LINEAR STABILITY AT THE ONSET OF ROTATING CONVECTION IN THE PRESENCE OF MAGNETIC FIELD

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ABSTRACT

The phenomena of convection are one of the most interesting problems in fluid dynamics. In this paper we shall study the case of linear stability of a rotating electrically conducting viscous layer heated from below lying in a uniform magnetic field based on the Boussinesq approximation. We restrict our study to the case when the direction of magnetic field and rotation are parallel; the discussion is focused on the case of large Taylor number T and Chandrasekhar number Q. Generally, magnetic field facilitates convection in a rapidly rotating frame breaking the rotational constraints. The numerical solutions for stationary convection showed that at fixed large T and as we increase Q, The critical Rayleigh number Ra_c stayed fixed until Q reached a special value, then as we increase Q, Rayleigh number continue to decrease reaching its minimum before starting to increase again, two minimum values are determined at large T. A further analysis done on the stationary convection is finding the critical value of Q which give the same critical Rayleigh number at large Taylor number T.

Keywords: *Chandrasekhar Number Q, Convection, Rotating Convection, Magnetic Field, Rayleigh Number Ra, Stationary Convection, Stability of convection, Taylor Number T.*

1. INTRODUCTION

One of the most important examples of fluid motion is convection. It considers one of the methods of transferring heat energy via a fluid medium. The main cause of motion is gravity that acts on density variations associated with temperature variation, where hot fluid rises while cold fluid sinks. Rotating convection is considered a significant example of convection in all planetary and stellar bodies, impacting on many other significant observed features such as the generation of magnetic fields. There are several contexts in which rotating and hydro-magnetic convection is significant, in atmospheres, oceans and planetary mantles. For example, in the Earth's lower atmosphere, convection leads to the formation of thunderstorms; moreover, the atmospheric circulation, the large scale air movement that distributes energy over the surface of the earth is driven by convection. [13]

In the sun, convection is responsible for generating a strong and complex magnetic field by dynamo action. The cause of the sun's magnetic field is the movement of the convection cells consisting of electrically conducting plasma which circulates in a way that helps create the solar field. [13]

In the Earth's outer core, it is now accepted that magnetic fields are generated and destroyed by the movement of fluid in the depths of the earth. Moreover, it is believed that the Earth's outer core is filled with a vigorously convecting and conducting fluid, where motions of the fluid across magnetic field produce electric currents that induce magnetic field against the effect of dissipation. In the Earth's mantle, very slow convection leads to plate tectonics and hence earthquakes. [5, 6]

Moreover, convection is used in engineering practices to provide the desired temperature for heating systems in homes and cooling devices.

The phenomenon of convection is quite old, but the first quantitative experiment was performed by Henri B’enard in 1900. B’enard made an experiment on thermal convection. He melted some wax in a metal dish heating it from below. Firstly B'enard noted no motion of the melted fluid wax, but at a critical value of the temperature, B'enard saw a hexagonal pattern on the surface of the melted wax. He had discovered the presence of convection cells below. [7]

B'enard determined some properties of convection such as the profile of the interface, the spatial periodicity of the hexagonal patterns and their variation. [7]

In 1916 Rayleigh modeled this problem again. Rayleigh assumed that there was an infinite layer of a fluid bounded by stationary horizontal boundaries $z = 0$ and $z = d$ and he assumed that both boundaries heated at a constant uniform temperature. He noted that the fluid develops a regular pattern of convection cells and he developed a complete linear stability theory assuming free surface boundaries for the velocity and perfectly conducting boundaries. [11]

In this paper, we shall study the linear stability at the onset of rotating convection in the presence of magnetic field, following Rayleigh's ideas. We will analyze the onset of rotating convection in the presence of magnetic.

2. THE BASIC PROBLEM AND THE PROCESS OF SOLUTION

Consider an infinite horizontal layer of conducting fluid and let be the uniform angular velocity of the rotation in a magnetic field B_0, which is constant in the rotating frame. Suppose that the lower boundary surface at

$z = (-1/2)d$ is heated to a higher temperature than the top surface $z = (1/2)d$, on the assumption that all material properties of the fluid are constant, then heat can be conducted across the layer along a constant temperature gradient $-\beta$ in the z-direction. If β is large enough, then the conduction solution is unstable to convective motions [1]. In this paper, we shall determine the marginal stability state at which convection supposed to be of small amplitude can occur.

The equations required are:

Momentum equation added Lorentz force and Coriolis force for incompressible fluid.

$$\frac{D\underline{u}}{Dt} + 2\underline{\Omega} \times \underline{u} = -\frac{1}{\rho_0}\nabla p + \frac{\rho g}{\rho_0}\hat{\underline{z}} + \nu\nabla^2 \underline{u} + \frac{1}{\rho}\underline{J} \times \underline{B} \qquad (1)$$

Heat equation for incompressible fluid.

$$\frac{\partial T}{\partial t} + (\underline{u} \cdot \times \nabla)T = \kappa\nabla^2 \mathrm{T} \qquad (2)$$

Induction equation with constant magnetic diffusivity (i.e. $\eta = \text{constant}$ **).**

$$\frac{\partial \underline{B}}{\partial t} = \nabla \times (\underline{u} \times \underline{B}) + \eta\nabla^2 \underline{B} \qquad (3)$$

And

$$\nabla \cdot \underline{u} = \nabla \cdot \underline{B} = 0 \qquad (4)$$

With basic states

$$\Theta = T_0 - \beta z, \quad \underline{u} = \underline{0}, \quad P = P_0 - g\rho_0\left(z + \frac{\alpha\beta z^2}{2}\right), \quad \underline{B} = \underline{B}_0.$$

Now, we perturb the basic states as follows: $\underline{u} = 0 + \underline{u}'$, $T = \Theta + \theta'$, $p = P + p'$, and $\underline{B} = \underline{B}_0 + \underline{B}'$ and we assume that the perturbed states satisfies the governing equations. By substitution in equations (1-4), we get:

$$\frac{\partial \underline{u}'}{\partial t} + 2\underline{\Omega} \times \underline{u}' = \frac{1}{\rho_0}\nabla p' + g\alpha\theta'\underline{\hat{z}} + \upsilon\nabla^2\underline{u}' + \frac{1}{\mu_0\rho_0}\left[(\underline{B}_0 \cdot \nabla)\underline{B}'\right], \qquad (5)$$

$$\frac{\partial \theta'}{\partial t} - \beta\omega' = \kappa\nabla^2\theta' \qquad (6)$$

$$\frac{\partial \underline{B}'}{\partial t} = (\underline{B}_0 \cdot \nabla)\underline{u}' + \eta\nabla^2\underline{B}', \qquad (7)$$

$$\nabla \cdot \underline{B}' = \nabla \cdot \underline{u}' = \underline{0} \qquad (8)$$

We have dropped non-linear terms such as $\left(\underline{u}' \cdot \nabla\right)\underline{u}', (\nabla \times \underline{B}') \times \underline{B}', \nabla \times (\underline{u}' \times \underline{B}')$ and $(\underline{u}' \cdot \nabla)\theta'$.

Since $\underline{B}_0$ is a constant vector, this makes simplification in the equations above as $(\nabla \times \underline{B}_0) \times \underline{B}' = 0, (\nabla \times \underline{B}_0) \times \underline{B}_0 = 0$ and we made more simplification using the vector identity:

$$(\nabla \times \underline{B}') \times \underline{B}_0 = \left[(\underline{B}_0 \cdot \nabla)\underline{B}'\right] - \left[(\underline{B}' \cdot \nabla)\underline{B}_0\right] + \underline{B}'(\nabla \cdot \underline{B}_0) - \underline{B}_0(\nabla \cdot \underline{B}') = \left[(\underline{B}_0 \cdot \nabla)\underline{B}'\right],$$

$$(\nabla \times \underline{u}') \times \underline{B}_0 = \left[(\underline{B}_0 \cdot \nabla)\underline{u}'\right] - \left[(\underline{u}' \cdot \nabla)\underline{B}_0\right] + \underline{u}'(\nabla \cdot \underline{B}_0) - \underline{B}_0(\nabla \cdot \underline{u}') = \left[(\underline{B}_0 \cdot \nabla)\underline{u}'\right],$$

To eliminate the pressure we take the curl of equation (5) and we drop primes

$$\frac{\partial \underline{\omega}}{\partial t} = g\alpha\nabla\theta\times\hat{\underline{z}} + \nu\nabla^2\underline{\omega} + 2(\underline{\Omega}\cdot\nabla)\underline{u} + \frac{1}{\rho_0}\left[(\underline{B}_0\cdot\nabla)\underline{J}\right] \qquad (9)$$

Where $\underline{\omega} = \nabla\times\underline{u}$, $\mu_0\underline{J} = \nabla\times\underline{B}$, and since $\underline{\Omega}$ is a constant vector, then

$$\nabla\times(\underline{\Omega}\times\underline{u}) = (\underline{u}\cdot\nabla)\underline{\Omega} - (\underline{\Omega}\cdot\nabla)\underline{u} + \underline{\Omega}(\nabla\cdot\underline{u}) - \underline{u}(\nabla\cdot\underline{\Omega}) = -(\underline{\Omega}\cdot\nabla)\underline{u}$$

Taking another curl of equation (9), we obtain

$$\frac{\partial\nabla^2\underline{u}}{\partial t} = g\alpha\nabla_h^2\theta + \nu\nabla^4\underline{u} - 2(\underline{\Omega}\cdot\nabla)\underline{\omega} + \frac{1}{\mu_0\rho_0}\left[(\underline{B}_0\cdot\nabla)\nabla^2\underline{B}\right] \qquad (10)$$

Taking the z-component of equations (9) and (10)

$$\frac{\partial\zeta}{\partial t} = \nu\nabla^2\zeta + 2(\underline{\Omega}\cdot\nabla)w + \frac{1}{\rho_0}(\underline{B}_0\cdot\nabla)J_z \qquad (11)$$

$$\frac{\partial\nabla^2 w}{\partial t} = g\alpha\nabla_h^2\theta + \nu\nabla^4 w - 2(\underline{\Omega}\cdot\nabla)\zeta + \frac{1}{\mu_0\rho_0}\left[(\underline{B}_0\cdot\nabla)\nabla^2 B_z\right] \qquad (12)$$

2.1.The process of solution when $\underline{B}_0$ and $\underline{\Omega}$ are both vertical

We shall restrict our analysis to the case when $\underline{B}_0$ and $\underline{\Omega}$ are both vertical due to limited time, then our basic equations become:

The heat equation

$$\frac{\partial\theta}{\partial t} - \beta w = \kappa\nabla^2\theta \qquad (13)$$

The z-component of the induction equation

$$\frac{\partial b_z}{\partial t} = B_0\frac{\partial w}{\partial z} + \eta\nabla^2 b_z \qquad (14)$$

The z-component of curl of induction equation

$$\frac{\partial j_z}{\partial t} = \frac{1}{\mu_0} B_0 \frac{\partial \zeta}{\partial z} + \eta \nabla^2 j_z \qquad (15)$$

The z-component of curl of momentum equation

$$\frac{\partial \zeta}{\partial t} = \nu \nabla^2 \zeta + 2\Omega \frac{\partial w}{\partial z} + \frac{1}{\rho_0} B_0 \frac{\partial j_z}{\partial z} \qquad (16)$$

The z-component of double curl of momentum equation

$$\frac{\partial \nabla^2 w}{\partial t} = g\alpha \nabla_h^2 \theta + \nu \nabla^4 w - 2\Omega \frac{\partial \zeta}{\partial z} + \frac{1}{\mu_0 \rho_0} B_0 \frac{\partial}{\partial z} \nabla^2 b_z \qquad (17)$$

2.2. Dimensionless equations

Now we switch to dimensionless variables, consider a length scale d, time scale d^2/κ, velocity scale κ/d temperature scale βd, and magnetic field scale B_0, substituting in equations (13-17):

$$\frac{\partial \theta}{\partial t} - w = \nabla^2 \theta \qquad (18)$$

$$\frac{\partial b_z}{\partial t} = \frac{\partial w}{\partial z} + \frac{\Pr}{Pm} \nabla^2 b_z \qquad (19)$$

$$\frac{\partial j_z}{\partial t} = \frac{\partial \zeta}{\partial z} + \frac{\Pr}{Pm} \nabla^2 j_z \qquad (20)$$

$$\frac{\partial \zeta}{\partial t} = \Pr \nabla^2 \zeta + T^{1/2} \Pr \frac{\partial w}{\partial z} + \frac{\Pr^2 Q}{Pm} \frac{\partial j_z}{\partial z} \qquad (21)$$

$$\frac{\partial \nabla^2 w}{\partial t} = \Pr Ra \nabla_h^2 \theta + \Pr \nabla^4 w - T^{1/2} \Pr \frac{\partial \zeta}{\partial z} + \frac{\Pr^2 Q}{Pm} \frac{\partial}{\partial z} \nabla^2 b_z \qquad (22)$$

The resulting dimensionless parameters are:

$$Q = \frac{B_0^2 d^2}{\mu_0 \rho_0 \eta \nu}, \quad Ra = \frac{g\alpha d^4 \beta}{\nu\kappa}, \quad T = \frac{4\Omega^2 d^4}{\nu^2}, \quad \Pr = \frac{\nu}{\kappa}, \quad Pm = \frac{\nu}{\eta}. \quad (23)$$

2.3.Analysis into normal modes

In this section, we shall investigate solutions of equations (18-22) which satisfy the normal boundary conditions (i.e. stress free and no slip boundary conditions, in addition to the electrical fluid is non conducting), Expressing each variable in equations (18-22) as a normal mode of the form:

$$w(x,y,z,t) = W(z)\exp\left(st + i(kx+ly)\right), \qquad \theta(x,y,z,t) = \Theta(z)\exp\left(st + i(kx+ly)\right),$$

$$j_z(x,y,z,t) = J(z)\exp\left(st + i(kx+ly)\right), \qquad b_z(x,y,z,t) = B(z)\exp\left(st + i(kx+ly)\right),$$

$$\zeta(x,y,z,t) = Z(z)\exp\left(st + i(kx+ly)\right)$$

Where s is the complex growth rate $(s = \sigma + i\omega)$, call $D = \frac{\partial}{\partial z}$ and call $a^2 = k^2 + l^2$. Substitute these solutions in equations (18-22), we obtain:

$$\left[s - \left(D^2 - a^2\right)\right]\Theta = W \quad (24)$$

$$\left[s - \frac{\Pr}{Pm}\left(D^2 - a^2\right)\right]B = DW \quad (25)$$

$$\left[s - \frac{\Pr}{Pm}\left(D^2 - a^2\right)\right]J = DZ \quad (26)$$

$$\left[s - \Pr\left(D^2 - a^2\right)\right]Z = \frac{\Pr^2 Q}{Pm} DJ + T^{1/2} \Pr DW \quad (27)$$

$$\left[s - \Pr\left(D^2 - a^2\right)\right]\left(D^2 - a^2\right)W = -a^2 \Pr Ra\Theta - T^{1/2} \Pr DZ + \frac{\Pr^2 Q}{Pm} D\left(D^2 - a^2\right)B \tag{28}$$

Eliminating J between equations (26) and equation (27) by applying the operator $\left(s - \frac{\Pr}{Pm}\left(D^2 - a^2\right)\right)$ on equation (27)

$$\left(\left(s - \Pr\left(D^2 - a^2\right)\right)\left(s - \frac{\Pr}{Pm}\left(D^2 - a^2\right)\right) - \frac{\Pr^2 Q}{Pm} D^2\right)Z =$$
$$T^{1/2} \Pr D\left(s - \frac{\Pr}{Pm}\left(D^2 - a^2\right)\right)W \tag{29}$$

Now eliminate B between equations (25) and (28) by applying the operator $\left(s - \frac{\Pr}{Pm}\left(D^2 - a^2\right)\right)$ on equation (28)

$$\left(D^2 - a^2\right)\left(\left(s - \Pr\left(D^2 - a^2\right)\right)\left(s - \frac{\Pr}{Pm}\left(D^2 - a^2\right)\right) - \frac{\Pr^2 Q}{Pm} D^2\right)W =$$
$$-a^2 \Pr Ra\left(s - \frac{\Pr}{Pm}\left(D^2 - a^2\right)\right)\Theta - T^{1/2} \Pr D\left(s - \frac{\Pr}{Pm}\left(D^2 - a^2\right)\right)Z \tag{30}$$

Now eliminate Z using equation (29), we obtain

$$\begin{bmatrix} \left(D^2-a^2\right)\left(\left(s-\Pr\left(D^2-a^2\right)\right)\left(s-\frac{\Pr}{Pm}\left(D^2-a^2\right)\right)-\frac{\Pr^2 Q}{Pm}D^2\right)^2 + \\ T\Pr^2 D^2\left(s-\frac{\Pr}{Pm}\left(D^2-a^2\right)\right)^2 \end{bmatrix} W =$$

$$-a^2 \Pr Ra\left(s-\frac{\Pr}{Pm}\left(D^2-a^2\right)\right)$$

$$\left(\left(s-\Pr\left(D^2-a^2\right)\right)\left(s-\frac{\Pr}{Pm}\left(D^2-a^2\right)\right)-\frac{\Pr^2 Q}{Pm}D^2\right)\Theta \qquad (31)$$

Eliminating Θ using equation (24), then equation (31) becomes:

$$\left(s-\left(D^2-a^2\right)\right)\begin{bmatrix} \left(D^2-a^2\right)\left(\left(s-\Pr\left(D^2-a^2\right)\right)\left(s-\frac{\Pr}{Pm}\left(D^2-a^2\right)\right)-\frac{\Pr^2 Q}{Pm}D^2\right)^2 \\ +T\Pr^2 D^2\left(s-\frac{\Pr}{Pm}\left(D^2-a^2\right)\right)^2 \end{bmatrix} W$$

$$=-a^2 \Pr Ra\left(s-\frac{\Pr}{Pm}\left(D^2-a^2\right)\right)$$

$$\left(\left(s-\Pr\left(D^2-a^2\right)\right)\left(s-\frac{\Pr}{Pm}\left(D^2-a^2\right)\right)-\frac{\Pr^2 Q}{Pm}D^2\right)W \qquad (32)$$

The solution of the characteristic value problem must satisfy the boundary conditions satisfied above.

2.4. Solutions for the case of stationary convection

In this section, we shall consider the case when instability sets in as stationary convection, which means the marginal state will be characterized by s = 0, so setting s = 0 in equation (32), we obtain

$$\left(D^2-a^2\right)\left(\left[\left(D^2-a^2\right)^2-QD^2\right]^2+TD^2\left(D^2-a^2\right)\right)W=$$
$$-a^2Ra\left[\left(D^2-a^2\right)^2-QD^2\right]W \qquad (33)$$

Solution of equation (33) must be sought to satisfy the boundary conditions satisfied before (i.e. stress free and no slip boundary conditions, in addition to the electrical fluid is non-conducting). Based on Chandrasekhar analysis, the proper solutions of equation (33) is

$$W=W_0\sin(n\pi z)$$

Substituting for W in equation (33), we obtain

$$Ra=\frac{\left(n^2\pi^2+a^2\right)\left(\left[\left(n^2\pi^2+a^2\right)^2+n^2\pi^2Q\right]^2+Tn^2\pi^2\left(n^2\pi^2+a^2\right)\right)}{a^2\left[\left(n^2\pi^2+a^2\right)^2+n^2\pi^2Q\right]} \qquad (34)$$

Define

$$x=\frac{a^2}{\pi^2},\quad Q_1=\frac{Q}{\pi^2},\quad T_1=\frac{T}{\pi^4},\quad R=\frac{Ra}{\pi^4} \qquad (35)$$

Then equation (34) for lowest unstable mode (i.e. $n=1$) can be written as

$$Ra=\pi^4\frac{(1+x)\left(\left((1+x)^2+Q_1\right)^2+T_1(1+x)\right)}{x\left((1+x)^2+Q\right)} \qquad (36)$$

Minimizing Rayleigh number over x, we obtain

$$x\left((1+x)^2+Q_1\right)\left(4(1+x)^2\left[(1+x)^2+Q_1\right]+\left[(1+x)^2+Q_1\right]^2+2T_1(1+x)\right)$$
$$-(1+x)\left(\left[(1+x)^2+Q_1\right]^2+T_1(1+x)\right)\left(\left[(1+x)^2+Q_1\right]+2x(1+x)\right)=0$$

$$\Rightarrow 2x(1+x)^2\left[(1+x)^2+Q_1\right]^2-\left[(1+x)^2+Q_1\right]^3-$$
$$T_1\left[(1+x)^2+Q_1\right](x^2-1)-2x(1+x)^3T_1=0,$$

$$\Rightarrow 2x^3 + 3x^2 - 1 = Q_1 + T_1 \frac{(1+x)^4 - Q_1(x^2-1)}{\left[(1+x)^2 + Q_1\right]^2} \qquad (37)$$

It is quite complicated to specify the minimum values of wave number x from equation (37) analytically, so we shall locate the Rayleigh minimum values numerically based on Maple work.

2.5. Numerical solutions for critical wave numbers and critical Rayleigh numbers

Firstly, looking at figure (1), we note that equation (37) has two minimum values for the Rayleigh number for various values of Q_1 and T_1. One minimum occurs at a large value of x, the other at values of x close to unity. For example when $T_1 = 10^5$ and at small Q_1, the larger value of x gives the smaller minimum, while at larger Q_1, the value of x close to unity gives the smaller minimum. So for given T_1 there is a certain value of Q_1, at which a jump from the larger x mode to the x of order unity mode occurs. For Q_1 greater than this critical value, the smaller x solution is preferred as Rayleigh number is increased. This behavior is detailed in table (1).

This double minimum behavior occurs only for large T_1. If T_1 is small, for example $T_1 = 1500$, then there is only a single minimum and the wave number will vary extremely and rapidly for a very small interval of Q_1. Table (2) gives the critical value of wave number a_c and Ra_c for various values of Q_1.

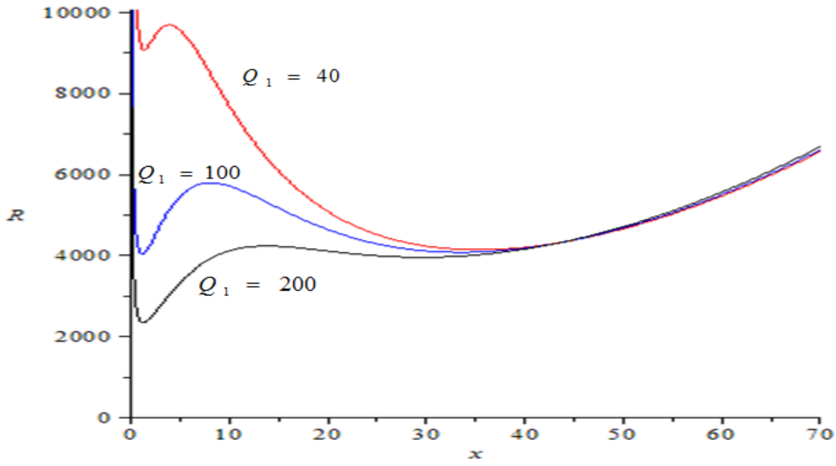

Fig. I. Onset of stationary convection for $T_1 = 10^5$ and $Q_1 = 40,100,200$ respectively

TABLE I THE CRITICAL WAVE NUMBERS WITH CRITICAL RAYLEIGH NUMBER FOR $T_1 = 10^5$ AND VARIOUS VALUES OF Q_1

Q_1	a_{c1}	a_{c2}	Ra_{c1}	Ra_{c2}
40	3.61	18.66	8.82×10^5	4.04×10^5
60	3.43	18.50	6.18×10^5	4.02×10^5
80	3.38	18.35	4.78×10^5	4.00×10^5
100	3.36	18.18	3.93×10^5	3.98×10^5
200	3.49	17.10	2.28×10^5	3.86×10^5
1000	5.68	-	1.83×10^5	-
10000	12.29	-	1.08×10^6	-
100000	18.90	-	1.02×10^7	-

Physically, this means that if convection starts with $T_1 = 10^5$ without magnetic field $Q_1 = 0$, and then we gradually increase the strength of magnetic field Q_1, and then cells appear at stationary convection will be elongated, however as we increase the strength of magnetic field corresponding to $Q_1 = 100$, then two different pattern of cells determined, one pattern will be highly elongated and the other pattern will be almost square. As we increase the strength of magnetic field, the critical Rayleigh number will start to decrease and pass through a minimum value, and eventually the inhibition due to the marginal field will predominate. All these results are valid for $T_1 > 2500$.

We are interested in the case where the value of Q_1 is such that the two critical Rayleigh numbers have the same value. This means that the two critical waves number both onsets at the same value of Ra_c.

TABLE II THE CRITICAL WAVE NUMBER WITH CRITICAL RAYLEIGH NUMBER FOR $T_1 = 1500$

Q_1	a_{crit}	Ra_{crit}
10	8.67	26190.20
20	7.79	25131.83
30	4.30	22548.18
50	4.33	19756.73
100	5.04	21682.51
1000	8.49	1.197×10^5
10000	12.80	1.065×10^6
100000	18.94	1.0148×10^7

2.6.Critical value of Q_1 for large T_1

Numerical investigation of equation (37) suggests there can be two minima for large T_1, with different values of x, say x_A and x_B. We assume that $x_A > x_B$ (i.e. where $x = a^2 / \pi^2$).

At large T_1, we seek a value of Q_1, at which these two extremal values x_A and x_B give the same critical value of Ra according to (36). This then defines a value of $Q_1 = Q_{crit}$, so for $Q_1 > Q_{crit}$, this minimum x_B will give a lower critical value of Ra_{crit} than the minimum x_A.

Moreover, if $Q_1 > Q_{crit}$, then the minima x_A will give the lower critical Ra_c.

Basically, finding Q_{crit} is not straightforward, however according to maple work, we have found that at $T_1 = 10^6$, which is large enough to have two critical wave numbers, there are two equal minima near $Q_1 = 212$, with $x_A = 1.0636$ which corresponds to $Ra_c = 1.84 \times 10^6$ and $x_B = 76.1$ which also corresponds to $Ra_c = 1.84 \times 10^6$.(see figure (II))

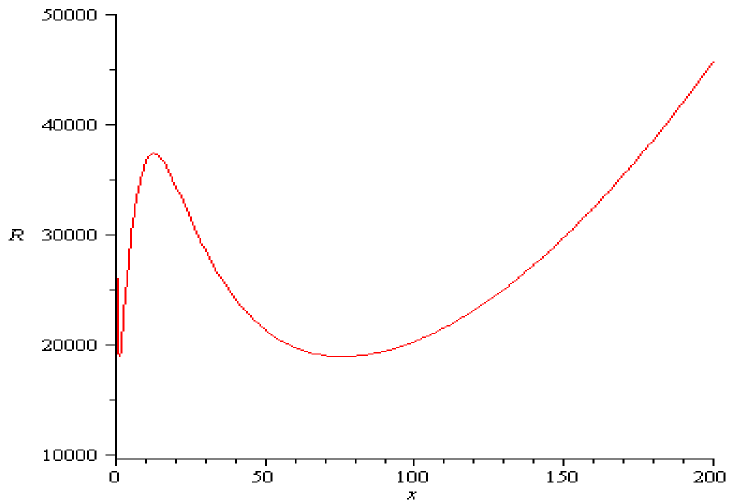

Fig. II Stationary convection for $T_1 = 10^6$ and Q1 = 212

In a non magnetic rotating convection where $Q_1 = 0$, then equation (37) gives $2x^3 \approx T_1$, so x sc09O88LIaled with $x \approx \left(\frac{T_1}{2}\right)^{1/3}$, then the critical Rayleigh number approximated as

$$R \approx \frac{\pi^4 x\left(x^4 + T_1 x\right)}{x^3} \approx \frac{\pi^4}{x}\left(x^3 + T_1\right) \approx \pi^4\left(\frac{2}{T_1}\right)^{1/3}\left(\frac{3T_1}{2}\right) = \frac{3\pi^4}{2^{2/3}} T^{2/3} \qquad (38)$$

This value of x and Ra correspond quite closely to the numerical values of the large x minimum in figure (II), suggesting that this minimum is determined by the rotation, but not the magnetic field.

For the smaller value of x, a significant result about this is that x_A is very close to 1, so the roots of $(1+x)^4 - Q_1\left(x^2 - 1\right)$ in the extremal equation (37) are not large, however it has order 1. In addition, $2x^3 + 3x^2 - 1$ is negligible, since x_A is close to 1, so the remaining terms in equation (37) give

$$Q_1^3 \approx T_1 \rightarrow Q_1 \approx T^{1/3} \qquad (39)$$

Thus $\left[(1+x)^2 + Q_1\right]^2$ in equation (36) is negligible to $T_1(1+x)$, so we are left with

$$Ra \approx 4\pi^4 \frac{T_1}{Q_1} \approx 4\pi^4 T^{2/3} \qquad (40)$$

For the large root x_B, terms including Q_1 which are of order $T_1^{1/3}$ 1 in the extremal equation (37) are negligible compare with terms include T_1 and x, so keeping terms of highest power of x and T_1, then x will be close to

$$x \approx \left(\frac{T_1}{2}\right)^{1/3} \qquad (41)$$

$$Ra \approx 3\pi^4\left(\frac{T_1}{2}\right)^{2/3} \qquad (42)$$

This is consistent with root of x in a nonmagnetic rapidly rotating convection. Substituting for x and Q_1 in equation (36), then we obtain

Since at Q_{crit}, the critical Rayleigh numbers are the same, this implies to

$$4\pi^4 \frac{T_1}{Q_1} \approx 3\pi^4 \left(\frac{T_1}{2}\right)^{2/3} \tag{43}$$

This simplifies to

$$Q_{crit} \approx \frac{2^{8/3}}{3} T^{1/3} = 2.1165 T^{1/3} \tag{44}$$

This is consistent with the values of Q_{crit} obtained for finite T_1 using Maple, the results being given in table (3), so at large T_1, $Q_1 = \frac{2^{3/8}}{3} T^{1/3}$ is the critical value of the Chandrasekhar number which divides the tall thin column modes from the order one magnetic wave number modes. So for $Q_1 < 2.1165T^{1/3}$, then large root of x is preferred, but for $Q_1 > 2.1165T^{1/3}$, then smaller x mode is preferred. Note that as Q_1 gets large compared to $T_1^{1/3}$, then the magnetic effect dominates and x become large again

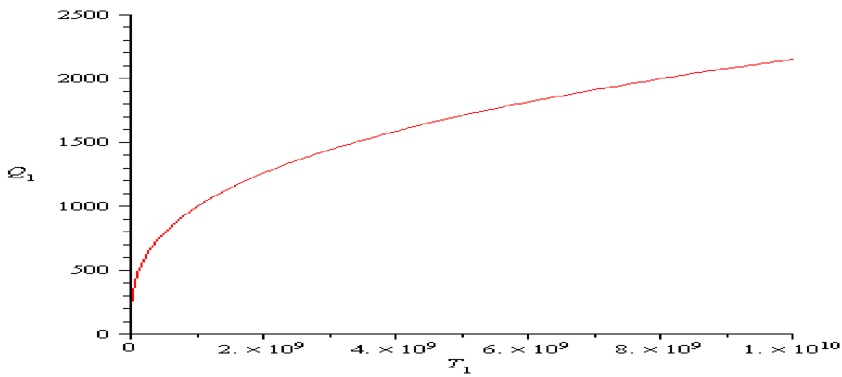

Fig. III Relation between T1 and critical values of Q1

TABLE III THE CRITICAL VALUE OF Q1 FOR VARIOUS VALUES OF T1 AND THEIR CORRESPONDING RAC

T_1	Q_{crit}	x_A	R_A	x_B	R_B
10^4	45.6	1.38	8.61×10^4	13.59	8.60×10^4
10^5	98.24	1.15	3.99×10^5	33.54	3.98×10^5
10^6	211.65	1.06	1.85×10^6	76.15	1.84×10^6
10^7	456	1.03	8.56×10^6	167.80	8.55×10^6
10^8	982.41	1.01	3.97×10^7	365.22	3.97×10^7
10^{10}	4559.94	1.00	8.55×10^8	1706.81	8.55×10^8

3. CONCLUDING REMARKS ON STATIONARY CONVECTION OF THE SYSTEM

In this paper we examined the linear stability of a rotating, electrically conducting viscous layer and lying in a uniform magnetic field B_0. We restricted our study to the case when the direction of the rotating axes and magnetic field are both vertical and the boundaries are free. The numerical analysis showed that at fixed T and as we increase strength of magnetic field Q, the critical Rayleigh number has high values until Q reached a specific value, and then Ra_c started to decrease reaching its minimum value at stationary convection.

Numerical investigation of the extremal equation at stationary convection showed that there are two exist minimum at large T, we determine the critical value of Chandrasekhar number which divides the tall thin column modes from the order one magnetic wave number modes on the fact that one of the x root is close to 1 and the other root is quite big and we conclude that

$$Q_{crit} = \frac{2^{8/3}}{3} T^{1/3}$$

So for $Q_1 < \left(2^{8/3}\right) / \left(3T^{1/3}\right)$, then large root of x is preferred, but for $Q_1 > \left(2^{8/3}\right) / \left(3T^{1/3}\right)$, then smaller x mode is preferred. Note that as Q_1 gets large compared to $T_1^{1/3}$, then the magnetic effect dominates and x become large again.

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