by LRF when the transmitted beam is totally reflected by the target. In order to measure the time of flight, an objective lens with diameter D is used to collect the reflected light onto a photodetector. If the transmitted beam is totally reflected by the target (e.g., R is small), we have

$$P_{Target} = P_{Transmitter} \quad (5)$$

By regarding the lens diameter as the detector area and taking Eq. (5) into account, eq. (3) can be rewritten as

$$P_{Detector} = \frac{D^2}{8R^2} \cdot P_{Transmitter} \quad (6)$$

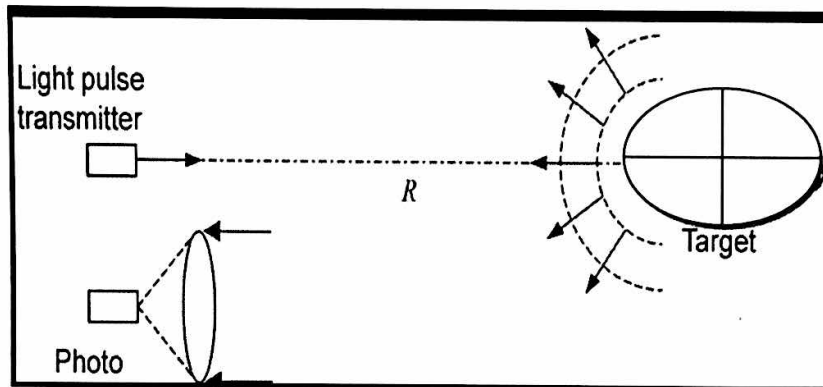


Fig. (1) Schematic diagram of range measurement using total incidence of pulse laser

However in many cases, for long range measurement, the transmitted laser beam diverges such that its diameter becomes larger than the target dimension.

Consequently, the beam is only partially reflected by the target as shown in Fig. (2). In this case, the target reflects only a portion of the transmitted power.

Assume that the transmitted light has a half angle of beam divergence θ . For small θ , the area covered by the light cone at distance R is $\pi(\theta R)^2$ as shown in Fig. (3). the power reflected by the target can be written as

$$P_{Target} = \frac{A_{Target}}{\pi\theta^2 R^2} \cdot P_{Transmitter} \quad (7)$$

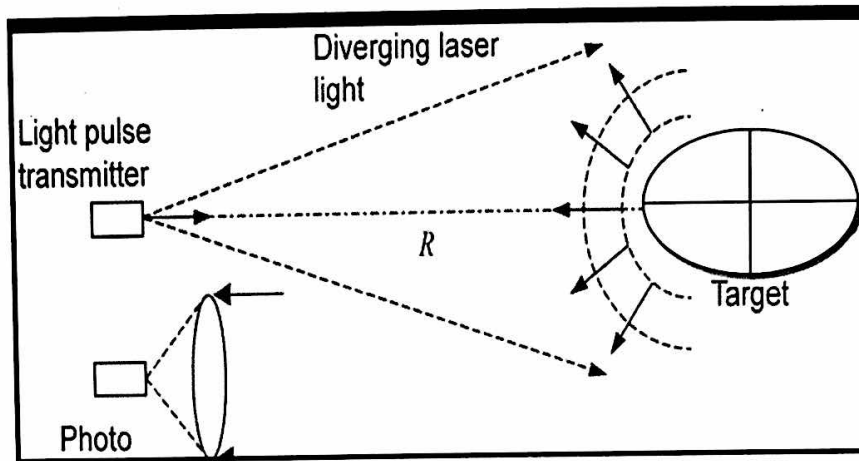


Fig. (2) Schematic diagram of range measurement using partial incidence of pulse laser.

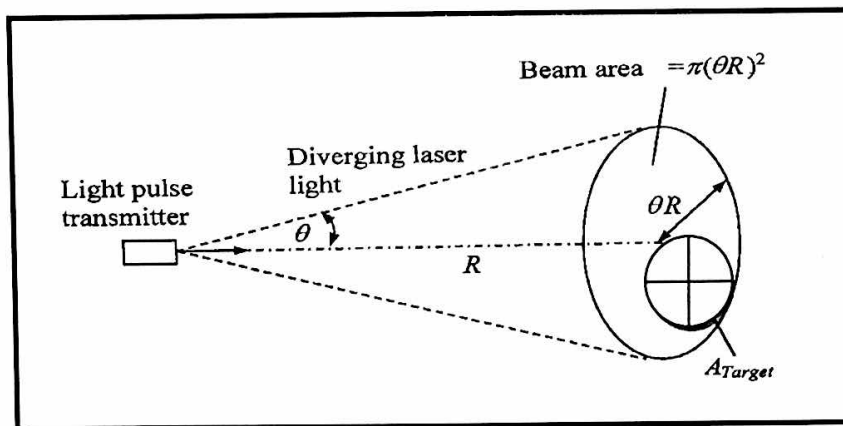


Fig. (3) Partial incidence of light beam on target

Where A is the area of the target that reflects the transmitted light beam.

As a consequence, eq. (6) becomes

$$P_{Detector} = \frac{D^2}{8R^2} \frac{A_{Target}}{\pi\theta^2 R^2} P_{Transmitter}$$

$$P_{Detector} = \frac{D^2 A_{Target}}{8\pi\theta^2 R^4} P_{Transmitter} \quad (8)$$

It should be noticed that we have assumed that the transmitted power is evenly distributed in a cone. Actually, the light beam is a Gaussian beam, which has peak intensity at the center. In the above discussion, we have actually assumed that the target is a diffuse object that is subject to Lambert's law [6] and its plane is parallel to the LRF.

According to the cosine law of Lambert, eq. (8) must be multiplied with $\cos\varphi$, which is the angle between the transmitted beam and the normal of the target [5].

$$P_{Detector} = \frac{D^2 A_{Target}}{8\pi\theta^2 R^4} P_{Transmitter} \cos\varphi \quad (9)$$

The overall resultant value of $\cos\varphi$ may be approximated by 0.5, because the target does not have a flat surface but a topographical structure. Accordingly, Eq. (9) reduces to.

$$P_{Detector} = \frac{D^2 A_{Target}}{16\pi\theta^2 R^4} P_{Transmitter} \quad (10)$$

2.2. Physical Range Equation

For actual power detection, the geometrical range equation given in eq. (10) needs to include physical attenuating coefficients such as: target reflectance ρ ; optical efficiency of the transmitter $\eta_{Transmitter}$; optical efficiency of the receiver $\eta_{Receiver}$; and atmosphere transmission factor given by [6]

$$T = e^{-\alpha R} \quad (11)$$

where α is the atmosphere extinction coefficient due to absorption and scattering. Consequently, the physical range equation for a partially reflected target can be rewritten as

$$P_{Detector} = \frac{T^2 \rho \eta_{transmitter} \eta_{receiver} D^2 A_{Target}}{16\pi\theta^2 R^4} P_{Transmitter} \quad (12)$$

Where as in accordance with eq. (6), the physical range equation for the totally reflected target becomes

$$P_{Detector} = \frac{T^2 \rho \eta_{transmitter} \eta_{receiver} D^2}{16R^2} P_{Transmitter} \quad (13)$$

Eqs. (12) and (13) are widely known range equations in LRF [1, 2, 7, 8].

2.3. Gaussian-Beam Range Equation

When a laser beam leaves an optical cavity containing a lasing material, its beam has a Gaussian profile. The intensity distribution of the Gaussian beam $I(\rho, z)$ is defined as [9]

$$I(\rho, z) = \frac{2P_{Beam}}{\pi w^2(z)} \exp\left[-\frac{2\rho^2}{w^2(z)}\right] \quad (14)$$

Where P_{Beam} is the total power carried by the beam given by

$$P_{Beam} = \int_0^{\infty} I(\rho, z) 2\pi\rho d\rho \quad (15)$$

The power contained within a circle of radius r_{Target} is

$$P_{Target} = \int_0^{r_{Target}} I(\rho, z) 2\pi\rho d\rho$$

$$P_{Target} = P_{Beam} \left[1 - \exp\left(-\frac{2r_{Target}^2}{w^2(z)}\right) \right] \quad (16)$$

By referring to $w=\theta.z$, for large z , such that $z=R$, the beam waist at this distance can be expressed as $w(z)=\theta.R$. Thus, eq. (16) becomes

$$P_{Target} = P_{Beam} \left[1 - \exp\left(-\frac{2r_{Target}^2}{\theta^2 R^2}\right) \right] \quad (17)$$

Since P_{Beam} is $P_{Transmitter}$, eq. (17) can be written as :-

$$P_{Target} = P_{Target} \left[1 - \exp\left(-\frac{2r_{Target}^2}{\theta^2 R^2}\right) \right] \quad (18)$$

Now we recall eq. (12) and replace $\frac{A_{Target}}{\pi\theta^2 R^2}$ with $\left[1 - \exp\left(-\frac{2r_{Target}^2}{\theta^2 R^2}\right) \right]$

This yield

$$P_{Detector} = \frac{T^2 \rho \eta_{transmitter} \eta_{receiver} D^2}{16R^2} \left[1 - \exp\left(-\frac{2r_{Target}^2}{\theta^2 R^2}\right) \right] * P_{Transmitter} \quad (19)$$

This is the new Gaussian beam range equation that derives by [5] which takes into account Gaussian profile of laser light such that the proposed range equation can be applied to all distances where reflected beams will be partially or totally reflected.

3. Simulation and Results

Simulation was carried out by matlab program to compare the measure values of the detector power as a function range under different attenuation coefficients. Some typical values of laser range finder are given in table (1)

Table (1) typical values of (LRF) system

Symbol	eq.(12)	eq.(13)	eq.(19)	meaning
P	0.21	0.21	0.21	Target reflectance
η_{trans}	0.85	0.85	0.85	Optical efficiency of the transmitter
η_{receiv}	0.45	0.45	0.45	Optical efficiency of the receiver
D	45mm	45mm	45mm	Diameter of the objective lens
A_{target}	$2.3*2.3m^2$	$2.3*2.3m^2$	$2.3*2.3m^2$	The area of the target
Θ	0.6 mrad	-	0.6 mrad	The half angle of divergence
P_{trans}	0.33Mw	0.33Mw	0.33Mw	The optical transmitter power
r_{target}	-	-	1.15m	The power contained with a circle of radius

The values of atmospheric attenuation (α) taken from [10] was in dB/km units and by the relation α (dB/km) = 4.343 α (km^{-1}) [11] converts into (km^{-1}) units, the values of atmospheric attenuation shown in table (2).

Table (2) the values of atmospheric attenuation under different weather [10]

weather	α (dB/km)	α (km^{-1})
Light fog	28.9	6.65

Very light fog	13.8	3.17
Rain	4.6	1.059
Light rain	2	0.46
Clear sky	0.47	0.1
Very clear sky	0.19	0.043

The simulation used to compare three range equations eq. (12), eq. (13) and eq. (19) using numerical data mention above. The following figures shows the logarithmic scale plots of the detector power as a function of the range R at different weather conditions

figure (4, 5) shows the relationship power detector and range which gives the behavior very close of the three range finder equation which are more like a straight line, which means that the three range finder equation benefit because it gives values very close in the case light fog (6.65 km^{-1}) and very light fog (3.17 km^{-1}). While in figure (6, 7) shows the relationship power detector and range was close values of the range equation. (13, 19) limit to 1900 (m), then the values of the range equation (13) and eq. (19) start difference after 1900 (m). Therefore eq. (12) is valid only for large R or $R \gg R_0$ and eq. (13) is valid only for small R or $R \ll R_0$ where $R_0 = 1900$ (m) is the range where the transmitted light beam is approximately as large as the target, however eq. (19) of the proposed Gaussian beam range equation is valid for all values of R.

figure (8, 9) show the nonlinear relationship between power detector and measure range. However, it appears that eq. (12) is erratic, i. e., when the range is short such that the target area is large than the light cone, the detector power becomes unrealistically large. For R is smaller than 1900 (m), eq. (12) produces misleading results, while eq. (13) and (19) are very close, for R is larger than 1900 m eq. (13) will produce misleading results. It deviated from the plot of eq (19). Therefore it is interesting to note that eq. (19) of the Gaussian beam range equation produces correct results for all values of the measured of R.

The advantage of using the Gaussian beam range equation is that we do need to check its validity by evaluation the target area and the beam diameter as required

by conventional range equations of eq. (12) and eq. (13). Furthermore, there is no discontinuity when R is approaching R_0 from left ($R \square R_0$) and from right ($R \square R_0$). It is important to note that eq. (13) approach with eq. (19) to range 1900 (m), after this range the two range equation begins to move away because the

correction factor $\left[1 - \exp\left(-\frac{2r_{Target}^2}{\theta^2 R^2}\right) \right]$ which begin effect at the range 1900 (m).

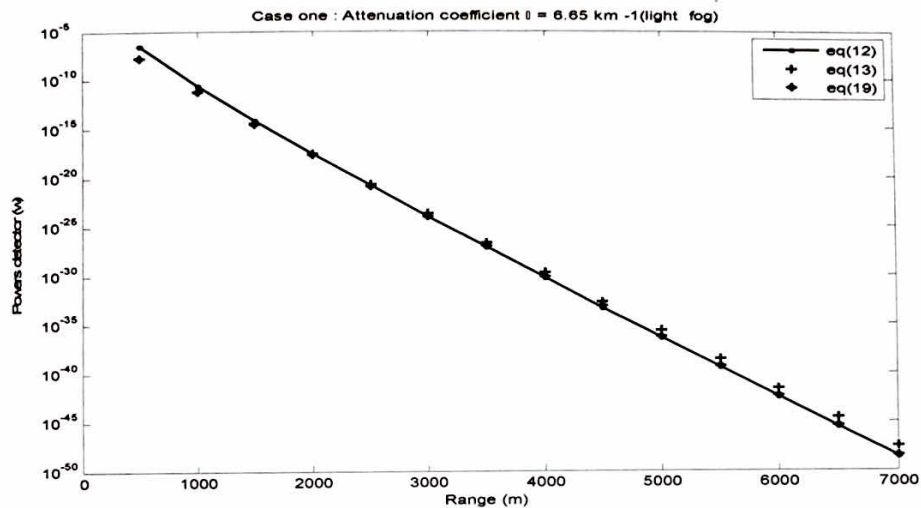


Figure (4) logarithmic plot of the calculated powers (W) as a function of the range (m) at attenuation coefficient = 6.65 km^{-1}

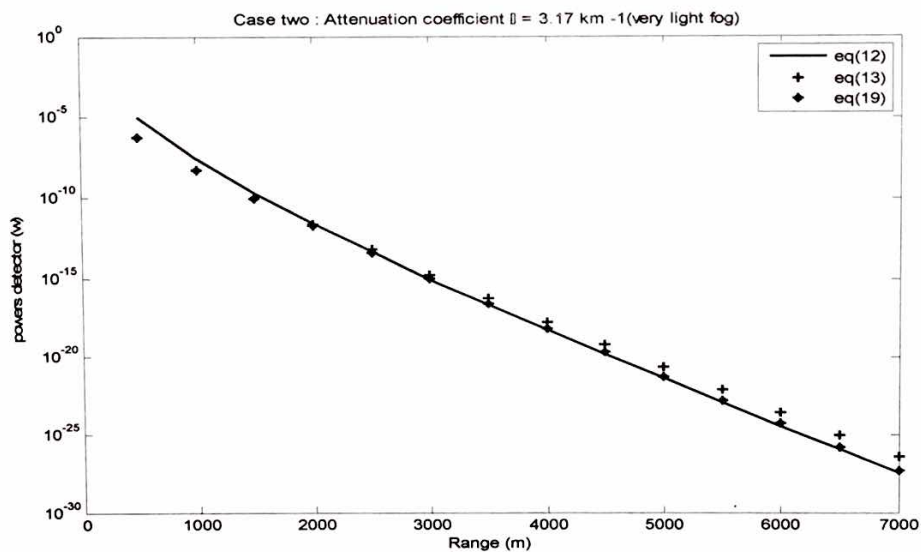


Figure (5) logarithmic plot of the calculated powers (W) as a function of the range (m) at attenuation coefficient = 3.17 km^{-1}

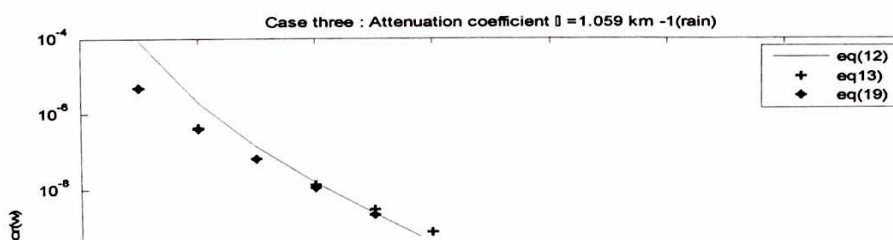


Figure (6) logarithmic plot of the calculated powers (W) as a function of the range (m) at attenuation coefficient = 1.059 km^{-1}

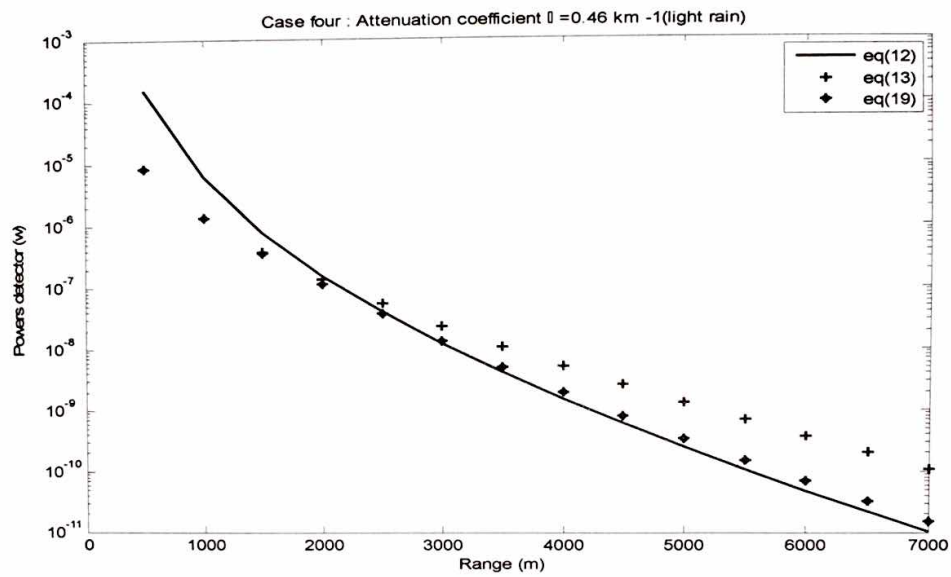


Figure (7) logarithmic plot of the calculated powers (W) as a function of the range (m) at attenuation coefficient = 0.1 km^{-1}

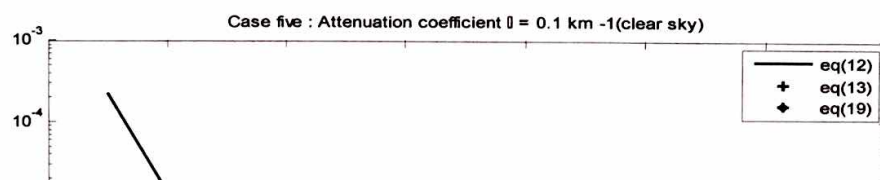


Figure (8) logarithmic plot of the calculated powers (W) as a function of the range (m) at attenuation coefficient = 0.1 km^{-1}

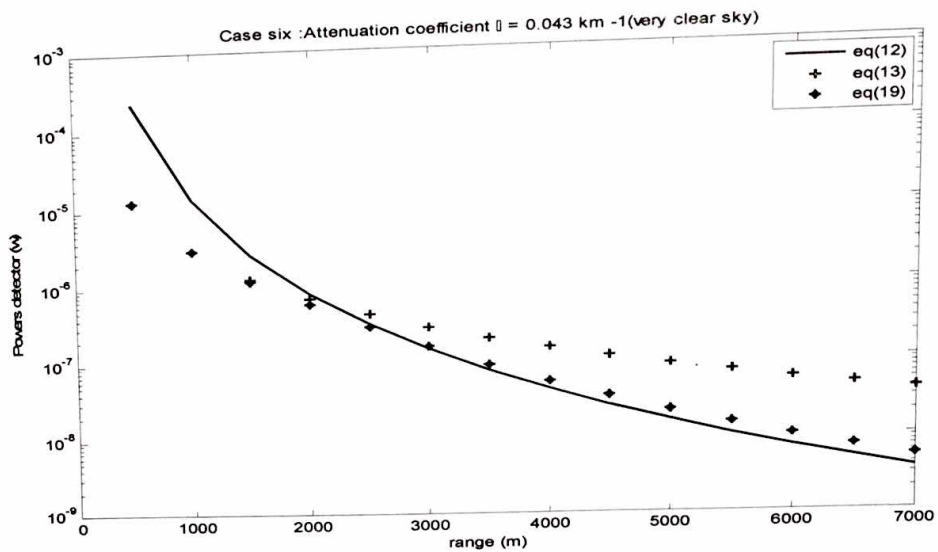


Figure (9) logarithmic plot of the calculated powers (W) as a function of the range (m) at attenuation coefficient = 0.043 km^{-1}

4. Conclusions

The simulation appears that the three range equation can be used to determine the range at all values of R when the atmospheric absorption coefficient has large values such as (rain, fog) because it has approach behavior a straight line where the power detector decreases with increase of range. In the case of (clear sky, very clear sky) have a small atmospheric absorption coefficient the range equations gives a different behavior, as a results eq. (12) used for large R or $R \gg R_0$, and eq. (13) used for small R or $R \ll R_0$; however the Gaussian beam range equation eq. (19) used for all values of R . the Gaussian beam range equation unifies eq. (12) and (13), is equally and directly verified for both domains of $R \gg R_0$ and $R \ll R_0$.

5. References

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