



The Neutrosophic Treatment for Multiple Storage Problem of Finite Materials and Volumes

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Abstract:

In this paper, we present a multi-inventory with limited size model to clarify the basic idea of multi-inventory systems in order to understand the relationships between the main variables, and examine the inventory's behaviour in a very broad range. In addition to the obvious applications in physical warehouses (such as electrical equipment, supplies, raw materials used in manufacturing, etc.), there are less predictable cases in which the multi-inventory model can be used. Such a model can be applied on the number of engineers and employees in a company, also on the number of students and professors in a university, as they constitute the processes of demand, hiring, and laying off which are types of compensation. Moreover, it may be useful at times not to look at physical goods as inventory as the prior examples are both types of inventory based on the space occupied as the available space can accommodate stored materials and is considered an inventory that must be compensated when depleted. The previous examples, in addition to many others, can be classified as inventory problems indicating the abundance of inventory models application, and the possibility of benefiting from the study of inventory theory in terms of clarifying the internal structure of the systems. In this study, we used the Neutrosophic logic to solve the problem of multi-inventory and limited size, depending on the fact that the optimal volume of materials to be stored is affected by the rate of demand for inventory. Moreover, this study is considered an expansion of one of

the known classical inventory models that depend on finite data and that is done by assigning a constant value to the inventory demand rate over the storage cycle time period, which does not correspond with the realistic application.

The limited application of classical inventory models was the motivating factor for this study as it deals with all data, whether specified or not in the inventory management process. Moreover, it takes into account all cases that the demand for inventory can go through, ranging from the cessation of demand for some stored materials to demand that exceeds the values provided by the real study.

Through this study, we developed mathematical relationships that we used to determine the necessary quantities of each of the materials to be stored based on the rate of demand and provide us with results that are more accurate. These results that can be utilized to store many materials in appropriate quantities and available volume, ensure that there is no shortage during the storage cycle period, and enables us to calculate all the necessary costs, which will achieve great profits.

Keyword: Inventory Management, Inventory Management Models, Neutrosophic Logic, Multiple Storage of Finite Materials and Volumes

1. Introduction

Inventory theory has gone through different stages since it was founded in 1920. The models were initially very simple and used a limited number of variables to determine the main parameters that affect the storage process. These models became more complex after adding more variables as they were more detailed.

One limitation was that it only dealt with one product. In contrast, the inventory that we encounter in the real world deals with different materials that are enormous and interactive to an extent that makes their management a complex problem and this led to the development of research to handle the new conditions of the inventory process.

Neutrosophic logic, which is a new vision of modeling developed by American mathematician Florentine Smarandache and is designed to effectively address the uncertainties inherent in the real world, as it came to replace the binary logic that recognizes right and wrong by introducing a third neutral state which can be interpreted as undefined or unconfirmed [5,7,8,9,11].

Florentine Smarandache presented Neutrosophic logic in 1995 as a generalization of fuzzy logic and an extension of the theory of fuzzy categories presented by Lotfi Zadeh [4] in 1965. In addition to that, Ahmed A. Salama presented the theory of Neutrosophic classical categories as a generalization of the theory of classical categories [10, 18] and developed, introduced and

formulated new concepts in the fields of mathematics, statistics, computer science, and others by neutrosophic logic [15-17,20,27]. The neutrosophic logic has significantly grown in recent years through its application in measurement, collections, graphs, while in optimization especially the neutrosophic geometric programming was an unfathomable area until Huda E. Khalid et al put the basic neutrosophic mathematical concepts of it since 2014, as well as the concepts of relation equations and many other scientific and practical fields [6,12-14,19,21-25,29-37].

In this paper, we shed the light to the application of neutrosophic logic to one of the inventory management models, which is the multi-inventory model for limited materials and volumes. This application will authorize dealing with inventory management models more accurately, contrary to what is known in the classical logic given that the demand for the stock of stored materials is not precisely defined. Rather, demand may fall within a range that includes all the cases that we can face, starting from the cessation of demand for some materials to the situation in which the demand for some materials exceeds the upper limit provided by the data and market study.

2. Discussion:

There are many reasons to keep inventory, including time efficiency and avoiding the cost and inconvenience caused by the continuous and infrequent compensation which requires much inventory and available space. This aids warehouse managers in production facilities or enterprises to determine the appropriate and optimal volume of inventory of each material, to secure the demand in a certain time, and to take advantage of the available space in a way that ensures securing materials throughout the duration of the storage cycle at the lowest possible cost.

Through the classic study of static inventory models, we found that the rate of demand for inventory is subject to a uniform probability distribution. Therefore, when the rate of demand for inventory is known indefinitely (not specified) during one time, we use the Neutrosophic uniform probability distribution that was previously studied [20].

This paper is a continuation of our previous study on inventory management [28].

Firstly, a summary of the studying problem according to classical logic will be presented.

The problem according to classical logic: [1]

Suppose that a production facility stores and sells an m items and the size of the storage warehouse is limited and equals to B unit. If one unit of the material i where $i = 1, 2, \dots, m$ occupies S_i from the volume of the repository. The requirement is determining the optimal amount of various inventory materials so that the production facility ensures the availability of that materials during the period of the storage cycle and at the lowest cost. To solve this problem, we build the appropriate mathematical model by setting the following appropriate hypotheses:

1. The rate of demand for an item i is constant and equal to λ_i in a unit of time.
2. The available initial quantity of the item i at the beginning of the storage cycle is equal to Q_i .
3. Whenever the storage level of the item i reaches to zero, it will be compensated with the same quantity Q_i .
4. The price of one unit of item i is equal to C_i .
5. There is a fixed cost K_i of preparing each demand of item i .
6. The cost of storing item i in the time unit is equal to h_i , and that each of these items is ordered at different times and independently of each other.

Mathematical model: to find

$$\text{Min } C(Q_i) = \sum_{i=1}^m \left(\frac{\lambda_i \cdot K_i}{Q_i} + \frac{h_i \cdot Q_i}{2} + \lambda_i \cdot C_i \right)$$

Subject to:

$$S_1 Q_1 + S_2 Q_2 + \dots + S_m Q_m \leq B$$

$$Q_i \geq 0 \quad ; i = 1, 2, \dots, m$$

The mathematical model is a nonlinear model used to find the optimal solution. To solve this model by one of the nonlinear programming methods, the method of Lagrange factors has been nominated by the authors. So, the following Lagrange function will be used: [1,2,3]

$$L(Q_i, \mu) = \sum_{i=1}^m \left(\frac{\lambda_i \cdot K_i}{Q_i} + \frac{h_i \cdot Q_i}{2} + \lambda_i \cdot C_i \right) + \mu (S_1 Q_1 + S_2 Q_2 + \dots + S_m Q_m - B)$$

The optimal solution is given by the following relationship

$$Q_i^* = \sqrt{\frac{2\lambda_i \cdot K_i}{h_i + 2\mu^* \cdot S_i}} \quad ; i = 1, 2, \dots, m \quad (1)$$

The optimal value of μ^* let to get the optimal value of Q_i^* satisfying the equality of the constrain $S_1 Q_1 + S_2 Q_2 + \dots + S_m Q_m \leq B$, that mean, $S_1 Q_1^* + S_2 Q_2^* + \dots + S_m Q_m^* = B$

From the relationship between the stored quantity and the rate of demand for stock, we must store quantities Q_i^* that meet demand, and at the same time, the space needed to store them does not exceed the warehouse space.

3. A Case Study: [1]

A factory stores and sells three raw materials A, B, C in its warehouse, which has an area of 700 m^2 only. The data for these materials are as follows:

indicator		material		
		A	B	C
λ_i	The rate of demand for the material i	5000	2000	10000
K_i	The cost of preparing the order of the material i	500	300	200
h_i	Storage cost per unit of material i	10	15	5
S_i	The space required for one unit in m^2	0.70	0.80	0.40

The Required:

Determine the optimal stock quantity of the three materials, so that the factory ensures the availability of materials during the storage cycle period and at the lowest cost.

Solution:

From studying data, we notice that the rate of inventory demand is given by specific values, therefore we must find the optimal quantities for the orders of these materials that correspond to the demand rate and are appropriate for the warehouse space. We will use the relationship (1) that gives us the optimal quantity that we have got from the above previous theoretical studying:

$$Q_i^* = \sqrt{\frac{2\lambda_i \cdot K_i}{h_i + 2\mu \cdot S_i}} ; i = 1, 2, \dots, m$$

By giving the qualitative values of the Lagrange factorial μ , from $\mu = 0$ then $\mu = 3, \mu = 5, \mu = 8$ to $\mu = 10$, it seems that $\mu = 10$ is the value at which the conditions of the problem are fulfilled and given the following optimal solution:

$$Q_1 = \sqrt{\frac{2\lambda_1 \cdot K_1}{h_1 + 2\mu \cdot S_1}} = \sqrt{\frac{(2) \cdot (5000) \cdot (500)}{10 + (2) \cdot (10) \cdot (0.7)}} = 456.54 \cong 457$$

$$Q_2 = \sqrt{\frac{2\lambda_2 \cdot K_2}{h_2 + 2\mu \cdot S_2}} = \sqrt{\frac{(2) \cdot (2000) \cdot (300)}{15 + (2) \cdot (10) \cdot (0.8)}} = 196.75 \cong 197$$

$$Q_3 = \sqrt{\frac{2\lambda_3 \cdot K_3}{h_3 + 2\mu \cdot S_3}} = \sqrt{\frac{(2) \cdot (10000) \cdot (200)}{5 + (2) \cdot (10) \cdot (0.4)}} = 554.7 \cong 555$$

$$B = (0.7) \cdot (456.54) + (0.8) \cdot (196.75) + (0.4) \cdot (554.7) = 699.5$$

The above area is approximately equal to the warehouse space $700m^2$, and thus we have reached the optimal solution that gives us the optimal quantities for orders from the three materials, which are: $Q_1^* = 457$, $Q_2^* = 197$, $Q_3^* = 555$

These quantities meet the demand for materials during the storage cycle and make the cost of storage as low as possible and occupy the space allocated for storage in the warehouse. However, it does not take into account fluctuations in the market and applies only to the specific case with the data contained in the text of the problem. Thus, it was necessary to find a study that gives results in which a margin of freedom takes into account the fluctuations of the market and takes the problem out of the frame of restriction and limitation.

Therefore, in this paper, we will address the previous problem according to the Neutrosophic logic that takes into account all definite and indefinite cases, through which we will be able to address market fluctuations resulting from the rate of demand for materials.

4. Reformulate the Above Case Study from a Neutrosophically perspective:

We assume that a certain facility stores and sells m material and that the volume of the storage warehouse is limited and equal to B unit, if one unit of the material i , where $i = 1, 2, \dots, m$, occupies a place S_i equal to the volume of the warehouse.

The Required:

To determine the optimal stock quantity of different materials, so that the factory ensures the availability of materials during the storage cycle period and at the lowest cost. It also takes into account that the rate of demand is indefinite. Thus, to solve this problem, we build the appropriate mathematical model by setting the following appropriate hypotheses:

1. The rate of demand for an item i is equal to λ_{iN} (indefinite) so that $\lambda_{iN} = [\lambda_{i2}, \lambda_{i2}]$ is an interval or $\lambda_{iN} = \{\lambda_{i2}, \lambda_{i2}\}$ is a set...etc.
2. The available initial quantity of the item i at the beginning of the storage cycle is equal to Q_i .
3. Whenever the storage level of the item i reaches to zero, it will be compensated with the same quantity Q_i .

4. The price of one unit of the item i is equal to C_i .
5. There is a fixed cost K_i of preparing each order of the item i .
6. The cost of storing item i in the time unit is equal to h_i , and that each of these items is ordered at different times and independently of each other.

5. Building the Mathematical Model:

Notice from the given problem that the main variable in the storage amount is the rate of demand in the condition that it does not exceed the volume of the warehouse. Therefore, we take λ_{iN} (indefinite) where $\lambda_{iN} = [0, \lambda_{i2}]$.

We denote the total cost of storage for material i by $C_i(Q_i)$ where we find

$$C_i(Q_i) = \frac{\lambda_{iN} \cdot K_i}{Q_i} + \frac{h_i \cdot Q_i}{2} + \lambda_{iN} \cdot C_i \quad \text{where } i = 1, 2, \dots, m$$

In which $C(Q_i)$ is the total cost of storage and gives the following formula:

$$C(Q_i) = \sum_{i=1}^m \left(\frac{\lambda_{iN} \cdot K_i}{Q_i} + \frac{h_i \cdot Q_i}{2} + \lambda_{iN} \cdot C_i \right)$$

In order for the amount of storage does not exceed the volume B of the warehouse, it must meet the following requirement:

$$S_1 Q_1 + S_2 Q_2 + \dots + S_m Q_m \leq B$$

From that we attain the following neutrosophic mathematical model:

$$\text{Min } C(Q_i) = \sum_{i=1}^m \left(\frac{\lambda_{iN} \cdot K_i}{Q_i} + \frac{h_i \cdot Q_i}{2} + \lambda_{iN} \cdot C_i \right)$$

Subject to

$$S_1 Q_1 + S_2 Q_2 + \dots + S_m Q_m \leq B$$

$$Q_i \geq 0 \quad ; i = 1, 2, \dots, m$$

The mathematical model is a nonlinear model used to find the optimal solution. We will use the method of Lagrange factors, so we form the following Lagrange function:

$$L(Q_i, \mu) = \sum_{i=1}^m \left(\frac{\lambda_{iN} \cdot K_i}{Q_i} + \frac{h_i \cdot Q_i}{2} + \lambda_{iN} \cdot C_i \right) + \mu (S_1 Q_1 + S_2 Q_2 + \dots + S_m Q_m - B)$$

To find the minimum limit, which is the optimal volume, we differentiate Lagrange function in regards to Q_i and μ where μ is the Lagrange factorial. We take its value to match the problem and we equate the differentiation to zero to get

$$\frac{\partial L(Q_i, \mu)}{\partial Q_i} = \frac{-\lambda_{iN} \cdot K_i}{Q_i^2} + \frac{h_i}{2} + \mu S_i = 0 \quad (*)$$

$$\frac{\partial L(Q_i, \mu)}{\partial Q_i} = S_1 Q_1 + S_2 Q_2 + \dots + S_m Q_m \rightarrow B = 0 \quad (**)$$

We solve the formulas (*) so m equation with m variable is Q_1, Q_2, \dots, Q_m and we get

$$Q_i = \sqrt{\frac{2\lambda_{iN} \cdot K_i}{h_i + 2\mu \cdot S_i}}; \quad i = 1, 2, \dots, m$$

Since $\lambda_{iN}, K_i, h_i, S_i, \mu \geq 0$ so the limitation $Q_i \geq 0$ is satisfied

We compensate for the limitation

$$S_1 Q_1 + S_2 Q_2 + \dots + S_m Q_m \leq B$$

We find

$$S_1 \sqrt{\frac{2\lambda_{1N} \cdot K_1}{h_1 + 2\mu \cdot S_1}} + S_2 \sqrt{\frac{2\lambda_{2N} \cdot K_2}{h_2 + 2\mu \cdot S_2}} + \dots + S_m \sqrt{\frac{2\lambda_{mN} \cdot K_m}{h_m + 2\mu \cdot S_m}} \leq B \quad (2)$$

In which the amount Q_1, Q_2, \dots, Q_m must satisfy the limitation (2), in the case that it does not

$$S_1 \sqrt{\frac{2\lambda_{1N} \cdot K_1}{h_1 + 2\mu \cdot S_1}} + S_2 \sqrt{\frac{2\lambda_{2N} \cdot K_2}{h_2 + 2\mu \cdot S_2}} + \dots + S_m \sqrt{\frac{2\lambda_{mN} \cdot K_m}{h_m + 2\mu \cdot S_m}} > B$$

We notice that the left side of the formula becomes smaller when the value of μ increases, since μ is an imagined value we can find one positive value for μ . For example μ^* and that is by gradually increasing the value until we satisfy the limitation (2). This means that the optimal demand value i material is given the following relationship

$$Q_i^* = \sqrt{\frac{2\lambda_{iN} \cdot K_i}{h_i + 2\mu^* \cdot S_i}}; \quad i = 1, 2, \dots, m \quad (3)$$

The optimal solution is given by the following relationship

If μ^* makes the constraint $S_1 Q_1 + S_2 Q_2 + \dots + S_m Q_m \leq B$ equal then the quantities Q_i^* achieve the following equality:

$$S_1 Q_1 + S_2 Q_2 + \dots + S_m Q_m = B$$

From the relationship between the stored quantity and the rate of demand for stock, we must store quantities Q_i^* that meet demand, and at the same time, the space needed to store them does not exceed the warehouse space.

We clarify the above aspects through the following example:

6. Practical Example

By taking the same case study that previously discussed in the classical point of view and put the rate of demand for materials indefinite, i.e. we took it in the form of intervals.

The context of the problem from neutrosophically perspective:

A factory stores and sells three raw materials A, B, C in its warehouse, which has an area of $700m^2$ only. The data for these materials were as follows:

Indicator		material		
		A	B	C
λ_{iN}	The rate of demand for the material i	[0,5000]	[0,2000]	[0,10000]
K_i	The cost of preparing the order of the material i	500	300	200
h_i	Storage cost per unit of material i	10	15	5
S_i	The space required for one unit in m^2	0.7	0.8	0.4

The required:

Determine the optimal stock quantity of the three materials, so that the factory ensures the availability of materials during the storage cycle period and at the lowest cost.

From the study of the data, we notice that the rate of demand for inventory is given by indeterminate (i.e. indefinite) values (i.e. intervals), and therefore we must find the optimal quantities for the orders of these materials that correspond to the demand rate and are appropriate for the warehouse space. We use the formula (3) that gives us the optimal quantity that we have reached in the theoretical study:

$$Q_i^* = \sqrt{\frac{2\lambda_{iN} \cdot K_i}{h_i + 2\mu \cdot S_i}} ; \quad i = 1, 2, \dots, m$$

By giving qualitative values of the Lagrange factorial μ , Firstly suppose that $\mu = 0$, then

$$Q_1 = \sqrt{\frac{2\lambda_{1N} \cdot K_1}{h_1 + 2\mu \cdot S_1}} = \sqrt{\frac{2[0,5000] \cdot (500)}{10}} = \sqrt{[0,500000]} = [0,707.15]$$

$$Q_2 = \sqrt{\frac{2\lambda_{2N} \cdot K_2}{h_2 + 2\mu \cdot S_2}} = \sqrt{\frac{2[0,2000] \cdot (300)}{15}} = \sqrt{[0,800000]} = [0,282.84]$$

$$Q_3 = \sqrt{\frac{2\lambda_{3N} \cdot K_3}{h_3 + 2\mu \cdot S_3}} = \sqrt{\frac{2[0,10000] \cdot (200)}{5}} = \sqrt{[0,800000]} = [0,894.45]$$

substitute the above values into the equation:

$$\begin{aligned} S_1 Q_1 + S_2 Q_2 + \dots + S_m Q_m &= B \\ 0.7[0,707.15] + 0.8[0,282.84] + 0.4[0,894.45] &= [0,495] + [0,226.27] + [0,357.78] \\ &= [0,1079.05] \end{aligned}$$

We need to ensure that the upper bound that we obtain from the above interval is less than or equal to the area of the warehouse $700m^2$. We note that the upper bound of the interval $[0,1079.05]$ is greater than the area of the warehouse (i.e. $1079.05 > 700 m^2$). Thus, we

must give a new value for μ that is greater than zero to reduce the stored quantity of a substance where $i = 1,2,3$. Assume that $\mu = 3$, by substituting this value into formula (3), we will obtain the following intervals:

$$Q_1 = \sqrt{\frac{2\lambda_{1N} \cdot K_1}{h_1 + 2\mu \cdot S_1}} = \sqrt{\frac{2[0,5000] \cdot (500)}{10 + (2) \cdot (3) \cdot (0.7)}} = \sqrt{[0,352000]} = [0,593.3]$$

$$Q_2 = \sqrt{\frac{2\lambda_{2N} \cdot K_2}{h_2 + 2\mu \cdot S_2}} = \sqrt{\frac{2[0,2000] \cdot (300)}{15 + (2) \cdot (3) \cdot (0.8)}} = \sqrt{[0,60600]} = [0,246.17]$$

$$Q_3 = \sqrt{\frac{2\lambda_{3N} \cdot K_3}{h_3 + 2\mu \cdot S_3}} = \sqrt{\frac{2[0,10000] \cdot (200)}{5 + (2) \cdot (3) \cdot (0.4)}} = \sqrt{[0,540500]} = [0,735.18]$$

$$0.7[0,593.3] + 0.8[0,246.17] + 0.4[0,735.18] = [0,415.31] + [0,196.94] + [0,294.07] \\ = [0,906.32]$$

It is clear that the upper bound of the interval $[0,906.32]$ is greater than the 700 m^2 area of the warehouse. Thus, we must give a new value for μ that is greater than 3 to reduce the stored quantity of a substance i where $i = 1,2,3$. For example, we substitute $\mu = 5$ into formula (3) and we obtain:

$$Q_1 = \sqrt{\frac{2\lambda_{1N} \cdot K_1}{h_1 + 2\mu \cdot S_1}} = \sqrt{\frac{2[0,5000] \cdot (500)}{10 + (2) \cdot (5) \cdot (0.7)}} = \sqrt{[0,294000]} = [0,542.22]$$

$$Q_2 = \sqrt{\frac{2\lambda_{2N} \cdot K_2}{h_2 + 2\mu \cdot S_2}} = \sqrt{\frac{2[0,2000] \cdot (300)}{15 + (2) \cdot (5) \cdot (0.8)}} = \sqrt{[0,52160]} = [0,228.39]$$

$$Q_3 = \sqrt{\frac{2\lambda_{3N} \cdot K_3}{h_3 + 2\mu \cdot S_3}} = \sqrt{\frac{2[0,10000] \cdot (200)}{5 + (2) \cdot (5) \cdot (0.4)}} = \sqrt{[0,444400]} = [0,666.63]$$

$$0.7[0,542.22] + 0.8[0,228.39] + 0.4[0,666.63] = [0,379.55] + [0,182.71] + [0,266.65] \\ = [0,828.91]$$

We note that the upper bound of the field $[0,828.91]$ is greater than the 700 m^2 area of the warehouse. Thus, we must give a new value for μ that is greater than 5 to reduce the stored quantity of a substance i where $i = 1,2,3$. For example, we substitute $\mu = 8$ into formula (3) and we obtain:

$$Q_1 = \sqrt{\frac{2\lambda_{1N} \cdot K_1}{h_1 + 2\mu \cdot S_1}} = \sqrt{\frac{2[0,5000] \cdot (500)}{10 + (2) \cdot (8) \cdot (0.7)}} = \sqrt{[0,235800]} = [0,485.59]$$

$$Q_2 = \sqrt{\frac{2\lambda_{2N} \cdot K_2}{h_2 + 2\mu \cdot S_2}} = \sqrt{\frac{2[0,2000] \cdot (300)}{15 + (2) \cdot (8) \cdot (0.8)}} = \sqrt{[0,43160]} = [0,207.75]$$

$$Q_3 = \sqrt{\frac{2\lambda_{3N} \cdot K_3}{h_3 + 2\mu \cdot S_3}} = \sqrt{\frac{2[0,10000] \cdot (200)}{5 + (2) \cdot (8) \cdot (0.4)}} = \sqrt{[0,350800]} = [0,592.28]$$

$$0.7[0, 485.59] + 0.8[0, 207.75] + 0.4[0, 592.28] = [0, 339.91] + [0, 166.2] + [0, 236.91] \\ = [0, 743.01]$$

Again, we note that the upper bound of the field $[0, 743.01]$ is greater than the $700m^2$ area of the warehouse. Thus, we must give a new value for μ that is greater than 8 to reduce the stored quantity of a substance i where $i = 1, 2, 3$. For example, we substitute $\mu = 10$ into formula (3) and we obtain:

$$Q_1 = \sqrt{\frac{2\lambda_{1N} \cdot K_1}{h_1 + 2\mu \cdot S_1}} = \sqrt{\frac{2[0, 5000] \cdot (500)}{10 + (2) \cdot (10) \cdot (0.7)}} = \sqrt{[0, 208350]} = [0, 456.45]$$

$$Q_2 = \sqrt{\frac{2\lambda_{2N} \cdot K_2}{h_2 + 2\mu \cdot S_2}} = \sqrt{\frac{2[0, 2000] \cdot (300)}{15 + (2) \cdot (10) \cdot (0.8)}} = \sqrt{[0, 38700]} = [0, 196.72]$$

$$Q_3 = \sqrt{\frac{2\lambda_{3N} \cdot K_3}{h_3 + 2\mu \cdot S_3}} = \sqrt{\frac{2[0, 10000] \cdot (200)}{5 + (2) \cdot (10) \cdot (0.4)}} = \sqrt{[0, 307700]} = [0, 554.7]$$

$$0.7[0, 456.45] + 0.8[0, 196.72] + 0.4[0, 554.7] = [0, 319.52] + [0, 157.38] + [0, 221.88] \\ = [0, 698.78]$$

We note that the upper limit of the field $[0, 698.78]$ is approximately equal $700m^2$ to the area of the warehouse, and thus we have reached the ideal solution that gives us the optimal quantities of orders from the three materials, which are:

$$Q_1^* = [0, 456.45], Q_2^* = [0, 196.72], Q_3^* = [0, 554.7]$$

These quantities meet the demand for materials during the storage cycle and make the cost of storage as low as possible and occupy the space allocated for storage in the warehouse.

Notes:

1. We note that the ideal quantities of orders from the three materials that we obtained from the classical study only represent one quantity from the range of optimal quantities obtained in the Neutrosophic study. This illustrates that using the Neutrosophic logic yields the best and most accurate results considering that the rate of demand for inventory changes and is affected by the market.
2. Any kind of imprecise data such as intervals, sets or other vague (i.e. indefinite) value can be selected for λ_{iN} , and meets the limitations so that the values we obtain for the required quantities do not exceed the space needed by the available space $700m^2$.

For example, if we assume that the indeterminacy belongs to an interval that takes into account that the demand for the stock of the three materials does not exist and does not exceed the upper limit that is in the data provided by the market study, the text of the issue becomes as follows:

7. A Case Study

A factory stores and sells three raw materials A, B, C in its warehouse, which has an area of $700m^2$ only. The data for these materials were as follows:

Indicator		material		
		A	B	C
λ_{iN}	The rate of demand for the material i	[2000,5000]	[1500,2000]	[4000,10000]
K_i	The cost of preparing the order of the material i	500	300	200
h_i	Storage cost per unit of material i	10	15	5
S_i	The space required for one unit in m^2	0.7	0.8	0.4

Required:

Obtaining the optimal demand values for the raw materials using the relationship that gives the optimal value, as denoted in the prior theoretical study:

$$Q_i^* = \sqrt{\frac{2\lambda_{iN} \cdot K_i}{h_i + 2\mu \cdot S_i}} ; \quad i = 1, 2, \dots, m$$

By giving hypothetical values for the Lagrange factorial μ from $\mu = 0$ then:

$$Q_1 = \sqrt{\frac{2\lambda_{1N} \cdot K_1}{h_1 + 2\mu \cdot S_1}} = \sqrt{\frac{2[2000,5000] \cdot (500)}{10}} = \sqrt{[20000,500000]} = [141.42, 223.60]$$

$$Q_2 = \sqrt{\frac{2\lambda_{2N} \cdot K_2}{h_2 + 2\mu \cdot S_2}} = \sqrt{\frac{2[1500,2000] \cdot (300)}{15}} = \sqrt{[60000,80000]} = [244.9, 282.8]$$

$$Q_3 = \sqrt{\frac{2\lambda_{3N} \cdot K_3}{h_3 + 2\mu \cdot S_3}} = \sqrt{\frac{2[5000,10000] \cdot (200)}{5}} = \sqrt{[400000,800000]} = [632.46, 894.43]$$

We insert these values into the formula:

$$S_1 Q_1 + S_2 Q_2 + S_3 Q_3 = 700$$

$$\begin{aligned} 0.7[141.42, 223.60] + 0.8[244.9, 282.8] + 0.4[632.46, 894.43] \\ = [98.99, 187.81] + [195.9, 226.24] + [252.98, 357.77] = [547.87, 771.82] \end{aligned}$$

We compare the upper bound 771,82 with the available space 700 and we note that:
 $771,82 > 700$

It is clear that the gained result from $\mu = 0$ is close to the available space value, therefore, we will take $\mu = 1$, predicting to get the optimal value as follow:

$$\begin{aligned} 0.7[132.45, 209.43] + 0.8[232.85, 268.87] + 0.4[525.23, 830.45] \\ = [92.72, 146.6] + [186.28, 215] + [210, 332.18] = [489, 693.78] \end{aligned}$$

We compare the upper bound of the interval = [489,693.78] with the available space $700m^2$ we note that:

$$693.78 \cong 700$$

This means that we have used almost the entire available space, and then the ideal quantities for orders on the three materials are as follows:

$$Q_1^* = [132.45, 209.43], Q_2^* = [232.85, 268.87], Q_3^* = [525.23, 830.45]$$

Note :

If we choose the Lagrange factorial and get a size smaller than the available, we continue to search for the optimal solution by taking a value of the Lagrange factorial that is smaller than the value that was used in order to increase the size. For example, if we take $\mu = 3$ we get [83.08,131.36] + [170.56,196.94] + [186,294.08] = [439.64,622.38]

We compare the upper bound of the interval [439.64,622.38] with the available space $700m^2$ and we note that:

$$622.38 < 700$$

This means that we have not used all of the available space, we must give a new value for μ that is less than 3 to enlarge the stored quantity of i substance where $i = 1,2,3$

Also, if we take $\mu = 2$ we obtain:

$$[276.7,138.35] + [177.9,205.42] + [196.95,311.4] = [593,655.17]$$

We compare the upper bound of the interval [593,655.17] with the available space $700m^2$ and we note that: $655.17 < 700$

This means that we have not used all of the available space, we must give a new value for μ that is less than 2 to enlarge the stored quantity of i substance where $i = 1,2,3$

Practical Fact

When choosing the Lagrange factorial, if we obtain a value greater than the actual value, we increase the Lagrange factorial to reduce the result. On the other hand, if we obtain a value smaller than the actual value, we reduce the Lagrange factorial to increase the result.

8. Conclusion and results:

Through the previous study, we note that the use of neutrosophic technique provides the production facilities, which depend on storing materials, a safe working environment that guarantees them the ability to address market fluctuations resulting from fluctuation in demand for inventory during the duration of the storage cycle. This is possible through the plans presented in this study, which included most of the cases that a production facility can go through, where the company can re-order the appropriate quantities according to the rate of demand for inventory and the volume available for it.

The results we reached through our study reflect the reality of the market, and this realistic application was lacking in the study according to the classical logic that deals with the rate of demand for inventory as a fixed amount throughout the duration of the storage cycle. The neutrosophic logic provides us with a more comprehensive study and allows us to compute the most accurate results possible.

In the near future, we look forward to studying the rest of the inventory management models according to the neutrosophic techniques, such as inventory models with a deficit of one item, dynamic models... etc.

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