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Contents

Artin Characters for the Special Linear Group SL(2, *p*) Where *p* Is a Prime Number ≤ 19

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Abstract

In this paper we calculate the induced character from the cyclic subgroups of the Special linear group SL(2,p) where p is a prime number ≤19,which is called Artin character. Furthermore, we found that Artin exponent for this group from Artin character which is equal to the order of SL(2,p) for any prime p ≤19.

الخلاصة

في هذا البحث قمنا بحساب الشخوص المحتثة من الزمر الجزئية للزمرة الخطية الخاصة SL(2,p) حيث ان p عدد أولي أقل أو يساوي 19 والتي تسمى بشواخص آرتن. إضافة لذلك قمنا بإيجاد أس آرتن لتلك الزمرة من شواخص آرتن والذي يكون مساوي لرتبة الزمرة SL(2,p) لأي عدد أولي أقل أو يساوي 19.

Introduction

Let G be a finite group, all characters of G induced from a principal character of cyclic subgroups of G are called Artin characters of G. Artin induction theorem [1] states that any rational valued character of G is a rational linear combination of the induced principal character of its cyclic subgroups. Lam [6] proved a sharp form of Artin's theorem, he determined the least positive integer A(G) such that A(G)χ is an integral linear combination of Artin character, for any rational valued character χ of G, and he called A(G) the Artin exponent of G and studied it extensively for many groups.

Let F be a field, the general linear group GL(n,F) is the group of all invertible (n×n) matrices with entries in F under matrix multiplication. The special linear group SL(n,F) is a subgroup of GL(n,F) which contains all matrices of determinant one.

In this work we take n=2 and choose F as a prime number p and we count all cyclic subgroup of SL(2,p), then we found Artin characters (induced characters) tables from these subgroups of all cases p, and write every rational valued character of SL(2,p) as an integral linear combination of Artin characters.

Our main goal is to find Artin exponent of SL(2,p) A(SL(2,p)) which is equal to the order of SL(2,p) for all prime $p \leq 19$.

§.1 Preliminaries

In this section, we recall some definitions and theorems which we needed.

Definition 1.1 **: [5]** A rational valued character χ of G is a character whose valued are in Z, that is $\chi(x) \in Z$, for all $x \in G$.

Definition 1.2 **: [4]** Let H be a subgroup of a group G, and ϕ be a class function of H. Then $\phi\uparrow^G$, the induced class function on G, is given by

$$\phi\uparrow(g) = \frac{1}{|H|}\sum_{x\in G}\phi^{\circ}(xgx^{-1}),$$

where ϕ° is defined by $\phi^{\circ}(h) = \phi(h)$ if $h \in H$ and $\phi^{\circ}(h) = 0$ if $h \notin H$.

Observe that $\phi\uparrow^G$ is a class function on G and $\phi\uparrow^G(1) = [G:H]\,\phi(1)$.

Another useful formula for computing $\phi\uparrow^G(g)$ explicitly is to choose representatives $x_1, x_2, \ldots, x_m$ for the m classes of H contained in the conjugacy class C_G in G which is given by

$$\phi\uparrow^G(g) = |C_G(g)|\sum_{i=1}^{m}\frac{\phi(x_i)}{|C_H(x_i)|}$$

where it is understood that $\varphi\uparrow^G(C_\alpha) = 0$ if $H \cap Cl(g)=\emptyset$. This formula is immediate from the definition of $\phi\uparrow^G$ since as x runs over G, $xgx^{-1} = x_i$ for exactly $|C_G(g)|$ values of x. If H is a cyclic subgroup then

$$\phi\uparrow^G(g) = \frac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\phi(x_i) \qquad ....(1\text{-}1)$$

Definition 1.3: [7] The character induced from the principal character of cyclic subgroups of G is called Artin character.

Definition 1.4 : [7] Let G be a finite group and let χ be any rational valued character on G. The smallest positive number n such that,

$$n\chi = \sum_{c} a_c \phi_c$$

where $a_c \in Z$ and ϕ_c is Artin character, is called the Artin exponent of G and denoted by A(G).

Theorem 1.5 : [7] Let 1 denote the principal character of G and $d \in Z$, then d is an Artin exponent of G if there exists (uniquely) integers $a_k \in Z$ such that

$$d \cdot 1 = \sum_{k=1}^{q} a_k \mu_k$$

where $\mu_1, \ldots, \mu_q$ are the Artin characters.

Theorem 1.6 : [7] For a subgroup H in G, A(H) divides A(G).

§.2 The Special Linear Group SL(2,p)

In this section, we introduce some concepts about the Special linear group.

Definition 2.1 : [7] The general linear group is the group of invertible n×n matrices over a field F denoted by GL(n,F). The determinant of these matrices is a homomorphism from GL(n,F) into F^* and we denote the kernel of this homomorphism by SL(n,F), the special linear group. Thus SL(n,F) is the subgroup of GL(n,F) which contains all matrices of determinant one.

In this work we are interested in finite special linear group, and we choose F to be finite, we consider the case when n=2 and F = p, where p =2, 3, 5, 7, 11, 13, 17 and 19.

Theorem 2.2 : [3] The order of $SL(2,p^k)$ is $p^k(p^{2k}-1)$.

Theorem 2.3 : [3] $G = SL(2,p^k)$ has exactly $p^k + 4$ conjugacy classes :

$$1, z, c, d, zc, zd, a, a^2, \ldots, a^{\frac{p^k-3}{2}}, b, b^2, \ldots, b^{\frac{p^k-1}{2}}$$

Let ν be the generator of the cyclic multiplicative group F^*, $1 \le \ell \le (p^k-3)/2$, $1 \le m \le (p^k-1)/2$.Thus this conjugacy classes is satisfied.

So table (2.1) represents the conjugacy classes of $SL(2,p^k)$

Table (2.1)

$g \in G$	Notation	C_g	$\lvert C_g \rvert$	$\lvert C_G(g) \rvert$
$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	1	C_1	1	$p^k(p^{2k}-1)$
$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	z	C_z	1	$p^k(p^{2k}-1)$
$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$	c	C_c	$(p^{2k}-1)/2$	$2p^k$
$\begin{pmatrix} 1 & 0 \\ \nu & 1 \end{pmatrix}$	d	C_d	$(p^{2k}-1)/2$	$2p^k$
$\begin{pmatrix} -1 & 0 \\ -1 & -1 \end{pmatrix}$	zc	C_{zc}	$(p^{2k}-1)/2$	$2p^k$
$\begin{pmatrix} -1 & 0 \\ -\nu & -1 \end{pmatrix}$	zd	C_{zd}	$(p^{2k}-1)/2$	$2p^k$
$\begin{pmatrix} \nu^\ell & 0 \\ 0 & \nu^{-\ell} \end{pmatrix}$	a^ℓ	$C_{a\ell}$	$p^k(p^k+1)$	p^k-1
Element of order$(p^k+1)m$	b^m	C_{bm}	$p^k(p^k-1)$	p^k+1

Now, in the following notation we refer to how the rational valued character of SL(2,p) is obtained.

Notation 2.4 **: [2]** Let G = SL(2,p^k) for some prime $p \neq 2$, e and e' denote divisors of $p^k - 1$ such that $e<\frac{p^k-1}{2}$ and $e'<\frac{p^k-1}{2}$, f and f' denote the divisors of $p^k + 1$ such that $f<\frac{p^k+1}{2}$ and $f'<\frac{p^k+1}{2}$, ρ_e is a primitive $(\frac{p^k-1}{e})$-th root of unity, σ_f is a primitive $(\frac{p^k+1}{f})$-th root of unity, 1, z, c, d, a, b are as in theorem (2.3), $\varepsilon=(-1)^{(p^k-1)/2}$, $\rho \in$ C be a $(p^k - 1)$-th root of unity and $\sigma \in$ C be a $(p^k + 1)$-th root of unity.

$$B(k)=\begin{cases}1 & if\ k\ is\ even\\ 2 & otherwise\end{cases} \qquad E(p^k)=\begin{cases}1 & if\ p^k\equiv 3 \bmod 4\\ 2 & otherwise\end{cases}$$

$$A(e)=\frac{1}{2}\Phi(\frac{p^k-1}{e}) \qquad C(f)=\frac{1}{2}\Phi(\frac{p^k+1}{f})$$

$\tau_1(e,e') = \sum_{\alpha\in\Gamma}(\rho_e^{e'}+\rho_e^{-e'})^{\alpha}=\dfrac{\Phi(\frac{p^k-1}{e})}{\Phi(\frac{p^k-1}{ee'})}\mu(\frac{p^k-1}{ee'})$, where $\Gamma = \Gamma(Q(\chi_e):Q)$. [Note that $\Gamma = \Gamma(Q(\rho_e + \rho_e^{-1}):Q)$].

$\tau_2(f,f') = \sum_{\alpha\in\Gamma_1}(\sigma_f^{f'}+\sigma_f^{-f'})^{\alpha}=\dfrac{\Phi(\frac{p^k+1}{f})}{\Phi(\frac{p^k+1}{ff'})}\mu(\frac{p^k+1}{ff'})$,where $\Gamma_1 = \Gamma(Q(\theta_f):Q)$. [Note that $\Gamma = \Gamma(Q(\sigma_f + \sigma_f^{-1}):Q)$]. $\chi_e = B(e)\sum_{\alpha\in\Gamma}\chi_i^{\alpha}$ where $e = (i,p^k - 1)$. $\theta_j = B(f)\sum_{\alpha\in\Gamma_1}\theta_j^{\alpha}$ where $f = (i,p^k + 1)$. ξ' and η' denote the irreducible characters of the rational representations of G arising from ξ_1 (or ξ_2) and η_1 (or η_2) respectively where k is odd. Also we know that the column for the class zc is obtained from the relation $\chi(zc)=\frac{\chi(z)}{\chi(1)}\chi(c)$ where χ is an irreducible character of G.

The character table of rational representations of SL(2,p^k), p is an odd prime, k is an odd is described in table (2.2).

Table (2.2)

Cg	1	z	c	zc	$a^{e'}$	$b^{f'}$
$\mid Cg \mid$	1	1	$(p^{2k}-1)/2$	$(p^{2k}-1)/2$	$p^k(p^k+1)$	$p^k(p^k-1)$
$\mid C_G(g) \mid$	$p^k(p^{2k}-1)$	$p^k(p^{2k}-1)$	$2p^k$	$2p^k$	p^k-1	p^k+1
1_G	1	1	1	1	1	1
ψ	p^k	p^k	0	0	1	-1
χ_e	$(p^k+1)A(e)B(e)$	$(-1)^e(p^k+1)A(e)B(e)$	$A(e)B(e)$	$(-1)^e A(e)B(e)$	$B(e)\tau_1(e,e')$	0
θ_f	$(p^k-1)C(f)B(f)$	$(-1)^f(p^k-1)C(f)B(f)$	$-C(f)B(f)$	$-(-1)^f C(f)B(f)$	0	$-B(e)\tau_2(f,f')$
ζ'	(p^k+1)	$\varepsilon(p^k+1)$	1	ε	$(-1)^{e'}2$	0
η'	$(p^k-1)E(p^k)$	$-\varepsilon(p^k-1)E(p^k)$	-1	ε	0	$(-1)^{f'+1}2E(p^k)$

§.3 Artin Character and Artin Exponent of SL(2,p)

In this section, we find the induced character (Artin character) of SL(2,p), for every $p \leq 19$ from the cyclic subgroups of SL(2,p) by using the formula (1.1) in definition (1.2), and we write the rational valued character of SL(2,p) as a linear combination of Artin character, and count the least positive integer A(SL(2,p)) such that A(SL(2,p))χ is an integral linear combination of Artin character for all rational valued character χ of SL(2,p).

(3.1) Artin Character and Artin Exponent of SL(2,2):

This group has 3 cyclic subgroups which are:
H_1, H_c and H_b.

The rational valued character table of SL(2,3) is given in the table (3.1.1).

Table (3.1.1)

C_g	1	c	b
$\vert C_g\vert$	1	2	3
$\vert C_G(g)\vert$	6	3	2
1_G	1	1	1
ψ	2	0	-1
θ	1	-1	1

We calculate the Artin character (induced character) of the cyclic subgroups by using the formula

$$\phi_H \uparrow^{SL(2,2)}(\alpha) = \frac{|C_{SL(2,2)}(x)|}{|C_H(\alpha)|} \sum_{\substack{\alpha \to x \\ H \to SL(2,2)}} \phi(\alpha)$$

and given it in table (3.1.2)

Table (3.1.2)

C_g	1	c	b
Φ_1	6	0	0
Φ_2	2	1	0
Φ_3	3	0	1

From tables (3.1.1) and (3.1.2) we can write the rational valued character of SL(2,2) as a linear combination of induced characters, and count the least positive integer A(SL(2,2)) (Artin exponent) such that A(SL(2,2)) χ_i is an integral linear combination(Z) of Φ_i , i=1,2,3 where χ_i is any rational valued character.

.

$$1=\Phi_3+\Phi_2-\frac{2}{3}\Phi_1$$

$$\psi=-\Phi_3+\frac{5}{6}\Phi_1$$

$$\theta=\Phi_3-\Phi_2$$

Note that $6\cdot\chi_i = Z\,\Phi_i$, i=1,2,3. So A(SL(2,2)) = 6.

(3.2) Artin Character and Artin Exponent of SL(2,3):

This group has 5 cyclic subgroups which are:
H_1, H_z , H_c , H_{zc} and H_b .

The rational valued character table of SL(2,3) is given in table (3.2.1), [5]

Table (3.2.1)

C_g	1	z	c	zc	b
$\|C_g\|$	1	1	4	4	6
$\|C_G(g)\|$	24	24	6	6	4
1_G	1	1	1	1	1
ψ	3	3	0	0	-1
θ_1	2	-2	-1	1	0
ξ'	4	-4	1	-1	0
η'	2	2	-1	-1	2

We calculate the Artin character (induced character) of the cyclic subgroups by using the following:-

$$\phi_H\uparrow^{SL(2,3)}(\alpha)=\frac{|C_{SL(2,3)}(x)|}{|C_H(\alpha)|}\sum_{\substack{\alpha\to x\\ H\to SL(2,3)}}\phi(\alpha)$$

and given it in table (3.2.2)

Table (3.2.2)

C_g	1	z	c	zc	b
Φ_1	24	0	0	0	0
Φ_2	12	12	0	0	0
Φ_3	8	0	2	0	0
Φ_4	4	8	0	2	0
Φ_5	6	12	0	0	2

From tables (3.2.1) and (3.2.2) we can writte the rational valued character of SL(2,3) as a linear combination of induced characters, and count the least positive integer A(SL(2,3)) (Artin exponent) such that A(SL(2,3)) χ_i is an integral linear combination(Z) of Φ_i, i=1,…,5 where χ_i is any rational valued character.

$$1 = \frac{1}{2}\Phi_5 + \frac{1}{2}\Phi_4 + \frac{1}{2}\Phi_3 - \frac{3}{4}\Phi_2 + \frac{1}{24}\Phi_1$$

$$\psi = -\frac{1}{2}\Phi_5 + \frac{3}{4}\Phi_2 - \frac{1}{8}\Phi_1$$

$$\theta_1 = \frac{1}{2}\Phi_4 - \frac{1}{2}\Phi_3 - \frac{1}{2}\Phi_2 + \frac{5}{12}\Phi_1$$

$$\zeta' = -\frac{1}{2}\Phi_4 + \frac{1}{2}\Phi_3 + \frac{1}{12}\Phi_1$$

$$\eta' = \Phi_5 - \frac{1}{2}\Phi_4 - \frac{1}{2}\Phi_3 - \frac{1}{2}\Phi_2 + \frac{1}{3}\Phi_1$$

Note that $24 \cdot \chi_i = (Z)\ \Phi_i$, i=1,…,5 . So A(SL(2,3)) = 24.

(3.3) Artin Character and Artin Exponent of SL(2,5):

This group has 7 cyclic subgroups which are:

H_1 , H_z , H_c , H_{zc} , H_a , H_b and H_{b^2} .

The rational valued character table of SL(2,5) is given in table (3.3.1), [5]

Table (3.3.1)

C_g	1	z	c	zc	a	b	b^2
$\|C_g\|$	1	1	12	12	30	20	20
$\|C_G(g)\|$	120	120	10	10	4	6	6
1_G	1	1	1	1	1	1	1
ψ	5	5	0	0	1	-1	-1
χ_1	6	-6	1	-1	0	0	0
θ_1	4	-4	-1	1	0	-1	1
θ_2	4	4	-1	-1	0	1	1
ξ'	6	6	1	1	-2	0	0
η'	4	-4	-1	1	0	2	-2

We calculate the Artin character (induced character) of the cyclic subgroups by using the following:-

$$\phi_H \uparrow^{SL(2,5)} (\alpha) = \frac{|C_{SL(2,5)}(x)|}{|C_H(\alpha)|} \sum_{\substack{\alpha \to x \\ H \to SL(2,5)}} \phi(\alpha)$$

and given it in table (3.3.2)

Table (3.3.2)

C_g	1	z	c	zc	a	b	b^2
Φ_1	120	0	0	0	0	0	0
Φ_2	60	60	0	0	0	0	0
Φ_3	24	0	2	0	0	0	0
Φ_4	12	36	3	3	0	0	0
Φ_5	30	60	0	0	2	0	0
Φ_6	20	40	0	0	0	2	0
Φ_7	40	0	0	0	0	0	2

From tables (3.3.1) and (3.3.2) we can write the rational valued character of SL(2,5) as a linear combination of induced characters, and count the least positive integer A(SL(2,5)) (Artin exponent) such that A(SL(2,5))χ_i is an integral linear combination(Z) of Φ_i, i=1,…,7 where χ_i is any rational valued character.

$$1=\frac{1}{2}\Phi_7+\frac{1}{2}\Phi_6+\frac{1}{2}\Phi_5+\frac{1}{3}\Phi_4-\frac{67}{69}\Phi_2-\frac{17}{120}\Phi_1$$

$$\psi=-\frac{1}{2}\Phi_7-\frac{1}{2}\Phi_6+\frac{1}{2}\Phi_5-\frac{1}{12}\Phi_2+\frac{5}{24}\Phi_1$$

$$\chi=-\frac{1}{3}\Phi_4+\Phi_3+\frac{1}{10}\Phi_2-\frac{1}{6}\Phi_1$$

$$\theta_1=\frac{1}{2}\Phi_7-\frac{1}{2}\Phi_6-\Phi_3+\frac{1}{3}\Phi_4-\frac{1}{15}\Phi_2+\frac{1}{12}\Phi_1$$

$$\theta_2=\frac{1}{2}\Phi_7+\frac{1}{2}\Phi_6-\frac{1}{2}\Phi_5-\frac{1}{2}\Phi_3+\frac{7}{30}\Phi_2-\frac{13}{120}\Phi_1$$

$$\zeta'=-\frac{1}{2}\Phi_5-\frac{2}{3}\Phi_4+\frac{3}{2}\Phi_3-\frac{37}{120}\Phi_1$$

$$\eta'=-\Phi_7+\Phi_6+\frac{1}{2}\Phi_5-\frac{1}{2}\Phi_3-\frac{37}{30}\Phi_2+\frac{19}{24}\Phi_1$$

Note that $120\cdot\chi_i$ = (Z) Φ_i, i=1,…,7. So A(SL(2,5)) = 120.

(3.4) Artin Character and Artin Exponent of SL(2,7):

This group has 8 cyclic subgroups which are:

H_1 , H_z , H_c , H_{zc} , H_a , H_{a^2} , H_b and H_{b^2} .

The rational valued character table of SL(2,7) is given in table (3.4.1), [5]

Table (3.4.1)

C_g	1	z	c	zc	a	a^2	b	b^2
$\lvert C_g \rvert$	1	1	24	24	56	56	42	42
$\lvert C_G(g) \rvert$	336	336	14	14	6	6	8	8
1_G	1	1	1	1	1	1	1	1
ψ	7	7	0	0	1	1	-1	-1
χ_1	8	-8	1	-1	1	-1	0	0
χ_2	8	8	1	1	-1	-1	0	0
θ_1	12	-12	-2	2	0	0	0	0
θ_2	6	6	-1	-1	0	0	0	2
ξ'	8	-8	1	-1	-2	2	0	0
η'	6	6	-1	-1	0	0	2	-2

We calculate the Artin character (induced character) of the cyclic subgroups by using the following:-

$$\phi_H \uparrow^{SL(2,7)}(\alpha) = \frac{|C_{SL(2,7)}(x)|}{|C_H(\alpha)|} \sum_{\substack{\alpha \to x \\ H \to SL(2,7)}} \phi(\alpha)$$

and given it in table (3.4.2)

Table (3.4.2)

C_g	1	z	c	zc	a	a^2	b	b^2
Φ_1	336	0	0	0	0	0	0	0
Φ_2	168	168	0	0	0	0	0	0
Φ_3	48	0	2	0	0	0	0	0
Φ_4	24	72	3	3	0	0	0	0
Φ_5	56	112	0	0	2	0	0	0
Φ_6	112	0	0	0	0	2	0	-0
Φ_7	42	84	0	0	0	0	2	0
Φ_8	84	168	0	0	0	0	0	4

From tables (3.4.1) and (3.4.2) we write the rational valued character of SL(2,7) as a linear combination of induced characters, and count the least positive integer A(SL(2,7)) (Artin exponent) such that A(SL(2,7))χ_i is an integral linear combination of Φ_i, i=1,…,8 whereχ_i is any rational valued character.

$$1=\frac{1}{4}\Phi_8+\frac{1}{2}\Phi_7+\frac{1}{2}\Phi_6+\frac{1}{2}\Phi_5+\frac{1}{3}\Phi_4-\frac{121}{168}\Phi_2+\frac{37}{336}\Phi_1$$

$$\psi=\frac{1}{4}\Phi_8-\frac{1}{2}\Phi_7+\frac{1}{2}\Phi_6+\frac{1}{2}\Phi_5+\frac{5}{24}\Phi_2-\frac{10}{24}\Phi_1$$

$$\chi_1=-\frac{1}{2}\Phi_6+\frac{1}{2}\Phi_5-\frac{1}{3}\Phi_4+\Phi_3-\frac{5}{21}\Phi_2+\frac{3}{28}\Phi_1$$

$$\chi_2=-\frac{1}{2}\Phi_6-\frac{1}{2}\Phi_5+\frac{1}{3}\Phi_4-\frac{3}{7}\Phi_2+\frac{1}{28}\Phi_1$$

$$\theta_2=\frac{2}{3}\Phi_4-2\Phi_3-\frac{25}{84}\Phi_2+\frac{71}{168}\Phi_1$$

$$\theta_2=\frac{1}{2}\Phi_8-\frac{1}{3}\Phi_4-\frac{9}{28}\Phi_2+\frac{19}{168}\Phi_1$$

$$\zeta'=\Phi_6-\Phi_5-\frac{1}{3}\Phi_4+\Phi_3+\frac{16}{21}\Phi_2-\frac{9}{14}\Phi_1$$

$$\eta'=-\frac{1}{2}\Phi_8+\Phi_7-\frac{1}{3}\Phi_4+\frac{3}{14}\Phi_2-\frac{1}{12}\Phi_1$$

Note that $336 \cdot \chi_i = (Z)\,\Phi_i$, i=1,...,8. So A(SL(2,7)) = 336.

(3.5) Artin Character and Artin Exponent of SL(2,11):

This group has 10 cyclic subgroups which are:

$H_1, H_z, H_c, H_{zc}, H_a, H_{a^2}, H_b, H_{b^2}, H_{b^3}$ and H_{b^4}.

The rational valued character table of SL(2,11) is given in the table (3.5.1), [5]

Table (3.5.1)

C_g	1	z	c	zc	a	a^2	b	b^2	b^3	b^4
$\lvert C_g \rvert$	1	1	60	60	132	132	110	110	110	110
$\lvert C_G(g) \rvert$	1320	1320	22	22	10	10	12	12	12	12
1_G	1	1	1	1	1	1	1	1	1	1
ψ	11	11	0	0	1	1	-1	-1	-1	-1
χ_1	24	-24	2	-2	1	-1	0	0	0	0
χ_2	24	24	2	2	-1	-1	0	0	0	0
θ_1	20	-20	-2	2	0	0	0	-2	0	2
θ_2	10	10	-1	-1	0	0	-1	1	2	1
θ_3	10	-10	-1	1	0	0	0	2	0	-2
θ_4	10	10	-1	-1	0	0	1	1	-2	1
ξ'	12	-12	1	-1	-2	2	0	0	0	0
η'	10	10	-1	-1	0	0	2	-2	2	-2

We calculate the Artin character (induced character) of the cyclic subgroups by using the following:-

$$\theta_4 = \frac{1}{4}\Phi_{10} - \frac{1}{3}\Phi_9 + \frac{1}{4}\Phi_8 + \frac{1}{2}\Phi_7 - \frac{1}{3}\Phi_4 + \frac{9}{22}\Phi_2 - \frac{2}{11}\Phi_1$$

$$\zeta' = \Phi_6 - \Phi_5 - \frac{1}{3}\Phi_4 + \Phi_3 + \frac{73}{165}\Phi_2 - \frac{64}{165}\Phi_1$$

$$\eta' = -\frac{1}{2}\Phi_{10} + \frac{1}{3}\Phi_9 - \frac{1}{2}\Phi_8 + \Phi_7 - \frac{1}{3}\Phi_4 - \frac{17}{66}\Phi_2 + \frac{31}{132}\Phi_1$$

Note that $1320\cdot\chi_i = (Z)\,\Phi_i$, i=1,…,10. So A(SL(2,11)) = 1320

3.6 Artin Character and Artin Exponent of SL(2,13):

This group has 10 cyclic subgroups which are:

$H_1, H_z, H_c, H_{zc}, H_a, H_{a^2}, H_{a^3}, H_{a^4}, H_b$ and H_{b^2}.

The rational valued character table of SL(2,13) is given in the table (3.6.1), [5]

Table (3.6.1)

C_g	1	z	c	zc	a	a^2	a^3	a^4	b	b^2
$\lvert C_g\rvert$	1	1	84	84	182	182	182	182	156	156
$\lvert C_G(g)\rvert$	2184	2184	26	26	12	12	12	12	14	14
1_G	1	1	1	1	1	1	1	1	1	1
ψ	13	13	0	0	1	1	1	1	-1	-1
χ_1	28	-28	2	-2	0	2	0	-2	0	0
χ_2	14	14	1	1	1	-1	-2	-1	0	0
χ_3	14	-14	1	-1	0	-2	0	2	0	0
χ_4	14	14	1	1	-1	-1	2	-1	0	0
θ_1	36	-36	-3	3	0	0	0	0	-1	1
θ_2	36	36	-3	-3	0	0	0	0	1	1
ξ'	14	14	1	1	-2	2	-2	2	0	0
η'	12	-12	-1	1	0	0	0	0	2	-2

We calculate the Artin character (induced character) of the cyclic subgroups by using the following:-

$$\phi_H \uparrow^{SL(2,13)}(\alpha) = \frac{|C_{SL(2,13)}(x)|}{|C_H(\alpha)|} \sum_{\substack{\alpha \to x \\ H \to SL(2,13)}} \phi(\alpha)$$

and given it in the table (3.6.2)

Table (3.6.2)

C_g	1	z	c	zc	a	a^2	a^3	a^4	b	b^2
Φ_1	2184	0	0	0	0	0	0	0	0	0
Φ_2	1092	1092	0	0	0	0	0	0	0	0
Φ_3	168	0	2	0	0	0	0	0	0	0
Φ_4	84	168	3	3	0	0	0	0	0	0
Φ_5	182	364	0	0	2	0	0	0	0	0
Φ_6	364	728	0	0	0	4	0	0	0	0
Φ_7	546	1092	0	0	0	0	6	0	0	0
Φ_8	728	0	0	0	0	0	0	4	0	0
Φ_9	156	312	0	0	0	0	0	0	2	0
Φ_{10}	312	0	0	0	0	0	0	0	0	2

From tables (3.6.1) and (3.6.2) we can write the rational valued character of SL(2,13) as a linear combination of induced characters, and count the least positive integer A(SL(2,13)) (Artin exponent) such that A(SL(2,13))χ_i is an integral linear combination(Z) of Φ_i, i=1,…,10 whereχ_i is any rational valued character.

$$1 = \frac{1}{2}\Phi_{10} + \frac{1}{2}\Phi_9 + \frac{1}{4}\Phi_8 + \frac{1}{6}\Phi_7 + \frac{1}{4}\Phi_6 + \frac{1}{2}\Phi_5 + \frac{1}{3}\Phi_4 - \frac{97}{156}\Phi_2 + \frac{41}{2184}\Phi_1$$

$$\psi = -\frac{1}{2}\Phi_{10} - \frac{1}{2}\Phi_9 + \frac{1}{4}\Phi_8 + \frac{1}{6}\Phi_7 + \frac{1}{4}\Phi_6 + \frac{1}{2}\Phi_5 - \frac{29}{84}\Phi_2 + \frac{13}{168}\Phi_1$$

$$\chi_1 = -\frac{1}{2}\Phi_8 + \frac{1}{2}\Phi_6 - \frac{2}{3}\Phi_4 + 2\Phi_3 - \frac{2}{39}\Phi_2 - \frac{37}{156}\Phi_1$$

$$\chi_2 = -\frac{1}{4}\Phi_8 - \frac{1}{3}\Phi_7 - \frac{1}{4}\Phi_6 + \frac{1}{2}\Phi_5 + \frac{1}{3}\Phi_4 + \frac{23}{165}\Phi_2 + \frac{1}{78}\Phi_1$$

$$\chi_3 = \frac{1}{2}\Phi_8 - \frac{1}{2}\Phi_6 - \frac{1}{3}\Phi_4 + \Phi_3 + \frac{29}{78}\Phi_2 - \frac{51}{156}\Phi_1$$

$$\chi_4 = -\frac{1}{4}\Phi_8 + \frac{1}{3}\Phi_7 - \frac{1}{4}\Phi_6 - \frac{1}{2}\Phi_5 + \frac{1}{3}\Phi_4 - \frac{1}{26}\Phi_2 + \frac{5}{52}\Phi_1$$

$$\theta_1 = \frac{1}{2}\Phi_{10} - \frac{1}{2}\Phi_9 + \Phi_4 - 3\Phi_3 - \frac{4}{91}\Phi_2 + \frac{71}{364}\Phi_1$$

$$\theta_2 = \frac{1}{2}\Phi_{10} + \frac{1}{2}\Phi_9 - \Phi_4 + \frac{4}{91}\Phi_2 - \frac{19}{364}\Phi_1$$

$$\zeta' = \frac{1}{2}\Phi_8 - \frac{1}{3}\Phi_7 + \frac{1}{2}\Phi_6 - \Phi_5 + \frac{1}{3}\Phi_4 + \frac{23}{78}\Phi_2 - \frac{37}{156}\Phi_1$$

$$\eta' = -\Phi_{10} + \Phi_9 + \frac{1}{3}\Phi_4 - \Phi_3 - \frac{95}{273}\Phi_2 + \frac{86}{273}\Phi_1$$

Note that $2184\cdot\chi_i = (Z)\ \Phi_i$, i=1,…,10. So A(SL(2,13)) = 2184.

***(3.7) Artin Character and Artin Exponent of SL(2,17)*:**

This group has 11 cyclic subgroups which are:
$H_1, H_z, H_c, H_{zc}, H_a, H_{a^2}, H_{a^4}, H_b, H_{b^2}, H_{b^3}$, and H_{b^4}.

The rational valued character table of SL(2,17) is given in table (3.7.1), [5]

Table (3.7.1)

C_g	1	z	c	zc	a	a^2	a^4	b	b^2	b^3	b^4
$\lvert C_g \rvert$	1	1	144	144	306	306	306	272	272	272	272
$\lvert C_G(g) \rvert$	4896	4896	34	34	16	16	16	18	18	18	18
1_G	1	1	1	1	1	1	1	1	1	1	1
ψ	17	17	0	0	1	1	1	-1	-1	-1	-1
χ_1	72	-72	4	-4	0	0	0	0	0	0	0
χ_2	36	36	2	2	0	0	-4	0	0	0	0
χ_4	18	18	1	1	0	-2	2	0	0	0	0
θ_1	48	-48	-3	3	0	0	0	0	0	-3	3
θ_2	48	48	-3	-3	0	0	0	0	0	3	3
θ_3	16	-16	-1	1	0	0	0	-1	1	2	-2
θ_6	16	16	-1	-1	0	0	0	1	1	-2	-2
ξ'	18	18	1	1	-2	2	2	0	0	0	0
η'	16	-16	-1	1	0	0	0	2	-2	2	-2

We calculate the Artin character (induced character) of the cyclic subgroups by using the folloeing:-

$$\phi_H \uparrow^{SL(2,17)}(\alpha) = \frac{|C_{SL(2,17)}(x)|}{|C_H(\alpha)|} \sum_{\substack{\alpha \to x \\ H \to SL(2,17)}} \phi(\alpha)$$

and given it in table (3.7.2)

Table (3.7.2)

C_g	1	z	c	zc	a	a^2	a^4	b	b^2	b^3	b^4
Φ_1	4896	0	0	0	0	0	0	0	0	0	0
Φ_2	2448	2448	0	0	0	0	0	0	0	0	0
Φ_3	288	0	2	0	0	0	0	0	0	0	0
Φ_4	144	432	3	3	0	0	0	0	0	0	0
Φ_5	306	612	0	0	2	0	0	0	0	0	0
Φ_6	612	1224	0	0	0	4	0	0	0	0	0
Φ_7	1224	2448	0	0	0	0	8	0	0	0	0
Φ_8	272	544	0	0	0	0	0	2	0	0	0
Φ_9	544	0	0	0	0	0	0	0	2	0	0
Φ_{10}	816	1632	0	0	0	0	0	0	0	6	0
Φ_{11}	544	0	0	0	0	0	0	0	0	0	2

From tables (3.7.1) and (3.7.2) we can write the rational valued character of SL(2,17) as a linear combination of induced characters, and count the least positive integer A(SL(2,17)) (Artin exponent) such that A(SL(2,17))χ_i is an integral linear combination of Φ_i, i=1,…,11 whereχ_i is any rational valued character.

$$1 = \frac{1}{2}\Phi_{11} + \frac{1}{6}\Phi_{10} + \frac{1}{2}\Phi_9 + \frac{1}{2}\Phi_8 + \frac{1}{8}\Phi_7 + \frac{1}{4}\Phi_6 + \frac{1}{2}\Phi_5 + \frac{1}{3}\Phi_4 - \frac{1061}{2448}\Phi_2 - \frac{701}{4896}\Phi_1$$

$$\psi = -\frac{1}{2}\Phi_{11} - \frac{1}{6}\Phi_{10} - \frac{1}{2}\Phi_9 - \frac{1}{2}\Phi_8 + \frac{1}{8}\Phi_7 + \frac{1}{4}\Phi_6 + \frac{1}{2}\Phi_5 - \frac{901}{2448}\Phi_2 + \frac{425}{1632}\Phi_1$$

$$\chi_1 = -\frac{4}{3}\Phi_4 + 4\Phi_3 + \frac{7}{34}\Phi_2 - \frac{29}{102}\Phi_1$$

$$\chi_2 = -\frac{1}{2}\Phi_7 + \frac{2}{3}\Phi_4 + \frac{27}{68}\Phi_2 - \frac{35}{408}\Phi_1$$

$$\chi_4 = \Phi_7 - \frac{1}{2}\Phi_6 + \frac{1}{3}\Phi_4 - \frac{7}{272}\Phi_2 + \frac{1}{51}\Phi_1$$

$$\theta_1 = \frac{2}{3}\Phi_{11} - \frac{1}{2}\Phi_{10} + \Phi_4 - 3\Phi_3 + \frac{7}{51}\Phi_2 + \frac{1}{204}\Phi_1$$

$$\theta_2 = \frac{3}{2}\Phi_{11} + \frac{1}{2}\Phi_{10} - \Phi_4 - \frac{7}{51}\Phi_2 - \frac{29}{204}\Phi_1$$

$$\theta_3 = -\Phi_{11} + \frac{1}{3}\Phi_{10} + \frac{1}{2}\Phi_9 - \frac{1}{2}\Phi_8 + \frac{1}{3}\Phi_4 - \Phi_3 - \frac{59}{153}\Phi_2 + \frac{133}{612}\Phi_1$$

$$\theta_6 = -\Phi_{11} - \frac{1}{3}\Phi_{10} + \frac{1}{2}\Phi_9 + \frac{1}{2}\Phi_8 - \frac{1}{3}\Phi_4 + \frac{9}{51}\Phi_2 + \frac{5}{612}\Phi_1$$

$$\zeta' = \frac{1}{4}\Phi_7 + \frac{1}{2}\Phi_6 - \Phi_5 + \frac{1}{3}\Phi_4 - \frac{41}{136}\Phi_2 + \frac{67}{816}\Phi_1$$

$$\eta' = -\Phi_{11} + \frac{1}{3}\Phi_{10} - \Phi_9 + \Phi_8 + \frac{1}{3}\Phi_4 - \Phi_3 - \frac{26}{51}\Phi_2 + \frac{64}{153}\Phi_1$$

Note that $4896\cdot\chi_i = (Z)\ \Phi_i$, i=1,…,11. So A(SL(2,17)) = 2184.

(3.8) Artin Character and Artin Exponent of SL(2,19):

This group has 11 cyclic subgroups which are:

$H_1, H_z, H_c, H_{zc}, H_a, H_{a^2}, H_{a^3}, H_{a^6}, H_b, H_{b^2}$ and H_{b^4}.

The rational valued character table of SL(2,19) is given in table (3.8.1), [5]

Table (3.8.1)

C_g	1	z	c	zc	a	a^2	a^3	a^6	b	b^2	b^4
$\lvert C_g \rvert$	1	1	180	180	380	380	380	380	342	342	342
$\lvert C_G(g) \rvert$	6840	6840	38	38	18	18	18	18	20	20	20
1_G	1	1	1	1	1	1	1	1	1	1	1
ψ	19	19	0	0	1	1	1	1	-1	-1	-1
χ_1	60	-60	3	-3	0	0	3	-3	0	0	0
χ_2	60	60	3	3	0	0	-3	-3	0	0	0
χ_3	20	-20	1	-1	1	-1	-2	2	0	0	0
χ_6	20	20	1	1	-1	-1	2	2	0	0	0
θ_1	72	-72	-4	4	0	0	0	0	0	-2	2
θ_2	72	72	-4	-4	0	0	0	0	0	2	2
θ_4	18	-18	-1	1	0	0	0	0	0	2	-2
ξ'	20	-20	1	-1	-2	2	-2	2	0	0	0
η'	18	18	-1	-1	0	0	0	0	2	-2	-2

We calculate the Artin character (induced character) of the cyclic subgroups by using the following:-

$$\phi_H \uparrow^{SL(2,19)}(\alpha) = \frac{\left|C_{SL(2,19)}(x)\right|}{\left|C_H(\alpha)\right|} \sum_{\substack{\alpha \to x \\ H \to SL(2,19)}} \phi(\alpha)$$

and given it in table (3.8.2)

Table (3.8.2)

C_g	1	z	c	zc	a	a^2	a^3	a^6	b	b^2	b^4
Φ_1	6840	0	0	0	0	0	0	0	0	0	0
Φ_2	3420	3420	0	0	0	0	0	0	0	0	0
Φ_3	360	0	2	0	0	0	0	0	0	0	0
Φ_4	180	540	3	3	0	0	0	0	0	0	0
Φ_5	342	684	0	0	2	0	0	0	0	0	0
Φ_6	760	0	0	0	0	2	0	0	0	0	0
Φ_7	1140	2280	0	0	0	0	6	0	0	0	0
Φ_8	2280	0	0	0	0	0	0	6	0	0	0
Φ_9	342	684	0	0	0	0	0	0	2	0	0
Φ_{10}	684	1368	0	0	0	0	0	0	0	4	0
Φ_{11}	342	684	0	0	0	0	0	0	0	0	2

From tables (3.8.1) and (3.8.2) we can write the rational valued character of SL(2,19) as a linear combination of induced characters, and count the least positive integer A(SL(2,19)) (Artin exponent) such that A(SL(2,19))χ_i is an integral linear combination of Φ_i, i=1,…,11 whereχ_i is any rational valued character.

$$1 = \frac{1}{2}\Phi_{11} + \frac{1}{4}\Phi_{10} + \frac{1}{2}\Phi_9 + \frac{1}{6}\Phi_8 + \frac{1}{6}\Phi_7 + \frac{1}{2}\Phi_6 + \frac{1}{2}\Phi_5 + \frac{1}{3}\Phi_4 - \frac{1927}{3420}\Phi_2 + \frac{13}{380}\Phi_1$$

$$\psi = -\frac{1}{2}\Phi_{11} - \frac{1}{4}\Phi_{10} - \frac{1}{2}\Phi_9 + \frac{1}{6}\Phi_8 + \frac{1}{6}\Phi_7 + \frac{1}{2}\Phi_6 + \frac{1}{2}\Phi_5 + \frac{17}{180}\Phi_2 - \frac{38}{285}\Phi_1$$

$$\chi_1 = -\frac{1}{2}\Phi_8 + \frac{1}{2}\Phi_7 - \Phi_4 + 3\Phi_3 - \frac{11}{57}\Phi_2 - \frac{13}{2280}\Phi_1$$

$$\chi_2 = \frac{-1}{2}\Phi_8 - \frac{1}{2}\Phi_7 + \Phi_4 + \frac{11}{57}\Phi_2 + \frac{31}{228}\Phi_1$$

$$\chi_3 = \frac{1}{3}\Phi_8 - \frac{1}{3}\Phi_7 - \frac{1}{2}\Phi_6 + \frac{1}{2}\Phi_5 - \frac{1}{3}\Phi_4 + \Phi_3 + \frac{289}{1710}\Phi_2 - \frac{451}{6840}\Phi_1$$

$$\chi_6 = \frac{1}{3}\Phi_8 + \frac{1}{3}\Phi_7 - \frac{1}{2}\Phi_6 - \frac{1}{2}\Phi_5 + \frac{1}{3}\Phi_4 - \frac{289}{1710}\Phi_2 - \frac{51}{6840}\Phi_1$$

$$\theta_1 = \Phi_{11} - \frac{1}{2}\Phi_{10} + \frac{4}{3}\Phi_4 - 4\Phi_3 - \frac{22}{95}\Phi_2 + \frac{258}{855}\Phi_1$$

$$\theta_2 = \Phi_{11} + \frac{1}{2}\Phi_{10} - \frac{4}{3}\Phi_4 - \frac{16}{95}\Phi_2 + \frac{51}{1710}\Phi_1$$

$$\theta_5 = -\Phi_{11} + \frac{1}{2}\Phi_{10} + \frac{1}{3}\Phi_4 - \Phi_3 - \frac{11}{190}\Phi_2 + \frac{43}{570}\Phi_1$$

$\zeta' = \frac{1}{3}\Phi_8 - \frac{1}{3}\Phi_7 + \Phi_6 - \Phi_5 - \frac{1}{3}\Phi_4 + \Phi_3 - \frac{401}{855}\Phi_2 - \frac{1341}{3420}\Phi_1$ Note that $6840 \cdot \chi_i \in$ (Z) Φ_i, i=1,…,11. So A(SL(2,19)) = 6840.

§.4 Conclusions

From this work we deduce the following results:

1- For every $p \leq 19$, $A(SL(2,p)) = p(p^2 - 1)$ = the order of SL(2,p).

2- For every prime number p

i- The order of cyclic subgroup $H_I = |SL(2,p)| = p(p^2 - 1)$.
ii- The order of cyclic subgroup $H_z = |SL(2,p)| / 2 = p(p^2 - 1) / 2$.
iii-The order of cyclic subgroup $H_c = |SL(2,p)| / p = (p^2 - 1)$.
iv-The order of cyclic subgroup $H_{zc} = |SL(2,p)| / 2p = (p^2 - 1) / 2$.
v- The order of cyclic subgroup $H_b = |SL(2,p)| / p(p - 1) = (p + 1)$.

3- For a prime p > 3

i- The order of cyclic subgroup $H_a = |SL(2,p)| / p(p+1) = (p - 1)$.
ii- The order of cyclic subgroup $H_{b^2} = |SL(2,p)| / 2p(p - 1) = (p + 1) / 2$.

4- For a prime p > 5 , the order of cyclic subgroup
$H_{a^2} = |SL(2,p)| / 2p(p + 1) = (p - 1)/2$.

5- For a prime p =11 and 17 the order of cyclic subgroup
$H_{b^3} = |SL(2,p)| / 3p(p-1) = (p + 1)/3$.

6- For a prime p =13 and 19 the order of cyclic subgroup
$H_{a^3} = |SL(2,p)| / 3p(p+1) = (p - 1)/3$.

Since there exist different orders of the cyclic subgroups of SL(2,p) we can not find general form of induced character table of SL(2,p) except that some orders of cyclic subgroups we given above .

References

1- E.Artin, Die Gruppentheoretisele Strukturder Diskriminaraten Algebraischer Zahlkörper, J.Reine Anger. Math., 1961, 164, pp.1-11.

2- H.Behravesh, The Rational Character Table of Special Linear Groups, J.Sc.I.R.Iran, 1998, 9(2), Spring, pp.173-180.

3- K.E. Gehles, Ordinary Characters of First Special Linear Groups, M.SC. Dissertation, Univ. of St. Andrews, August 2002.

4- I.M.Jsaacs, Characters Theory of Finite Groups, Academic Press, New York, 1976.

5- N.S.Jasim, Results of the Factor Group CF(C,Z)/R(G), M.Sc. Thesis, Univ. of Technology, 2005.

6- T.Y.Lam, Artin Exponent of Finite Groups, J. Algebra, New York, 1968, 9, pp.94-119.

7- L.E.Sigler, Algebra, Springer-Verlage, Berlin, 1976 .