Neutrosophic approach to Dynamic Programming on group Decision Making problems

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Abstract

In this article, complicated group decision-making situations where the preference data is represented by linguistic variables are addressed using the dynamic programming approach. Making conclusions clear through accurate figures is difficult for decision-makers due to the complexity and ambiguity of reality. Neutrosophic is used to encode the linguistic variables because they cannot be directly computed. Neutrosophic sets are used to manage indeterminacy in a practical situation. The relationships between single and interval Neutrosophic sets are then measured using novel distance and similarity models. The suggested dynamic programming interval-based clustering methodology is then used to group the decision-makers. Additionally, a novel method for computing the interval weights of decision-makers and clusters is described, accounting for both the cluster center and group size. A centroid-based ranking system is then used to compare and order the possibilities, and illustrated experiments are presented to demonstrate how effectively the suggested technique operates. Comparisons and discussions are also done to show its superiority.

Keywords: Dynamic programming; neutrosophic system; score function; clustering the decision makers.

1. Introduction

In order to approve the independent activities by considering the available sources, dynamic programming is the best way to take the correct decisions. Decision making problem is mainly affected by a criterion called dimension which is used to obtain the optimum solution for the problem. The decision with one criterion is called one dimensional problem otherwise it is multi-dimensional. The multi-dimensional problem was solved in [1],[2],[3].

The neutrosophic logic was founded by the American mathematician Smarandache in 1995. His interest in fuzzy will create the light to the neutrosopic logic [4],[5],[6]. The usefulness of this logic is spread fast in many researcher in various of applications [7] – [17] in the passageof time. In [18], neutrosophic fuzzy logic is used to solve the dynamic programming. The method used is to find the optimal profit of the problem. But it can be applicable for only the single phase diagram. If the decision makers were more and not in the regular form, the above method cannot be applicable. Hence our aim is to get the optimal path for the above problem which is also suitable for small number of decision makers also. In [19], we are going to take two different problems and going to clustering the decision makers in simple way.

Here our goal is use to single value neutrosopic system(SVNS) to cluster the decision makers for single diagram as well as the multiple diagram. Hence section 2 follows preliminaries followed by proposed method in section 3. In section 4 illustrative example and finally in section 5 conclusion is given.

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2.PRELIMINARIES:

In this section, some basic definitions of Neutrosophic(NS), Single value Neutrosophic(SVNS) ,score function were defined, which will be used in the forthcoming sections.

Definition 2.1:

TheNeutrosophicset [20] for the universal set $X$is defined as $N = \{(\hat{T}_S(x), \hat{I}_S(x), \hat{F}_S(x)), x \in X\}$ where $\hat{T}_S(x), \hat{I}_S(x), \hat{F}_S(x) \in [0,1]$ and $0 \le \hat{T}_S(x) + \hat{I}_S(x) + \hat{F}_S(x) \le 3$.

Definition 2.2:

The one dimensional logic equation for dynamic programming is defined as

$$\hat{Z}_i = \hat{F}_i(x_i) i = 1,2, \dots, n$$

Definition 2.3:

The relationship[18] between the current function $\widehat{Z}_\iota$ and the previous function $\hat{Z}_{i-1}$ at the stage i is given by

$$\hat{Z}_i = \widehat{g}_\iota(x_i, z_{i-1}) i = 1,2, \dots, n$$

Definition 2.4:

Let us we define the SVNS score function,

$$\mathbb{S}^*(d_i) = \mathbb{S}^*(u^T, v^I, w^F) = \frac{1}{2}\left(u^T - v^I + \left(\frac{w^F}{2}\right)\right) (1)$$

Where $d_i = (u^T, v^I, w^F)$.

3.ProposedNeutrosopic Dynamic Programming:

To solve the complex decision problems, first we have to invite the p decision makers $D^* = \{d_1, d_2, \dots, d_p\}$ with the similarity value is given $D^* = [\beta_{ij}]_{nXn}$. In each step we have to optimize the path which is calculated from backwards and the decision makers present in the path have been clusterd separately. Continue the process until all the decision makers have been allioted in any one of the clusters.

Step 1: Consider the problem of p decision makers $D^* = \{d_1, d_2, \dots, d_p\}$ with the similarity value is given $D^* = [\beta_{ij}]_{nXn}$.

Step 2: Fix the threshold for the similarity values as $\alpha = 0.5$. If the similarity value between d_i and d_j is greater that equal to the threshold value then the path is acceptable otherwise not. That is

$$d^*(d_i, d_j) \ge \alpha, i \ne j \ (2)$$

Step 3: Start from backwards consider d_p with the similarity value of remaining decision makers with the condition (2). Convert all the values to the score function using (1).

*Step 4:*Convert the problem in single value neutrosophic system.

Step 5: The proposed equation is

$$d_\gamma = max\{\mathbb{S}^*(d_i)\} \ where\ \gamma, i = 1,2, \dots, p-1, \gamma \ne i (3)$$

If the value is not equal to zero. Then the path is travelling fromd_p to $d_\gamma$otherwise not.

Step 6: If d_γ leads same value for two or more than two decision makers, both the decision makers were considered and repeat the Step 5 for both the decision makers.Among them the minimum value is considered.

Step 7: Consider d_γ with the similarity value satisfying (2) of other decision makers d_i, where $i = 1,2, \dots, \gamma - 1$. Then,

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$d_\delta = max\{\mathbb{S}^*(d_i)\} + d_\gamma \qquad where\ \delta, i = 1,2, \ldots, \gamma - 1 \qquad (4)$

Step 8: Continue the step 5 and step 6 until the path stops with the decision makers which satisfying (2).

Step 9: Then the decision makers in this path is grouped into a new cluster. Continue the procedure until all the decision makers should present in any one of the clusters.

4. Illustrative example:

Here we are going to discuss two problems present in [19]

Example 1:

Suppose there are five decision makers whose similarity value is

Table 1: similarity value of five decision makers

	d_1	d_2	d_3	d_4	d_5
d_1	1	0.6	0.25	0.7	0.3
d_2	0.6	1	0.9	0.5	0.46
d_3	0.25	0.9	1	0.38	0.4
d_4	0.7	0.5	0.38	1	0.5
d_5	0.3	0.46	0.4	0.5	1

Step 1: Consider the problem of 5 decision makers $D^* = \{d_1, d_2, \ldots, d_5\}$ with the similarity value is given $D^* = [\beta_{ij}]_{nXn}$ which is given by table 1.

Step 2: Fix the threshold for the similarity values as $\alpha = 0.5$ and consider only the value satisfying the condition (2). Then the table 1 becomes table 2 and the diagrammatic representation is given in fig 1.

Table 2: similarity value satisfying condition (2)

	d_1	d_2	d_3	d_4	d_5
d_1	1	0.6		0.7	
d_2	0.6	1	0.9	0.5	
d_3		0.9	1		
d_4	0.7	0.5		1	0.5
d_5				0.5	1

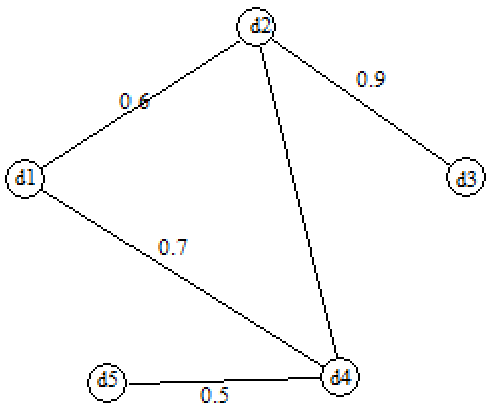

Figure 1: diagrammatic representation of table 2

Step 3,4,5: Consider the decision maker d_5 with the similarity value is given in table 3

Table 3: Neutrosophic and score functional value of d_5

Considered decision maker	Path moving to decision maker	Similarity value	Neutrosopic number	Score function value by (1)
d_5	d_4	0.5	(0.35,0.1,0.05)	0.2625

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Hence the path is moving from d_5 to d_4

*Step 7:*Consider the decision maker d_4 with the similarity value is given in table 4

Table 4: Neutrosophic and score functional value of d_4

Considered decision maker	Path moving to decision maker	Similarity value	Neutrosopic number	Score function value by (1)	d_δ by (4) is maximum of
d_4	d_1	0.7	(0.49,0.14,0.07)	0.3675	0.63
	d_2	0.5	(0.35,0.1,0.05)	0.2625	0.525

The maximum value is 0.63. hence the next path leads to d_1.

Hence the path is moving from d_5 to d_4 to d_1

Hence the cluster 1 groups $\{d_5, d_4, d_1\}$.

Step 8: Among the remaining decision makers, consider the next highest d_3 with the similarity value is given in table 5

Table 5: Neutrosophic and score functional value of d_3

Considered decision maker	Path moving to decision maker	Similarity value	Neutrosopic number	Score function value by (1)
d_3	d_2	0.9	(0.63,0.180.09)	0.4725

Hence the path is moving from d_3 to d_2. Since there were no decision makers left over. The cluster 2 groups $\{d_3, d_2\}$.

Step 9: Hence the problem concludes with the solution given in table 6 and diagrammatic representation is given in fig 2.

Table 6: solution for example 1

Cluster 1	$\{d_5, d_4, d_1\}$
Cluster 2	$\{d_3, d_2\}$

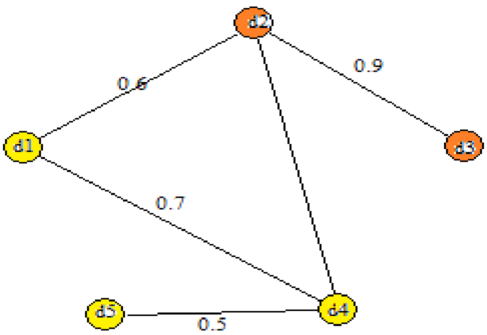

Fig 2: Diagrammatic representation of solution for example 1

Example 2:

Suppose there are twenty decision makers whose similarity value is

Table 7: similarity value of twenty decision makers

	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}	d_{11}	d_{12}	d_{13}	d_{14}	d_{15}	d_{16}	d_{17}	d_{18}	d_{19}	d_{20}
d_1	1	0.56	0.22	0.22	0.33	0.22	0.78	1	0.33	0.22	0.22	0.22	0.56	1	0.11	0.33	0.78	0.11	0.11	0.78

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d_2	0.56	1	0.11	0.33	0.44	0.22	0.44	0.56	0.33	0.11	0	0.11	1	0.33	0.22	0.33	0.56	0.11	0.22	0.78
d_3	0.22	0.11	1	0.44	0.44	0.78	0.11	0.22	0.44	0.89	1	0.78	0.11	0.22	0.56	0.44	0.11	0.56	0.11	0.11
d_4	0.22	0.33	0.44	1	1	0.67	0.33	0.33	0.89	0.56	0.56	0.56	0.33	0.22	0.67	1	0.33	0.56	0.56	0.33
d_5	0.33	0.44	0.44	1	1	0.67	0.22	0.44	1	0.56	0.56	0.56	0.33	0.33	0.67	1	0.33	0.56	0.44	0.33
d_6	0.22	0.22	0.78	0.67	0.67	1	0.22	0.22	0.78	1	0.67	1	0.22	0.22	0.78	0.67	0.22	1	0.78	0.22
d_7	0.78	0.44	0.11	0.22	0.22	0.22	1	0.89	0.33	0.22	0.22	0.22	0.78	0.78	0.11	0.33	0.78	0.11	0.22	0.89
d_8	1	0.56	0.22	0.44	0.44	0.22	0.89	1	0.33	0.22	0.33	0.22	0.56	0.67	0.11	0.44	0.89	0.22	0.33	0.56
d_9	0.33	0.33	0.44	1	1	0.78	0.33	0.33	1	0.56	0.56	0.67	0.22	0.33	0.67	0.67	0.22	1	0.89	0.22
d_{10}	0.22	0.11	0.89	0.56	0.56	1	0.22	0.22	0.56	1	0.78	1	0.11	0.22	0.89	0.67	0.22	0.89	0.89	0.22
d_{11}	0.22	0	1	0.56	0.56	0.67	0.22	0.33	0.56	0.78	1	0.11	0	0.22	0.44	0.56	0.22	0.33	0.67	0
d_{12}	0.22	0.11	0.78	0.56	0.56	1	0.22	0.22	0.67	1	0.11	1	0.11	0.22	0.89	0.67	0.22	1	1	0.55
d_{13}	0.56	1	0.11	0.33	0.33	0.22	0.78	0.56	0.22	0.11	0	0.11	1	0.56	0.22	0.33	0.67	0.11	0.11	1
d_{14}	1	0.33	0.22	0.22	0.33	0.22	0.78	0.67	0.33	0.22	0.22	0.22	0.56	1	0.22	0.33	0.89	0.22	0.11	0.78
d_{15}	0.11	0.22	0.56	0.67	0.67	0.78	0.11	0.11	0.67	0.89	0.44	0.89	0.22	0.22	1	0.67	0.22	1	0.67	0.22
d_{16}	0.33	0.33	0.44	1	1	0.67	0.33	0.44	0.67	0.67	0.56	0.67	0.33	0.33	0.67	1	0.33	0.67	0.56	0.33
d_{17}	0.78	0.56	0.11	0.33	0.33	0.22	0.78	0.89	0.22	0.22	0.22	0.22	0.67	0.89	0.22	0.33	1	0.22	0.22	0.67
d_{18}	0.11	0.11	0.56	0.56	0.56	1	0.11	0.22	1	0.89	0.33	1	0.11	0.22	1	0.67	0.22	1	0.78	0.22
d_{19}	0.11	0.22	0.11	0.56	0.44	0.78	0.22	0.33	0.89	0.89	0.67	1	0.11	0.11	0.67	0.56	0.22	0.78	1	0.11
d_{20}	0.78	0.78	0.11	0.33	0.33	0.22	0.89	0.56	0.22	0.22	0	0.22	1	0.78	0.22	0.33	0.67	0.22	0.11	1

Step 1: Consider the problem of 20 decision makers $D^* = \{d_1, d_2, \dots, d_{20}\}$ with the similarity value is given $D^* = [\beta_{ij}]_{nXn}$ which is given by table 7.

Step 2: Fix the threshold for the similarity values as $\alpha = 0.5$ and consider only the value satisfying the condition (2). Then the table 7 becomes table 8 and the diagrammatic representation is given in fig 3.

Table 8: similarity value satisfying condition (2)

	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}	d_{11}	d_{12}	d_{13}	d_{14}	d_{15}	d_{16}	d_{17}	d_{18}	d_{19}	d_{20}
d_1	1	0.56					0.78	1					0.56	1			0.78			0.78
d_2	0.56	1						0.56					1				0.56			0.78
d_3			1			0.78				0.89	1	0.78			0.56			0.56		
d_4				1	1	0.67			0.89	0.56	0.56	0.56			0.67	1		0.56	0.56	
d_5				1	1	0.67			1	0.56	0.56	0.56			0.67	1		0.56		
d_6			0.	0.	0.	1			0.	1	0.	1			0.	0.		1	0.	

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			78	67	67				78		67				78	67			78	
d_7	0.78						1	0.89					0.78	0.78			0.78			0.89
d_8	1	0.56					0.89	1					0.56	0.67			0.89			0.56
d_9				1	1	0.78			1	0.56	0.56	0.67			0.67	0.67		1	0.89	
d_{10}			0.89	0.56	0.56	1			0.56	1	0.78	1			0.89	0.67		0.89	0.89	
d_{11}			1	0.56	0.56	0.67			0.56	0.78	1					0.56			0.67	
d_{12}			0.78	0.56	0.56	1			0.67	1		1			0.89	0.67		1	1	0.55
d_{13}	0.56	1					0.78	0.56					1	0.56			0.67			1
d_{14}	1						0.78	0.67					0.56	1			0.89			0.78
d_{15}			0.56	0.67	0.67	0.78			0.67	0.89		0.89			1	0.67		1	0.67	
d_{16}				1	1	0.67			0.67	0.67	0.56	0.67			0.67	1		0.67	0.56	
d_{17}	0.78	0.56					0.78	0.89					0.67	0.89			1			0.67
d_{18}			0.56	0.56	0.56	1			1	0.89		1			1	0.67		1	0.78	
d_{19}				0.56		0.78			0.89	0.89	0.67	1			0.67	0.56		0.78	1	
d_{20}	0.78	0.78					0.89	0.56					1	0.78			0.67			1

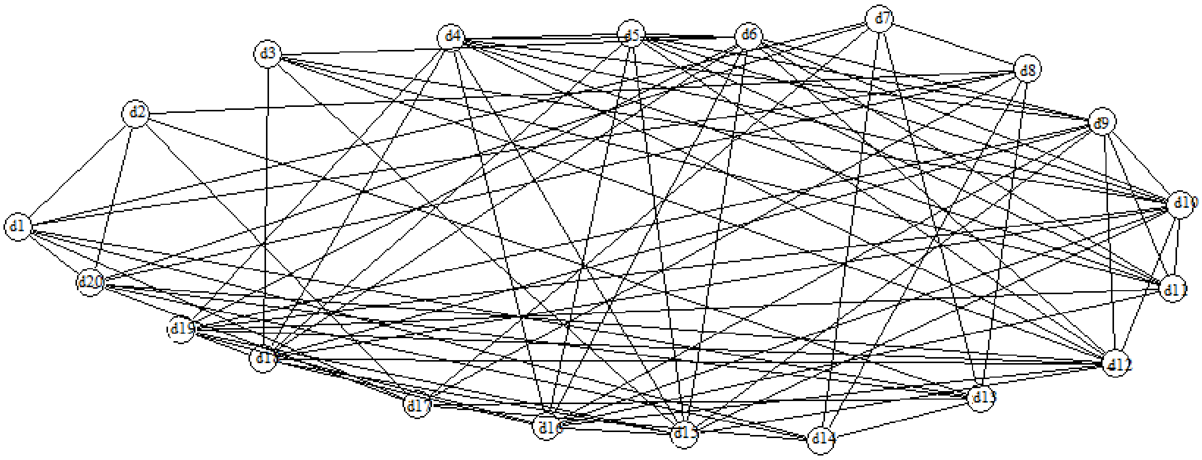

Figure 3: diagrammatic representation of table 8

Step 3,4,5,6,7,8:

Cluster 1

Consider the decision maker d_{20} with the similarity value is given in table 9

Table 9: Neutrosophic and score functional value of d_9

Considered decision maker	Path moving to decision maker	Similarity value	Neutrosopic number	Score function value by (1)
d_{20}	d_1	0.78	(0.546,0.156,0.078)	0.4095

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	d_2	0.78	(0.546,0.156,0.078)	0.4095
	d_7	0.89	(0.623,0.178,0.089)	0.46725
	d_8	0.56	(0.392,0.112,0.056)	0.294
	d_{13}	1	(0.7,0.2,0.1)	0.525
	d_{14}	0.78	(0.546,0.156,0.078)	0.4095
	d_{17}	0.67	(0.469,0.134,0.067)	0.35175

Since the maximum value is 0.525, the path is moving from d_{20} to d_{13}

Consider the decision maker d_{13} with the similarity value is given in table 10

Table 10: Neutrosophic and score functional value of d_{13}

Considered decision maker	Path moving to decision maker	Similarity value	Neutrosopic number	Score function value by (1)	d_δ by (4) is maximum of
d_{13}	d_1	0.56	(0.392,0.112,0.056)	0.294	0.819
	d_2	1	(0.7,0.2,0.1)	0.525	1.05
	d_7	0.78	(0.546,0.156,0.078)	0.4095	0.9345
	d_8	0.56	(0.392,0.112,0.056)	0.294	0.819

The maximum value is 1.05. hence the next path leads to d_2.

Hence the path is moving from d_{20} to d_{13} to d_2

Consider the decision maker d_2 with the similarity value is given in table 11

Table 11: Neutrosophic and score functional value of d_2

Considered decision maker	Path moving to decision maker	Similarity value	Neutrosopic number	Score function value by (1)	d_δ by (4) is maximum of
d_2	d_1	0.56	(0.392,0.112,0.056)	0.294	1.344

The maximum value is 1.344. hence the next path leads to d_1.

Hence the path is moving from d_{20} to d_{13} to d_2 to d_1.

Hence the cluster 1 groups $\{d_{20}, d_{13}, d_2, d_1\}$.

If we continue the same procedure for remaining decision makers, we get the solution for remaining clusters.

Step 9: Hence the problem concludes with the solution given in table 12 and diagrammatic representation is given in fig 4.

Table 12: solution for example 2

Cluster 1	$\{d_{20}, d_{13}, d_2, d_1\}$
Cluster 2	$\{d_{19}, d_{12}, d_6, d_3\}$
Cluster 3	$\{d_{18}, d_{15}, d_{10}, d_5, d_4\}$
Cluster 4	$\{d_{17}, d_{14}\}$
Cluster 5	$\{d_{16}, d_{11,} d_9\}$
Cluster 6	$\{d_8, d_7\}$

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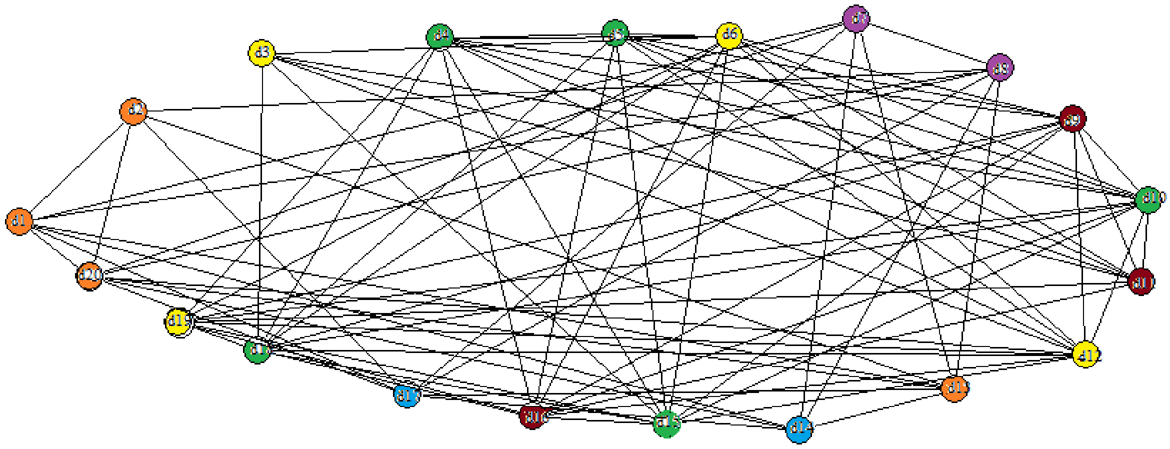

Figure 4: Diagrammatic representation of solution for example 2

Conclusion

Since large-scale decision-makers are involved in the decision-making process, we developed a dynamic programming Neutrosophic-based clustering model in this study to handle them. From a wide perspective, the recommended clustering model—which is a significant advancement over the preceding clustering models—might cluster the decision-makers. It was discovered that the weight vector of the clusters was crucial for the decision-making process, so a novel weight determination method that includes the network centre and the group size was created. By considering cluster size and, the new weight determination model can produce more rational weights. significance into consideration. Additionally, the Neutrosophic were ranked instantly using a centroid-based ranking technique, which can prevent information loss and generate more reliable and accurate findings. As decision-making processes become more complex in the actual world, it is projected that this method will be widely used. The effectiveness of the suggested method was demonstrated in two illustrative tests, and extensive comparisons with other methods were also made to demonstrate the system's superiority. The suggested approach can be expanded in the future to include fuzzy extensions sets

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