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Characteristic Zero Resolution of Weyl Module in the Case of the Partition (8,7,3)

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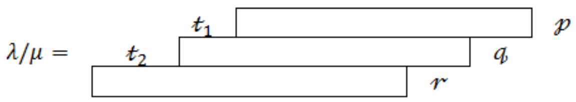
Abstract. In this paper, we studied the resolution of Weyl module for characteristic zero in the case of partition (8,7,3) by using mapping Cone which enables us to get the results without depended on the resolution of Weyl module for characteristic free for the same partition.

1. Introduction

Let R be a commutative ring with 1 and $\mathcal{F}$ is a free R-module and $\mathcal{D}_i\mathcal{F}$ be the divided power algebra of degree i.

The resolution of partition $(p+t_1+t_2, q+t_2, r)$ which represented by below diagram and in our case $t_1 = t_2 = 0$.



Authors in [1 - 6] discussed the resolution of Weyl module for characteristic free for the partitions (4,4,4), (3,3,2), (6,6,3), (6,5,3), (7,6,3) and (8,7,3), respectively. Haytham R.H. and Niran S.J in [7] exhibit the terms and the exactness of the Weyl resolution in the case of partition (8,7). As well in [8] they illustrate the terms of characteristic-free resolution and Lascoux resolution of the partition (8,7,3).

Buchsbaum D.A. and Rota G.C. in [9] define the Capelli identities as:

Let $i, j, k, \ell \in \mathcal{P}^+$, then the divided powers of the place polarizations satisfy the following identities:

(1) If $k \neq j$, then

$$\partial_{ij}^{(r)}\partial_{jk}^{(s)} = \sum_{\alpha\geq 0}\partial_{jk}^{(s-\alpha)}\partial_{ij}^{(r-\alpha)}\partial_{ik}^{(\alpha)}$$

$$\partial_{jk}^{(s)}\partial_{ij}^{(r)} = \sum_{\alpha\geq 0}(-1)^{\alpha}\partial_{ij}^{(r-\alpha)}\partial_{jk}^{(s-\alpha)}\partial_{ik}^{(\alpha)}$$

(2) If $i \neq k$ and $j \neq \ell$ then $\partial_{ik}^{(s)}\partial_{i\ell}^{(r)} = \partial_{i\ell}^{(r)}\partial_{ik}^{(s)}$

In this work we survey the resolution of Weyl module for characteristic zero in the case of partition (8,7,3) by using mapping Cone without depending on the resolution of Weyl module for characteristic free for the same partition.

2. Characteristic-zero resolution of Weyl module with mapping Cone in the case of partition (8,7,3)

Before we study the resolution of Weyl module for characteristic-zero in isolation of characteristic-free, we need the definition of mapping Cone we review that as in [10]

Consider the following commute diagram

$$\begin{array}{lccccccccc}
C_0: & C_{n-1} & \xrightarrow{d_{n-1}} & C_n & \xrightarrow{d_n} & C_{n+1} & \xrightarrow{d_{n+1}} & C_{n+2} & \cdots \\
 & \downarrow f_{n-1} & & \downarrow f_n & & \downarrow f_{n+1} & & \downarrow f_{n+2} & \\
D_0: & D_{n-1} & \xrightarrow{d'_{n-1}} & D_n & \xrightarrow{d'_n} & D_{n+1} & \xrightarrow{d'_{n+1}} & D_{n+2} & \cdots
\end{array}$$

If the rows sequence are exact and

$\partial n-1: Cn \otimes Dn-1 \longrightarrow Cn+1 \otimes Dn$ defined by

$(\alpha,b) \longmapsto (-dn(\alpha), d'_{n-1}(b)\, t + fn(\alpha))$ such hat $\partial n-1 \circ \partial n = 0; \forall\, n \in \mathbb{Z}^+$

Then the sequence

$$Cn-1 \xrightarrow{\partial_{n-1}} Cn \otimes Dn-1 \xrightarrow{\partial_n} Cn+1 \otimes Dn \xrightarrow{\partial_{n+1}} Cn+2 \otimes Dn+1 \xrightarrow{\partial_{n+2}} \ldots,$$

is exact.

Consider the complex of Lascoux in our partition (8,7,3) as the following diagram:

$$
\begin{array}{ccccc}
0 \longrightarrow & \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} & \xrightarrow{h_1} & \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} & \\
& f_1 \downarrow & \text{A} & \downarrow g_1 & \\
0 \longrightarrow & \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} & \xrightarrow{h_2} & \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} & \\
& f_2 \downarrow & \text{B} & \downarrow g_2 & \\
0 \longrightarrow & \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} & \xrightarrow{h_3} & \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} & \\
& & & \downarrow d' & \\
& & & \mathcal{K}_{(8,7,3)}(\mathcal{F}) & \\
& & & \downarrow & \\
& & & 0 &
\end{array}
$$

Diagram (2.1)

Where $h_1(v) = \partial_{21}(v)$; $v \in \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_1\mathcal{F}$

$f_1(v) = \partial_{32}(v)$; $v \in \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_1\mathcal{F}$

$h_2(v) = \partial_{21}^{(2)}(v)$; $v \in \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_2\mathcal{F}$

$h_3(v) = \partial_{21}(v)$; $v \in \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}$ and

$g_2(v) = \partial_{32}(v)$; $v \in \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_2\mathcal{F}$

So we need to define g_1 which make the diagram A commute, i.e

$(\partial_{21}^{(2)}\,\partial_{32})(v) = (g_1 \circ \partial_{21})(v)$

From Capelli identities, we know that

$\partial_{21}^{(2)}\,\partial_{32} = \partial_{32}\,\partial_{21}^{(2)} - \partial_{21}\,\partial_{31}$ and $\partial_{21}\,\partial_{31} = \partial_{31}\,\partial_{21}$

Then

$$
\begin{aligned}
\partial_{21}^{(2)}\,\partial_{32} &= \tfrac{1}{2}\partial_{32}\partial_{21}\partial_{21} - \partial_{21}\partial_{31} \\
&= \tfrac{1}{2}\partial_{32}\partial_{21}\partial_{21} - \partial_{31}\partial_{21} \\
&= \left(\tfrac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)\partial_{21}
\end{aligned}
$$

So we get $g_1(v) = \left(\tfrac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)(v)$; $v \in \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_1\mathcal{F}$

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Now if we use the mapping Cone to the following diagram

$$\begin{array}{ccccc} 0 \longrightarrow & \mathcal{D}_{10}\mathcal{F}\otimes\mathcal{D}_7\mathcal{F}\otimes\mathcal{D}_1\mathcal{F} & \xrightarrow{\hbar_1} & \mathcal{D}_9\mathcal{F}\otimes\mathcal{D}_8\mathcal{F}\otimes\mathcal{D}_1\mathcal{F} \\ & \downarrow f_1 & & \downarrow g_1 \\ & \mathcal{D}_{10}\mathcal{F}\otimes\mathcal{D}_6\mathcal{F}\otimes\mathcal{D}_2\mathcal{F} & \xrightarrow{\hbar_2} & \mathcal{D}_8\mathcal{F}\otimes\mathcal{D}_8\mathcal{F}\otimes\mathcal{D}_2\mathcal{F} \end{array}$$

We get the subcomplex

$$0 \longrightarrow \mathcal{D}_{10}\mathcal{F}\otimes\mathcal{D}_7\mathcal{F}\otimes\mathcal{D}_1\mathcal{F} \xrightarrow{\varphi_3} \begin{array}{c}\mathcal{D}_9\mathcal{F}\otimes\mathcal{D}_8\mathcal{F}\otimes\mathcal{D}_1\mathcal{F}\\ \oplus \\ \mathcal{D}_{10}\mathcal{F}\otimes\mathcal{D}_6\mathcal{F}\otimes\mathcal{D}_2\mathcal{F}\end{array} \xrightarrow{\delta_1} \mathcal{D}_8\mathcal{F}\otimes\mathcal{D}_8\mathcal{F}\otimes\mathcal{D}_2\mathcal{F} \tag{1}$$

Where $\varphi_3(x) = (-\partial_{21}(x), \partial_{32}(x))$ and

$$\delta_1(x_1, x_2) = (\ \partial_{21}^{(2)}(x_2) + \left(\frac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)(x_1)$$

Proposition (2.1):

$$\delta_1 \circ \varphi_3 = 0$$

Proof:

$$\begin{aligned}\delta_1 \circ \varphi_3(b) &= \delta_1(-\partial_{21}(b), \partial_{32}(b)) \\ &= \partial_{21}^{(2)}(\partial_{32}(b)) + \left(\tfrac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)(-\partial_{21}(b)) \\ &= \left(\partial_{21}^{(2)}\partial_{32}\right)(b) - \left(\tfrac{1}{2}\partial_{32}\partial_{21}\partial_{21}\right)(b) + (\partial_{31}\partial_{21})(b) \\ &= \left(\partial_{21}^{(2)}\partial_{32}\right)(b) - \left(\partial_{32}\,\partial_{21}^{(2)}\right)(b) + (\partial_{31}\partial_{21})(b)\end{aligned}$$

But from Capelli identities we have

$$\partial_{21}^{(2)}\,\partial_{32} = \partial_{32}\,\partial_{21}^{(2)} - \partial_{21}\,\partial_{31} \quad \text{and} \quad \partial_{31}\,\partial_{21} = \partial_{21}\,\partial_{31}$$

Then

$$\begin{aligned}\delta_1 \circ \varphi_3(b) &= \left(\partial_{32}\,\partial_{21}^{(2)}\right)(b) - (\partial_{21}\partial_{31})(b) - \left(\partial_{32}\,\partial_{21}^{(2)}\right)(b) + (\partial_{21}\partial_{31})(b) \\ &= 0 \quad \blacksquare\end{aligned}$$

By employing a mapping Cone again on the subcomplex (1) and the rest of diagram (2.1) we have

$$
\begin{array}{ccccccc}
0 \longrightarrow \mathcal{D}_{10}\mathcal{F}\otimes\mathcal{D}_{7}\mathcal{F}\otimes\mathcal{D}_{1}\mathcal{F} & \xrightarrow{\varphi_3} & \begin{matrix}\mathcal{D}_{9}\mathcal{F}\otimes\mathcal{D}_{8}\mathcal{F}\otimes\mathcal{D}_{1}\mathcal{F}\\ \oplus \\ \mathcal{D}_{10}\mathcal{F}\otimes\mathcal{D}_{6}\mathcal{F}\otimes\mathcal{D}_{2}\mathcal{F}\end{matrix} & \xrightarrow{\delta_1} & \mathcal{D}_{8}\mathcal{F}\otimes\mathcal{D}_{8}\mathcal{F}\otimes\mathcal{D}_{2}\mathcal{F} \\
 & & \downarrow \delta_2 & C & \downarrow g_2 \\
 & & \mathcal{D}_{9}\mathcal{F}\otimes\mathcal{D}_{6}\mathcal{F}\otimes\mathcal{D}_{3}\mathcal{F} & \xrightarrow{h_3} & \mathcal{D}_{8}\mathcal{F}\otimes\mathcal{D}_{7}\mathcal{F}\otimes\mathcal{D}_{3}\mathcal{F} \\
 & & & & \downarrow \\
 & & & & \mathcal{K}_{(8,7,3)}(\mathcal{F}) \\
 & & & & \downarrow \\
 & & & & 0
\end{array}
$$

Diagram (2.2)

Now we define

$$
\delta_2 : \begin{matrix}\mathcal{D}_{9}\mathcal{F}\otimes\mathcal{D}_{8}\mathcal{F}\otimes\mathcal{D}_{1}\mathcal{F}\\ \oplus \\ \mathcal{D}_{10}\mathcal{F}\otimes\mathcal{D}_{6}\mathcal{F}\otimes\mathcal{D}_{2}\mathcal{F}\end{matrix} \longrightarrow \mathcal{D}_{9}\mathcal{F}\otimes\mathcal{D}_{6}\mathcal{F}\otimes\mathcal{D}_{3}\mathcal{F} \quad \text{by}
$$

$$
\delta_2(a,b) = \partial_{32}^{(2)}(a) + \left(\tfrac{1}{2}\partial_{21}\partial_{32} + \partial_{31}\right)(b)
$$

Proposition (2.2):

The diagram C in diagram (2.2) is commute.

Proof:

To prove the diagram is commute it is sufficient to prove that

$$(g_2 \circ \delta_1)(a,b) = (h_3 \circ \delta_2)(a,b)$$

$$
\begin{aligned}
(g_2 \circ \delta_1)(a,b) &= g_2\left(\partial_{21}^{(2)}(b)\right) + \left(\tfrac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)(a) \\
&= \partial_{32}\left(\partial_{21}^{(2)}(b)\right) + \left(\tfrac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)(a) \\
&= \left(\partial_{32}\,\partial_{21}^{(2)}\right)(b) + \left(\tfrac{1}{2}\partial_{32}\partial_{32}\partial_{21} - \partial_{32}\partial_{31}\right)(a) \\
&= \left(\partial_{32}\,\partial_{21}^{(2)}\right)(b) + \left(\partial_{32}^{(2)}\partial_{21} - \partial_{32}\partial_{31}\right)(a)
\end{aligned}
$$

But from Capelli identities we have

$\partial_{32}^{(2)}\partial_{21} = \partial_{21}\partial_{32}^{(2)} + \partial_{32}\partial_{31}$ and $\partial_{32}\partial_{21}^{(2)} = \partial_{21}^{(2)}\partial_{32} - \partial_{21}\partial_{31}$

So we get

$$(\mathcal{g}_2 \circ \delta_1)(a,b) = \left(\partial_{21}^{(2)}\partial_{32} + \partial_{21}\partial_{31}\right)(b) + \left(\partial_{21}\partial_{32}^{(2)} + \partial_{32}\partial_{31} - \partial_{32}\partial_{31}\right)(a)$$

$$= \left(\tfrac{1}{2}\partial_{21}\partial_{21}\partial_{32} + \partial_{21}\partial_{31}\right)(b) + \left(\partial_{21}\partial_{32}^{(2)}\right)(a)$$

$$= \partial_{21}\left[\left(\tfrac{1}{2}\partial_{21}\partial_{32} + \partial_{31}\right)(b) + \partial_{32}^{(2)}(a)\right]$$

$$= (h_3 \circ \delta_2)(a,b) \quad \blacksquare$$

Hence from the mapping Cone, we have the following complex

$$0 \longrightarrow \mathcal{D}_{10}\mathcal{F}\otimes\mathcal{D}_7\mathcal{F}\otimes\mathcal{D}_1\mathcal{F} \xrightarrow{\varphi_3} \begin{matrix}\mathcal{D}_9\mathcal{F}\otimes\mathcal{D}_8\mathcal{F}\otimes\mathcal{D}_1\mathcal{F}\\ \oplus \\ \mathcal{D}_{10}\mathcal{F}\otimes\mathcal{D}_6\mathcal{F}\otimes\mathcal{D}_2\mathcal{F}\end{matrix} \xrightarrow{\varphi_2} \begin{matrix}\mathcal{D}_8\mathcal{F}\otimes\mathcal{D}_8\mathcal{F}\otimes\mathcal{D}_2\mathcal{F}\\ \oplus \\ \mathcal{D}_9\mathcal{F}\otimes\mathcal{D}_6\mathcal{F}\otimes\mathcal{D}_3\mathcal{F}\end{matrix}$$

$$\xrightarrow{\varphi_1} \mathcal{D}_8\mathcal{F}\otimes\mathcal{D}_7\mathcal{F}\otimes\mathcal{D}_3\mathcal{F} \xrightarrow{d'_{(8,7,3)}(\mathcal{F})} \mathcal{K}_{(8,7,3)}(\mathcal{F}) \longrightarrow 0$$

where

$$\varphi_2(a,b) = (-\delta_1(a,b), \delta_2(a,b))$$

$$= \left(-\partial_{21}^{(2)}(b) - \left(\tfrac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)(a), \partial_{32}^{(2)}(a) + \left(\tfrac{1}{2}\partial_{21}\partial_{32} + \partial_{31}\right)(b)\right)$$

$$\varphi_1(a,b) = \partial_{32}(a) + \partial_{21}(b)$$

Proposition (2.3):

$$\varphi_2 \circ \varphi_3 = 0$$

Proof:

$(\varphi_2 \circ \varphi_3)(a) = \varphi_2(-\partial_{21}(a), \partial_{32}(a))$; $a \in \mathcal{D}_{10}\mathcal{F}\otimes\mathcal{D}_7\mathcal{F}\otimes\mathcal{D}_1\mathcal{F}$

$$= \left(\left(-\partial_{21}^{(2)}\partial_{32}\right)(a) + \left(\tfrac{1}{2}\partial_{32}\partial_{21}\partial_{21} - \partial_{31}\partial_{21}\right)(a),\right.$$

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$$\left(-\partial_{32}^{(2)}\partial_{21}\right)(a)+\left(\tfrac{1}{2}\partial_{21}\partial_{32}\partial_{32}+\partial_{31}\partial_{32}\right)(a)\Big)$$

$$=\Big(\left(-\partial_{21}^{(2)}\partial_{32}\right)(a)+\left(\partial_{32}\partial_{21}^{(2)}-\partial_{31}\partial_{21}\right)(a),\left(-\partial_{32}^{(2)}\partial_{21}\right)(a)+$$

$$\left(\partial_{21}\partial_{32}^{(2)}+\partial_{31}\partial_{32}\right)(a)\Big)$$

But from Capelli identities we have

$\partial_{32}\partial_{21}^{(2)}=\partial_{21}^{(2)}\partial_{32}+\partial_{21}\partial_{31}$, $\partial_{21}\partial_{32}^{(2)}=\partial_{32}^{(2)}\partial_{21}-\partial_{32}\partial_{31}$,

$\partial_{21}\partial_{31}=\partial_{31}\partial_{21}$ and $\partial_{32}\partial_{31}=\partial_{31}\partial_{32}$

Which implies that

$(\varphi_2\circ\varphi_3)(a)$

$$=\Big(\left(-\partial_{21}^{(2)}\partial_{32}\right)(a)+\left(\partial_{21}^{(2)}\partial_{32}\right)(a)+(\partial_{21}\partial_{31})(a)-(\partial_{21}\partial_{31})_{31}(a),$$

$$\left(-\partial_{32}^{(2)}\partial_{21}\right)(a)+\left(\partial_{32}^{(2)}\partial_{21}\right)(a)-(\partial_{32}\partial_{31})(a)+(\partial_{32}\partial_{31})(a)\Big)$$

= (0,0) ■

Proposition (2.4):

$$\varphi_1\circ\varphi_2=0$$

Proof:

$$(\varphi_1\circ\varphi_2)(a,b)=\varphi_1\Big(-\partial_{21}^{(2)}(b)-\left(\tfrac{1}{2}\partial_{32}\partial_{21}-\partial_{31}\right)(a),\partial_{32}^{(2)}(a)+$$

$$\left(\tfrac{1}{2}\partial_{21}\partial_{32}+\partial_{31}\right)(b)\Big)$$

$$=\left(-\partial_{32}\partial_{21}^{(2)}\right)(b)-\left(\tfrac{1}{2}\partial_{32}\partial_{32}\partial_{21}\right)(a)-(\partial_{31}\partial_{32})(a)+$$

$$\left(\partial_{21}\partial_{32}^{(2)}\right)(a)+\left(\tfrac{1}{2}\partial_{21}\partial_{21}\partial_{32}\right)(b)+(\partial_{21}\partial_{31})(b)$$

$$=\left(-\partial_{32}\partial_{21}^{(2)}\right)(b)-\left(\partial_{32}^{(2)}\partial_{21}\right)(a)-(\partial_{31}\partial_{32})(a)+$$

$$\left(\partial_{21}\partial_{32}^{(2)}\right)(a)+\left(\partial_{21}^{(2)}\partial_{32}\right)(b)+(\partial_{21}\partial_{31})(b)$$

Again from Capelli identities we get

$(\varphi_1 \circ \varphi_2)(a, b) =$
$\left(-\partial_{21}^{(2)}\partial_{32}\right)(b) - (\partial_{21}\partial_{31})(b) - \left(\partial_{21}\partial_{32}^{(2)}\right)(a) - (\partial_{32}\partial_{31})(a) +$
$(\partial_{32}\partial_{31})(a) + \left(\partial_{21}\partial_{32}^{(2)}\right)(a) + \left(\partial_{21}^{(2)}\partial_{32}\right)(b) + (\partial_{21}\partial_{31})(b)$
$= 0$ ■

Finally, we present the following theorem which shows that the complex of Lascoux in the case of partition (8,7,3) is exact.

Theorem (2.5):
The complex

$$0 \longrightarrow \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \xrightarrow{\varphi_3} \begin{matrix} \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \\ \oplus \\ \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \end{matrix} \xrightarrow{\varphi_2} \begin{matrix} \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \\ \oplus \\ \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \end{matrix}$$

$$\xrightarrow{\varphi_1} \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \xrightarrow{d'_{(8,7,3)}(\mathcal{F})} \mathcal{K}_{(8,7,3)}(\mathcal{F}) \longrightarrow 0$$

Is exact.

Proof:

Since the diagrams, A and B in a diagram (2.1) are commutes and each of the maps

$h_1 : \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \longrightarrow \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_1\mathcal{F}$; where $h_1(v) = \partial_{21}(v)$,

and

$h_2 : \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \longrightarrow \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_2\mathcal{F}$; where $h_2(v) = \partial_{21}^{(2)}(v)$,

are injective [9] and [11], then we have a commuting diagram with an exact row. But from proposition (2.1) we have $\delta_1 \circ \varphi_3 = 0$ which implies that the mapping Cone conditions are satisfied and the complex

$$0 \longrightarrow \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \xrightarrow{\varphi_3} \begin{matrix} \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \\ \oplus \\ \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \end{matrix} \xrightarrow{\delta_1} \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_2\mathcal{F}$$

Is exact.

Now consider the diagram (2.2), since diagram C is commute and $\hbar_3 : \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \longrightarrow \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}$; where $\hbar_3(v) = \partial_{21}(v)$ is injective [9] and [11], so we have diagram (2.2) commute with exact rows. But $\varphi_2 \circ \varphi_3 = 0$ (proposition (2.3)) and $\varphi_1 \circ \varphi_2 = 0$ (proposition (2.4)) then again the mapping Cone conditions are satisfied, so the complex

$$0 \longrightarrow \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \xrightarrow{\varphi_3} \begin{matrix} \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \\ \oplus \\ \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \end{matrix} \xrightarrow{\varphi_2} \begin{matrix} \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \\ \oplus \\ \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \end{matrix}$$

$$\xrightarrow{\varphi_1} \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \xrightarrow{d'_{(8,7,3)}(\mathcal{F})} \mathcal{K}_{(8,7,3)}(\mathcal{F}) \longrightarrow 0$$

Is exact. ■

Conclusions

By using mapping Cone we can find the resolution of Weyl module for characteristic zero in the case of partition (8,7,3) without depending on the resolution of Weyl module for characteristic free for the same partition.

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