A Numerical Solution for Sine-Gordon Type System

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Abstract

A numerical solution for Sine-Gordon type system was done by the use of two finite difference schemes, the first is the explicit scheme and the second is the Crank-Nicholson scheme. A comparison between the two schemes showed that, the explicit scheme is easier and has faster convergence than the Crank-Nicholson scheme which is more accurate . The MATLAB system was used for the numerical computations.

1. Introduction

One of the most gratifying features of the solution of partial differential equations by difference methods is that many of the methods and proofs based on linear equations with constant coefficients can over directly to non-linear equations. Thus many of the simplest explicit and implicit two-level methods can be used for non-linear equations. A satisfactory explicit difference replacement of equation is. This difference approximation is very simple to use, but suffers from the disadvantage that the ratio of the time step to the square of the space increment is strictly limited. The stability limitation can be removed by using an implicit difference method of Crank–Nicholson type [7]. Ercolani et al [3] developed a homoclinic geometric structure of the integrable Sine-Gordon equation under periodic boundary conditions. Ablowtiz et al [1] investigated the numerical behavior of a double discrete, completely integrable discretization of the Sine-Gordon equation. Ablowtiz et al [2] used the nonlinear spectrum as a basis for comparing the effectiveness of symplectic and nonsymplectic integrators in capturing infinite dimensional phase space dynamics. Speight and Ward [10] described a spatially-discrete Sine-Gordon system with some novel features.Wei [11] used the discrete singular convolution (DSC) algorithm to solve Sine-Gordon equation. Nakagiri and Ha [8] studied the uniqueness and existence of the solutions of the coupled Sine-Gordon equations by the use of the variational method. They solved the quadratic optimal control problems for the control systems described by coupled Sine-Gordon equations. Lu and Schmid [5] presented a class of schemes based on symplectic integrators for the computation of solutions of Sine-Gordon type systems. Khusnutdinova and Pelinovsky [4] considered a system of coupled Klein-Gordon equations; they found both linear and nonlinear solutions involving the exchange in the energy between the different components of the system.

In this paper, the Sine-Gordon type system was solved by using two finite difference schemes.

2. The Mathematical Model

One of the well-known nonlinear wave equations is Klein-Gordon equation

$$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = -f(u) \qquad (1)$$

which consider one of the most important nonlinear wave equations.

When $f(u) = \delta u$, where δ is constant then equation (1), becomes the linear Klein-Gordon equation

$$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = -\delta u \qquad (2)$$

Which is the simplest model of the equations which describe the normally dispersive linear waves. When $\delta = 0$, then equation (2), reduced to the classical wave equation.

When $f(u) = \delta \sin u$, then equation (1), becomes the Sine-Gordon equation

$$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = -\delta \sin u \qquad (3)$$

Equation (1) was introduced by Klein and Gordon in 1920s as a model of the nonlinear wave equation. Equation (3) can be generalized to a system of two coupled Sine-Gordon equations

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = -\delta^2 \sin(u - w)$$

$$\frac{\partial^2 w}{\partial t^2} - c^2 \frac{\partial^2 w}{\partial x^2} = \sin(u - w) \qquad (4)$$

$$u(x,0) = A\cos x,\; u_t(x,0) = 0,\; w(x,0) = 0,\; w_t(x,0) = 0$$

$$u(0,t) = 1,\; u(\pi,t) = -1,\; w(0,t) = 0,\; w(\pi,t) = 0,\; 0 \le x \le \pi,\; 0 \le t \le a$$

Where A is constant.

3. Derivation of the Explicit Scheme Formula for the Sine – Gordon System

Assume that the rectangle [7]: $R = \{(x,t): 0 \le x \le \pi ,\, 0 \le t \le a\}$ is subdivided into $(n-1)(m-1)$ rectangles with sides $\Delta x = h$, $\Delta t = k$. Start at the bottom rows, where $t = t_1 = 0$, and the initial condition is [4] :

$$\left.\begin{aligned} u(x_i,t_1) &= u(x_i,0) = f_1(x_i) = A\cos(x_i) \;,\quad i = 2,3,\dots,n-1 \\ w(x_i,t_1) &= w(x_i,0) = f_2(x_i) = 0 \;,\quad i = 2,3,\dots,n-1 \end{aligned}\right\} \qquad (5)$$

To complete the grid points in the second rows, we use

$$\left.\begin{aligned} u_{tt}(x_i,0) &= u_{xx}(x_i,0) - \delta^2 \sin(u(x_i,0) - w(x_i,0)) \\ w_{tt}(x_i,0) &= c^2 w_{xx}(x_i,0) + \sin(u(x_i,0) - w(x_i,0)) \end{aligned}\right\} \Rightarrow$$

By using Taylor's formula of order 2, we have

$$\left.\begin{aligned} u(x,k) &= u(x,0)+u_t(x,0)k+\frac{u_{tt}(x,0)k^2}{2}+O(k^3) \\ w(x,k) &= w(x,0)+w_t(x,0)k+\frac{w_{tt}(x,0)k^2}{2}+O(k^3) \end{aligned}\right\} \quad (6)$$

Applying formula (6) at $x = x_i$, together with $u(x_i,0)= f_{1i}$, $u_t(x_i,0)= g_1(x_i)= g_{1i}$, $w(x_i,0)= f_{2i}$, $w_t(x_i,0)= g_2(x_i)= g_{2i}$, we get

$$\left.\begin{aligned} u(x_i,k) &= f_{1i}+kg_{1i}+\frac{r^2}{2}\left(f_{1i+1}-2f_{1i}+f_{1i-1}\right)-\frac{k^2\delta^2}{2}\sin\left(f_{1i}-f_{2i}\right)+O(h^2)O(k^2)+O(k^3) \\ w(x_i,k) &= f_{2i}+kg_{2i}+\frac{c^2r^2}{2}\left(f_{2i+1}-2f_{2i}+f_{2i-1}\right)+\frac{k^2}{2}\sin\left(f_{1i}-f_{2i}\right)+O(h^2)O(k^2)+O(k^3) \end{aligned}\right\} \quad (7)$$

Where $r = \frac{k}{h}$, formula (7) can be simplified to obtain the difference formula for the second rows [6]:

$$\left.\begin{aligned} u_{i,2} &= (1-r^2)f_{1i}+kg_{1i}+\frac{r^2}{2}\left(f_{1i-1}+f_{1i+1}\right)-\frac{k^2\delta^2}{2}\sin\left(f_{1i}-f_{2i}\right) \\ w_{i,2} &= (1-c^2r^2)f_{2i}+kg_{2i}+\frac{c^2r^2}{2}\left(f_{2i-1}+f_{2i+1}\right)+\frac{k^2}{2}\sin\left(f_{1i}-f_{2i}\right) \end{aligned}\right\} \quad (8)$$

for $i = 2,3,...,n-1$

A method for computing the approximating to $u(x,t)$ at grid points in successive rows will be developed,

$\{u(x_i,t_j)\ ,\ i=2,...,n-1\}\ ,\ j=3,4,...,m$

$\{w(x_i,t_j)\ ,\ i=2,...,n-1\}\ ,\ j=3,4,...,m$

The difference formulas used for $u_{tt}(x,t), u_{xx}(x,t), w_{tt}(x,t)$ and $w_{xx}(x,t)$ are:

$$\left.\begin{aligned} u_{tt}(x,t) &= \frac{u(x,t+k)-2u(x,t)+u(x,t-k)}{k^2}+O(k^2) \\ u_{xx}(x,t) &= \frac{u(x+h,t)-2u(x,t)+u(x-h,t)}{h^2}+O(h^2) \\ w_{tt}(x,t) &= \frac{w(x,t+k)-2w(x,t)+w(x,t-k)}{k^2}+O(k^2) \\ w_{xx}(x,t) &= \frac{w(x+h,t)-2w(x,t)+w(x-h,t)}{h^2}+O(h^2) \end{aligned}\right\} \quad (9)$$

Where the grid points are:

$x_{i+1} = x_i + h\ ,\ x_{i-1} = x_i - h\ ,\ t_{j+1} = t_j + k\ ,\ t_{j-1} = t_j - k$

Neglecting the terms $O(k^2)$ and $O(h^2)$, and use approximations $u_{i,j}$ and $w_{i,j}$ for $u(x_i,t_j)$ and $w(x_i,t_j)$ in (9), which are in terms substituted in (4), we get

$$\left.\begin{aligned}&\frac{u_{i,j+1}-2u_{i,j}+u_{i,j-1}}{k^2}-\frac{u_{i+1,j}-2u_{i,j}+u_{i-1,j}}{h^2}=-\delta^2\sin\left(u_{i,j}-w_{i,j}\right)\\&\frac{w_{i,j+1}-2w_{i,j}+w_{i,j-1}}{k^2}-c^2\,\frac{w_{i+1,j}-2w_{i,j}+w_{i-1,j}}{h^2}=\sin\left(u_{i,j}-w_{i,j}\right)\end{aligned}\right\}\qquad(10)$$

After some mathematical manipulation , we obtain

$$\left.\begin{aligned}&u_{i,j+1}=\left(2-2r^2\right)u_{i,j}+r^2\left(u_{i-1,j}+u_{i+1,j}\right)-u_{i,j-1}-k^2\delta^2\sin\left(u_{i,j}-w_{i,j}\right)\\&w_{i,j+1}=\left(2-2c^2r^2\right)w_{i,j}+c^2r^2\left(w_{i-1,j}+w_{i+1,j}\right)-w_{i,j-1}+k^2\sin\left(u_{i,j}-w_{i,j}\right)\end{aligned}\right\}\qquad(11)$$

Formula (11) represents the explicit finite difference formula for the Sine – Gordon system in (4). Formula (11) is employed to create $(j+1)\,th$ rows across the grid, assuming that approximations in the jth and $(j-1)th$ rows are known. Notice that this formula explicitly gives the values $u_{i,j+1}$, $w_{i,j+1}$ in terms of $u_{i-1,j}$, $u_{i,j}$, $u_{i+1,j}$, $w_{i-1,j}$, $w_{i,j}$ and $w_{i+1,j}$.

4. Derivation of the Crank–Nicholson Formula for the Sine – Gordon System

The diffusion terms u_{xx} and w_{xx} in this method are represented by central differences, with their values at the current and previous time steps averaged [7] :

$$\left.\begin{aligned}&u_{xx}=\left[\frac{u(x-h,t+k)-2u(x,t+k)+u(x+h,t+k)}{2h^2}+\frac{u(x-h,t-k)-2u(x,t-k)+u(x+h,t-k)}{2h^2}\right]\\&u_{tt}=\frac{u(x,t+k)-2u(x,t)+u(x,t-k)}{k^2}\\&w_{xx}=\left[\frac{w(x-h,t+k)-2w(x,t+k)+w(x+h,t+k)}{2h^2}+\frac{w(x-h,t-k)-2w(x,t-k)+w(x+h,t-k)}{2h^2}\right]\\&w_{tt}=\frac{w(x,t+k)-2w(x,t)+w(x,t-k)}{k^2}\end{aligned}\right\}(12)$$

By using the approximations $u_{i,j}$ and $w_{i,j}$ for $u(x_i,t_j)$ and $w(x_i,t_j)$ in (12), which are in turn substituted into system (4), we have

$$\left.\begin{aligned}&\frac{u_{i,j+1}-2u_{i,j}+u_{i,j-1}}{k^2}-\frac{u_{i+1,j+1}-2u_{i,j+1}+u_{i-1,j+1}}{2h^2}-\frac{u_{i+1,j-1}-2u_{i,j-1}+u_{i-1,j-1}}{2h^2}=-\delta^2\sin\left(u_{i,j}-w_{i,j}\right)\\&\frac{w_{i,j+1}-2w_{i,j}+w_{i,j-1}}{k^2}-c^2\,\frac{w_{i+1,j+1}-2w_{i,j+1}+w_{i-1,j+1}}{2h^2}-c^2\,\frac{w_{i+1,j-1}-2w_{i,j-1}+w_{i-1,j-1}}{2h^2}=\sin\left(u_{i,j}-w_{i,j}\right)\end{aligned}\right\}\Rightarrow$$

$$\left.\begin{aligned}&-r^2u_{i-1,j+1}+\left(2+2r^2\right)u_{i,j+1}-r^2u_{i+1,j+1}=r^2u_{i-1,j-1}-\left(2+2r^2\right)u_{i,j-1}+r^2u_{i+1,j-1}\\&+4u_{i,j}-2k^2\delta^2\sin\left(u_{i,j}-w_{i,j}\right)\\&-c^2r^2w_{i-1,j+1}+\left(2+2c^2r^2\right)w_{i,j+1}-c^2r^2w_{i+1,j+1}=c^2r^2w_{i-1,j-1}-\left(2+2c^2r^2\right)w_{i,j-1}+c^2r^2w_{i+1,j-1}\\&+4w_{i,j}+2k^2\sin\left(u_{i,j}-w_{i,j}\right)\end{aligned}\right\}\qquad(13)$$

for $i = 2,3,...,n-1$

Formula (13) represents the Crank–Nicholson formula for system (4).

The terms on the right hand side of (13) are all known. Hence, the equations in (13) form a tridiagonal linear algebraic systems $A_1 X_1 = B_1$ and $A_2 X_2 = B_2$. The boundary conditions are used in the first and last equations only i.e.

$$u_{1,j-1} = u_{1,j+1} = b_1 ,\ u_{n,j-1} = u_{n,j+1} = b_2 ,\ w_{1,j-1} = w_{1,j+1} = c_1 \ and \ w_{n,j-1} = w_{n,j+1} = c_2 \ , \forall j.$$

Equations in (13) are especially pleasing to view in their tridiagonal matrix forms $A_1 X_1 = B_1$, $A_2 X_2 = B_2$, where A_1 and A_2 are the coefficient matrices, X_1 and X_2 are the unknown vectors and B_1, B_2 are the known vectors as shown below:

$$A_1 X_1 = \begin{bmatrix} 2+2r^2 & -r^2 & & & & & \\ -r^2 & 2+2r^2 & -r^2 & & & & \\ & & \ddots & & & & \\ & & -r^2 & 2+2r^2 & -r^2 & & \\ & & & & \ddots & & \\ & & & -r^2 & 2+2r^2 & -r^2 \\ & & & & -r^2 & 2+2r^2 \end{bmatrix} \begin{bmatrix} u_{2,j+1} \\ u_{3,j+1} \\ \vdots \\ u_{p,j+1} \\ \vdots \\ u_{n-2,j+1} \\ u_{n-1,j+1} \end{bmatrix} = B_1 =$$

$$\begin{bmatrix} 2b_1 r^2 - \left(2+2r^2\right) u_{2,j-1} + r^2 u_{3,j-1} + 4u_{2,j} - 2k^2\delta^2 \sin\left(u_{2,j} - w_{2,j}\right) \\ r^2 u_{2,j-1} - \left(2+2r^2\right) u_{3,j-1} + r^2 u_{4,j-1} + 4u_{3,j} - 2k^2\delta^2 \sin\left(u_{3,j} - w_{3,j}\right) \\ \\ r^2 u_{p-1,j-1} - \left(2+2r^2\right) u_{p,j-1} + r^2 u_{p+1,j-1} + 4u_{p,j} - 2k^2\delta^2 \sin\left(u_{p,j} - w_{p,j}\right) \\ \\ 2b_2 r^2 + r^2 u_{n-2,j-1} - \left(2+2r^2\right) u_{n-1,j-1} + 4u_{n-1,j} - 2k^2\delta^2 \sin\left(u_{n-1,j} - w_{n-1,j}\right) \end{bmatrix} \quad (14)$$

$$A_2 X_2 = \begin{bmatrix} 2+2c^2r^2 & -r^2 & & & & & \\ -r^2 & 2+2c^2r^2 & -r^2 & & & & \\ & & \ddots & & & & \\ & & -r^2 & 2+2c^2r^2 & -r^2 & & \\ & & & & \ddots & & \\ & & & -r^2 & 2+2c^2r^2 & -r^2 \\ & & & & -r^2 & 2+2c^2r^2 \end{bmatrix} \begin{bmatrix} w_{2,j+1} \\ w_{3,j+1} \\ \vdots \\ w_{p,j+1} \\ \vdots \\ w_{n-2,j+1} \\ w_{n-1,j+1} \end{bmatrix} = B_2 =$$

$$\begin{bmatrix} 2c_1c^2r^2 - (2+2c^2r^2)w_{2,j-1} + c^2r^2w_{3,j-1} + 4w_{2,j} + 2k^2\sin(u_{2,j}-w_{2,j}) \\ c^2r^2w_{2,j-1} - (2+2c^2r^2)w_{3,j-1} + c^2r^2w_{4,j-1} + 4w_{3,j} + 2k^2\sin(u_{3,j}-w_{3,j}) \\ \\ c^2r^2w_{p-1,j-1} - (2+2c^2r^2)w_{p,j-1} + c^2r^2u_{p+1,j-1} + 4u_{p,j} + 2k^2\sin(u_{p,j}-w_{p,j}) \\ \\ 2c_2c^2r^2 + r^2w_{n-2,j-1} - (2+2c^2r^2)w_{n-1,j-1} + 4u_{n-1,j} + 2k^2\sin(u_{n-1,j}-w_{n-1,j}) \end{bmatrix} \quad (15)$$

When the Crank–Nicholson Scheme is implemented with a computer, the linear systems $A_1X_1 = B_1$ and $A_2X_2 = B_2$ can be solved by either direct means or by iteration. In this paper, the Gaussian elimination method (direct method) has been used to solve the algebraic systems $A_1X_1 = B_1$ and $A_2X_2 = B_2$ [9].

5. Algorithm of Explicit Scheme

1. Input a, b, n, m, c, δ .
2. Evaluate h=a/(n-1), k=b/(m-1), r=k/h.
3. Save dimensions to u and v as matrices.
4. Evaluate the boundary conditions of u and v:
   $u(0,t)=1,\ u(\pi,t)=-1,\ w(0,t)=0,\ w(\pi,t)=0,\ 0 \le x \le \pi,\ 0 \le t \le a.$
5. For i=2 to n-1, evaluate the initial conditions:
   $u(x,0) = A\cos x,\ u_t(x,0)=0,\ w(x,0)=0,\ w_t(x,0)=0.$
6. End
7. For j=3:m, for i=2:n-1evaluate the formula (11).
8. End

6. Algorithm of Crank-Nicholson Scheme

1. Input a, b, n, m, c, δ .
2. Evaluate h=a/(n-1), k=b/(m-1), r=k/h.
3. Save dimensions to u and v as matrices.
4. Evaluate the boundary conditions of u and v:
   $u(0,t)=1,\ u(\pi,t)=-1,\ w(0,t)=0,\ w(\pi,t)=0,\ 0 \le x \le \pi,\ 0 \le t \le a.$
5. For i=2 to n-1, evaluate the initial conditions:
   $u(x,0) = A\cos x,\ u_t(x,0)=0,\ w(x,0)=0,\ w_t(x,0)=0$
6. End.
7. Input the principle diagonals and off diagonals of the coefficient matrices A_1 and A_2 as row vectors.
8. For j=3:m, for i=2:n-1evaluate the vectors B_1 and B_2 and solve the tridiagonal systems $A_1X_1 = B_1$ and $A_2X_2 = B_2$.
9. End.

Table (1) shows a comparison between explicit scheme and Crank-Nicholson scheme of u and w when h=0.3142, k=0.05, r=0.1592, c=1, $\delta = 1$, a=1, and A=1, for sine-Gordon system in equation (4).

Explicit Scheme of u when h=0.3142, k=0.05, r=0.1592	Crank-Nicholson Scheme of u when h=0.3142, k=0.05, r=0.1592	Explicit Scheme of w when h=0.3142, k=0.0, r=0.15925	Crank-Nicholson Scheme of w when h=0.3142, k=0.05 r=0.1592
0.9511	0.9511	0	0
0.9489	0.9489	0.0010	0.0010
0.9423	0.9429	0.0040	0.0040
0.9317	0.9319	0.0090	0.0089
0.9172	0.9176	0.0156	0.0155
0.8994	0.8999	0.0238	0.0236
0.8787	0.8794	0.0333	0.0330
0.8558	0.8566	0.0438	0.0434
0.8312	0.8321	0.0549	0.0545
0.8055	0.8066	0.0664	0.0659
0.7795	0.7806	0.0780	0.0775
0.7536	0.7547	0.0893	0.0888
0.7285	0.7294	0.1001	0.0997
0.7045	0.7052	0.1102	0.1099
0.6820	0.6825	0.1194	0.1192
0.6613	0.6616	0.1272	0.1274
0.6425	0.6426	0.1345	0.1345
0.6259	0.6257	0.1402	0.1404
0.6114	0.6110	0.1447	0.1450

0.5990	0.5984	0.1481	0.1484
0.5886	0.5879	0.1502	0.1506

Note: The results in table (1) had been obtained from the algorithms of the explicit and Crank-Nicholson schemes which had been mentioned above after converting them to two programs in MATLAB.

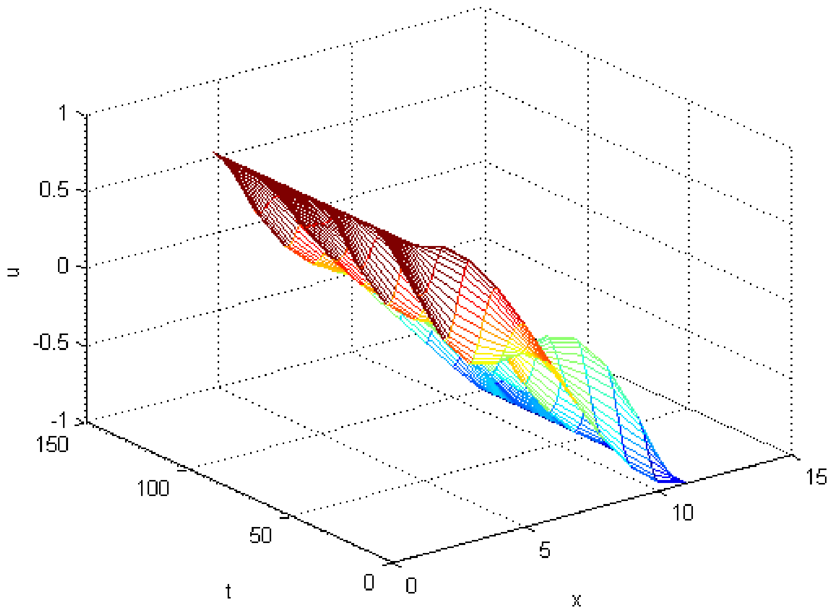

Figure (1) shows a solution curves of Crank-Nicholson scheme of u when h=0.3142, k=0.1, r=0.3183, c=1, $\delta = 1$, a=1, and A=1, for sine-Gordon system in equation (4).

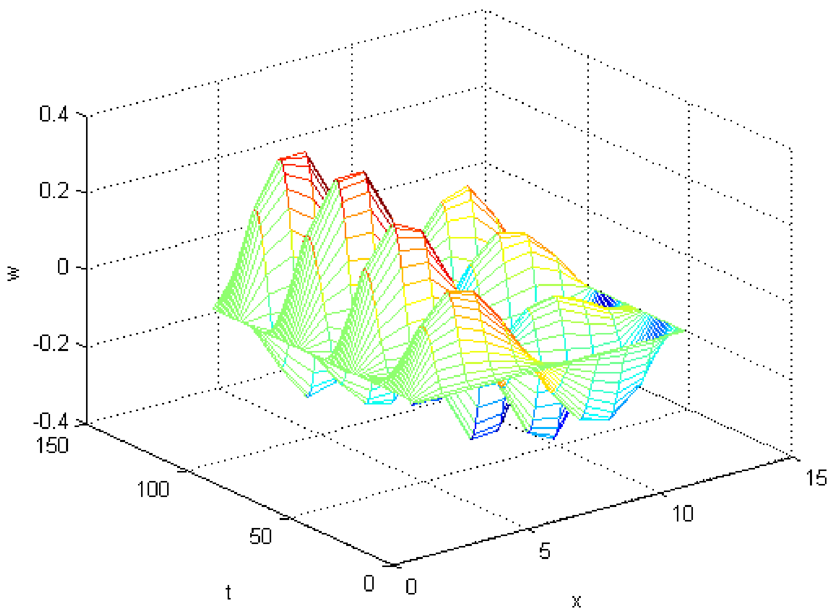

Figure (2) shows a solution curves of Crank-Nicholson scheme of w when h=0.3142, k=0.1, r=0.3183, c=1, $\delta = 1$, a=1, and A=1, for sine-Gordon system in equation (4).

7. Conclusions

We concluded from the comparison between the two schemes that the explicit scheme is easier and has faster convergence than the Crank–Nicholson scheme which is more accurate.

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الحل العددي لنظام من النوع Sine–Gordon

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الخلاصة

لقد تم حل نظام Sine-Gordon عددياً باستخدام طريقتين من طرائق الفروقات المنتهية، الأولى هي الطريقة الصريحة والثانية هي طريقة Crank-Nicholson. إذ بينت المقارنة بين الطريقتين إن الطريقة الصريحة هي الأسهل والأسرع تقارباً في حين كانت طريقة Crank-Nicholson هي الأكثر دقة، وقد استخدام نظام MATLAB في إيجاد الحسابات العددية.